SECTION – A

MULTIPLE CHOICE QUESTION (MCQ)

Q.1 – Q.10 carry one mark each.

1. Let $\{a_n\}_{n\geq 1}$ be a sequence of real numbers such that $a_1 = 2$ and, for $n \geq 1$,

$$a_{n+1} = \frac{2\,a_n + 1}{a_n + 1}$$

Then

(A) $1.5 \leq a_n \leq 2$, for all natural number $n \geq 1$

(B) there exist a natural number $n \geq 1$ such that $a_n > 2$

(C) there exist a natural number $n \geq 1$ such that $a_n < 1.5$

(D) there exist a natural number $n \geq 1$ such that $a_n > \frac{1+\sqrt{5}}{2}$

2. The value of

$$\lim_{n\to\infty}\left(1+\frac{2}{n}\right)^{n^2} e^{-2n}$$

Is

(A) e^{-2}

(B) e^{-1}

(C) e

(D) e^{2}

3. Let $\{a_n\}_{n\geq 1}$ and $\{b_n\}_{n\geq 1}$ be two convergent sequence of real numbers. For $n \geq 1$, define $u_n = \max\{a_n, b_n\}$ and $v_n = \min\{a_n, b_n\}$. Then

(A) neither $\{u_n\}_{n\geq 1}$ nor $\{v_n\}_{n\geq 1}$ converges

(B) $\{u_n\}_{n\geq 1}$ converges but $\{v_n\}_{n\geq 1}$ does not converge

(C) $\{u_n\}_{n\geq 1}$ does not converge but $\{v_n\}_{n\geq 1}$ converges

(D) both $\{u_n\}_{n\geq 1}$ and $\{v_n\}_{n\geq 1}$ converge

4. Let $M = \begin{bmatrix} \frac{1}{4} & \frac{3}{4} \\ \frac{3}{5} & \frac{2}{5} \end{bmatrix}$. If I is the 2 x 2 identity matrix and 0 is the 2 x 2 zero matrix, then

(A) $20M^2 - 13M + 7I = 0$

(B) $20M^2 - 13M - 7I = 0$

(C) $20M^2 + 13M + 7I = 0$

(D) $20M^2 + 13M - 7I = 0$

5. Let X be a random variable with the probability density function

$$f(x)=\begin{cases}\frac{a^p}{\Gamma(p)}e^{-ax}x^{p-1} & x\geq 0, a>0, p>0\\ 0, & otherwise.\end{cases}$$

If $E(X)=20$ and $Var\ (X)=10$, then (a,p) is

(A) (2, 20)

(B) (2, 40)

(C) (4, 20)

(D) (4, 40)

6. Let X be a random variable with the distribution function

$$F(x)=\begin{cases}0, & x<0,\\ \frac{1}{4}+\frac{4x-x^2}{8} & 0\leq x\leq 2,\\ 1, & x\leq 2.\end{cases}$$

Then $P(X=0)+P(X=1.5)+P(X=2)+P(X\geq 1)$

Equals

(A) $\frac{3}{8}$

(B) $\frac{5}{8}$

(C) $\frac{7}{8}$

(D) 1

7. Let X_1, X_2 and X_3 be i.i.d. $U(0,1)$ random variables. Then $E\left(\frac{X_1+X_2}{X_1+X_2+X_3}\right)$ equals

(A) $\frac{1}{3}$

(B) $\frac{1}{2}$

(C) $\frac{2}{3}$

(D) $\frac{3}{4}$

8. Let $x_1=0, x_2=1, x_3=2, x_4=3$ and $x_5=0$ be the observed value of a random sample of size 5 from a discrete distribution with the probability mass function

$$f(x:\theta)=P(X=x)=\begin{cases}\frac{\theta}{3}, & x=0,\\ \frac{2\theta}{3}, & x=1,\\ \frac{1-\theta}{2}, & x=2,3,\end{cases}$$

Where $\theta\in[0,1]$ is the unknown parameter. Then the maximum likelihood estimate of θ is

(A) $\frac{2}{5}$

(B) $\frac{3}{5}$

(C) $\frac{5}{7}$

(D) $\frac{5}{9}$

9. **Consider four coins labelled as 1, 2, 3 and 4. Suppose that the probability of obtaining a 'head' in a single toss of the i^{th} coins is $\frac{i}{4}, i = 1, 2, 3, 4$. A coin is chosen uniformly at random and flipped. Given that the flip resulted in a 'head', the conditional probability that the coin was labelled either 1 or 2 equals**

(A) $\frac{1}{10}$

(B) $\frac{2}{10}$

(C) $\frac{3}{10}$

(D) $\frac{4}{10}$

10. **Consider the linear regression model $y_i = \beta_0 + \beta_1 x_i + \epsilon_i; i = 1, 2, \dots, n,$ where $\epsilon_i's$ are i.i.d. standard normal variables. Given that**

$\frac{1}{n}\sum_{i=1}^{n} x_i = 3.2$, $\frac{1}{n}\sum_{i=1}^{n} y_i = 4.2$, $\frac{1}{n}\sum_{i=1}^{n}\left(x_j - \frac{1}{n}\sum_{i=1}^{n} x_i\right)^2 = 1.5$ and $\frac{1}{n}\sum_{i=1}^{n}\left(x_j - \frac{1}{n}\sum_{i=1}^{n} x_i\right)\left(y_j - \frac{1}{n}\sum_{i=1}^{n} y_i\right)$ = 1.7,

The maximum likelihood estimates of β_0 and β_1, respectively are

(A) $\frac{17}{15} and \frac{32}{75}$

(B) $\frac{32}{75} and \frac{17}{15}$

(C) $\frac{17}{15} and \frac{43}{75}$

(D) $\frac{43}{75} and \frac{17}{15}$

Q.11 – Q.30 carry two marks each.

11. **Let $f: [-1, 1] \to \mathbb{R}$ be defined by $f(x) = \frac{x^2 + [sin\pi x]}{1+|x|}$, where [y] denotes the greatest integer has than or equal to y . Then**

(A) f is continuous at $-\frac{1}{2}, 0, 1$

(B) f is discontinuous at $-1, 0, \frac{1}{2}$

(C) f is discontinuous at $-1, -\frac{1}{2}, 0, \frac{1}{2}$

(D) f is continuous everywhere except at 0

12. **Let $f, g: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2 - \frac{\cos x}{2}$ and $g(x) = \frac{x \sin x}{2}$. Then**

(A) $f(x) = g(x)$ for more than two values of x

(B) $f(x) \neq g(x)$ for all x in $\mathbb{R}$

(C) $f(x) = g(x)$ for exactly one values of x

(D) $f(x) = g(x)$ for exactly two values of x

13. **Consider the domain $D = \{(x, y) \in \mathbb{R}^2 : x \leq y\}$ and the function $h: D \to \mathbb{R}$ defined by $h((x, y)) = (x-2)^4 + (y-1)^4, (x, y) \in D$.**

Then the minimum value of h on D equals

(A) $\frac{1}{2}$

(B) $\frac{1}{4}$

(C) $\frac{1}{8}$

(D) $\frac{1}{16}$

14. **Let $M = [X \;\; Y \;\; Z]$ be an orthogonal matrix with $X, Y, Z \in \mathbb{R}^3$ as its column vectors. Then $Q = XX^T + YY^T$**

(A) is a skew –symmetric matrix

(B) is the 3 x 3 identity matrix

(C) satisfies $Q^2 = Q$

(D) satisfies $QZ = Z$

15. **Let $f: [0, 3] \to \mathbb{R}$ be defined by**

$$f(x) = \begin{cases} 0, & 0 \le x < 1 \\ e^{x^2} - e & 1 \le x < 2 \\ e^{x^2} + 1 & 2 \le x \le 3 \end{cases}$$

Now, define $F: [0, 3] \to \mathbb{R}$ by

$$F(0) = 0 \text{ and } F(x) = \int_0^x f(t)\,dt, \text{for } 0 < x \le 3$$

Then

(A) F is differentiable at $x = 1$ and $F'(1) = 0$

(B) F is differentiable at $x = 2$ and $F'(2) = 0$

(C) F is not differentiable at $x = 1$

(D) F is differentiable at $x = 2$ and $F'(2) = 1$

16. **If x, y and z are real numbers such that $4x + 2y + z = 31$ and $2x + 4y - z = 19$, then the value of $9x + 7y + z$**

(A) cannot be computed from the given information

(B) equals $\frac{281}{3}$

(C) equals $\frac{182}{3}$

(D) equals $\frac{218}{3}$

17. **Let $M = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & -1 \end{bmatrix}$. If**

$$V = \left\{(x, y, 0) \in \mathbb{R}^3 : M\begin{bmatrix} x \\ y \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ and } W = \left\{(x, y, z) \in \mathbb{R}^3 : M\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}\right\},$$

Then

(A) the dimension of V equals 2

(B) the dimension of W equals 2

(C) the dimension of V equals 1

(D) $V \cap W = \{(0, 0, 0)\}$

18. **Let M be a 3 x 3 non-zero, skew- symmetric real matrix. If I is the 3 x 3 identity matrix, then**

(A) M is invertible

(B) the matrix $I + M$ is invertible

(C) there exist a non-zero real numbers a such that $al + m$ is not invertible

(D) all the eigenvalues of M are real

19. Let X be a random variable with the moment generating function

$$M_X(t) = \frac{6}{\pi^2}\sum_{n\geq 1}\frac{e^{t^2/2n}}{n^2}, t \in \mathbb{R}$$

Then $P(X \in \mathbb{Q})$, where $\mathbb{Q}$ is the set of rational numbers, equals

(A) 0

(B) $\frac{1}{4}$

(C) $\frac{1}{2}$

(D) $\frac{3}{4}$

20. Let X be a discrete random variables with the moment generating function

$$M_X(t) = \frac{(1+3e^t)^2(3+e^t)^3}{1024}, t \in \mathbb{R}$$

Then

(A) $E(X) = \frac{9}{4}$

(B) $P(X \geq 1) = \frac{27}{1024}$

(C) $Var(X) = \frac{15}{32}$

(D) $P(X = 5) = \frac{3}{1024}$

21. Let $\{X_n\}_{n\geq 1}$ be a sequence of independent random variables with X_n having the probability density function as

$$f(x) = \begin{cases} \frac{1}{2^{n/2}\Gamma\left(\frac{n}{2}\right)} & x > 0, \\ 0, & otherwise \end{cases}$$

Then $\lim_{n\to\infty}\left[P\left(X_n > \frac{3}{4}n\right) + P(X_n > n + 2\sqrt{2n})\right]$

Equals

(A) $1 + \Phi(2)$

(B) $1 - \Phi(2)$

(C) $\Phi(2)$

(D) $2 - \Phi(2)$

22. Let X be a poisson random variables with mean $\frac{1}{2}$. Then $E((X+1)!)$ equals

(A) $2e^{-\frac{1}{2}}$

(B) $4e^{-\frac{1}{2}}$

(C) $4e^{-1}$

(D) $2e^{-1}$

23. Let X be a standard normal random variable. Then $P(X^3 - 2X^2 - X + 2 > 0)$ equals

(A) $2\Phi(1) - 1$

(B) $1 - \Phi(2)$

(C) $2\Phi(1) - \Phi(2)$

(D) $\Phi(2) - \Phi(1)$

24. Let X and Y have joint probability density function

$$f(x,y)=\begin{cases}2, & 0\le x\le y\le 1,\\ 0, & otherwise.\end{cases}$$

Let $a=E(Y|X=\frac{1}{2})$ and $b=Var(Y|X=\frac{1}{2})$. Then (a,b) is

(A) $\left(\frac{3}{4},\frac{7}{12}\right)$

(B) $\left(\frac{1}{4},\frac{1}{48}\right)$

(C) $\left(\frac{1}{4},\frac{7}{12}\right)$

(D) $\left(\frac{3}{4},\frac{1}{48}\right)$

25. Let X and Y have the joint probability mass function

$$P(X=m,Y=n)=\begin{cases}\frac{m+n}{21}, & m\ 1,2,3;n=1,2,\\ 0, & otherwise.\end{cases}$$

Then $P(X=2|Y=2)$ equals

(A) $\frac{1}{3}$

(B) $\frac{2}{3}$

(C) $\frac{1}{2}$

(D) $\frac{1}{4}$

26. Let X and Y be two independent standard normal random variables. Then the probability density function of $Z=\frac{|X|}{|Y|}$ is

(A) $f(z)=\begin{cases}\frac{\sqrt{\frac{1}{2}}}{\sqrt{\pi}}e^{\frac{z}{2}}z^{\frac{1}{2}} & z>0\\ 0, & otherwise\end{cases}$

(B) $f(z)=\begin{cases}\frac{2}{\sqrt{2\pi}}e^{-z^2/2} & z>0\\ 0, & otherwise\end{cases}$

(C) $f(z)=\begin{cases}e^{-z} & z>0,\\ 0, & otherwise\end{cases}$

(D) $f(z)=\begin{cases}\frac{2}{\pi}\frac{1}{(1+z^2} & z>0,\\ 0, & otherwise\end{cases}$

27. Let X and Y have the joint probability density function

$$f(x,y)=\begin{cases}e^{-y} & 0<x<y<\infty\\ 0, & otherwise.\end{cases}$$

Then the correlation coefficient between X and Y equals

(A) $\frac{1}{3}$

(B) $\frac{1}{\sqrt{3}}$

(C) $\frac{1}{\sqrt{2}}$

(D) $\frac{2}{\sqrt{3}}$

28. Let $x_1 = 2, x_2 = 1$ *and* $x_3 = -1$ be the observed value of a random sample of size three from a discrete distribution with the probability mass function

$$f(x;\theta) = P(X = x) = \begin{cases} \frac{1}{2\theta+1} & x \in \{-\theta, -\theta+1, \dots 0, .., \theta\} \\ 0, & otherwise, \end{cases}$$

Where $\theta \in \circledast = \{1, 2, \dots..\}$ is the unknown parameter. Then the method of moment estimate of $\theta$is

(A) 1

(B) 2

(C) 3

(D) 4

29. Let X be a random sample from a discrete distribution with the probability mass function

$$f(x;\theta) = P(X = x) = \begin{cases} \frac{1}{\theta} & x = 1, 2, \dots, \theta, \\ 0, & otherwise, \end{cases}$$

Where $\theta \in \circledast = \{20, 40\}$ is the unknown parameter. Consider testing

$H_0: \theta = 40$ against $H_0: \theta = 20$

At a level of significance $a = 0.1$. Then the uniformly most powerful test rejects H_0 if and only if

(A) $X \leq 4$

(B) $X > 4$

(C) $X \geq 3$

(D) $X < 3$

30. Let X_1 and X_2 be a random sample of size 2 from a discrete distribution with the probability mass function

$f(x;\theta) = P(X = x) = \begin{cases} \theta, & x = 0, \\ 1-\theta & x = 1, \end{cases}$

Where $\theta \in \circledast = \{0.2, 0.4\}$ is the known parameters. For testing $H_0: \theta = 0.2$ against $H_1: \theta = 0.4$, consider a test with the critical region

$C = \{(x_1, x_2) \in \{0, 1\}$ x $\{0, 1\} : x_1 + x_2 < 2\}$.

Let a *and* β denote the probability of Type I error and power of the test, respectively

Then (a, β) is

(A) (0.36, 0.74)

(B) (0.64, 0.36)

(C) (0.05, 0.64)

(D)(0.36, 0.64)

SECTION- B
MULTIPLE SELECT QUESTION (MSQ)

Q.31 – Q.40 carry two marks each

31. Let $\{a_n\}_{n\geq 1}$ be a sequence of real numbers such that

$a_n = \sum_{k=n+1}^{2n} \frac{1}{k}$, $n \geq 1$.

Then which of the following statements (s) is (are) true?

(A) $\{a_n\}_{n\geq 1}$ is an increasing sequence

(B) $\{a_n\}_{n\geq 1}$ is bounded below

(C) $\{a_n\}_{n\geq 1}$ is bounded above

(D) $\{a_n\}_{n\geq 1}$ is a convergent sequence

32. Let $\sum_{n\geq 1}(a_n)^2$ be a convergent series of positive real numbers. Then which of the following statements id (are) true?

(A) $\sum_{n\geq 1}(a_n)^2$ is always convergent

(B) $\sum_{n\geq 1}\sqrt{a_n}$ is always convergent

(C) $\sum_{n\geq 1}\frac{\sqrt{a_n}}{n}$ is always convergent

(D) $\sum_{n\geq 1}\frac{\sqrt{a_n}}{n^{1/4}}$ is always convergent

33. Let $\{a_n\}_{n\geq 1}$ be a sequence of real numbers such that $a_1 = 3$ and for $n \geq 1$,

$$a_{n+1} = \frac{a_n^2 - 2a_n + 4}{2}$$

Then which of the following statements (s) is (are) true?

(A) $\{a_n\}_{n\geq 1}$ is a monotone sequence

(B) $\{a_n\}_{n\geq 1}$ is a bounded sequence

(C) $\{a_n\}_{n\geq 1}$ does not have finite limit, as $n \to \infty$

(D) $\lim_{n\to\infty} a_n = 2$

34. Let $f: \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} x^4(2 + sin\frac{1}{x}) & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Then which of the following statement (s) is (are) true?

(A) f attains its minimum at 0

(B) f is monotone

(C) f is differentiable at 0

(D) $f(x) > 2x^4 + x^3, for\ all\ x > 0$

35. Let P be a probability function that assigns the same weight to each of the points of the sample space Ω= {1, 2, 3, 4}. Consider the events $E = \{1, 2\}, F = \{1, 3\}\ and\ G = \{3, 4\}$. Then which of the following statements (s) is (are) true?

(A) E and F are independent

(B) E and G are independent

(C) E and G are independent

(D) E, F and G are independent

36. Let $X_1, , X_2, \dots, X_n, n \geq 5$, be a random sample from a distribution with the probability density function

$$f(x;\theta) = \begin{cases} e^{-(x-\theta),} & x \geq \theta \\ 0, & otherwise, \end{cases}$$

Where $\theta \in \mathbb{R}$ is the unknown parameter. Then which of the following statements (s) is (are) true?

(A) A 95% confidence interval of θ has to be of finite length

(B) $(\min\{X_1, X_2, \dots, X_n\} + \frac{1}{n}1n(0.05), \min\{X_1, X_2, \dots, X_n\}$ is a 95% confidence interval of θ

(C) A 95 % confidence interval of θ can be of length 1

(D) A 95 % confidence interval of θ can be of length 2

37. Let $\{X_1, X_2, \dots, X_n\}$ be a random sample from $U(0, \theta)$, where $\theta > 0$ is the unknown parameter. Let $X_{(n)} = max\{X_1, X_2, \dots, X_n\}$. Then which of the following is (are) consistent estimator (s) of θ^3?

(A) $8X_n^3$

(B) $X_{(n)}^3$

(C) $\left(\frac{2}{n}\sum_{i=5}^{n} X_i\right)^3$

(D) $\frac{nX_{(n)}^5+1}{n+1}$

38. Let $\{X_1, X_2, \dots, X_n\}$ be a random sample from a distribution with the probability density function

$$f(x:\theta) = \begin{cases} c(\theta)e^{-(x-\theta)} & x \geq 2\theta \\ 0, & otherwise, \end{cases}$$

Where $\theta \in \mathbb{R}$ is the unknown parameter. Then which of the following statements (s) is (are) true?

(A) The maximum likelihood estimator of θ is $\frac{\min\{X_1, X_2, \dots, X_n\}}{2}$

(B) $c(\theta) = 1$, for all $\theta \in \mathbb{R}$

(C) the maximum likelihood estimator of θ is $\min\{X_1, X_2, \dots, X_n\}$

(D) The maximum likelihood estimator of θ does not exist

39. Let $\{X_1, X_2, \dots, X_n\}$ be a random sample from a distribution with the probability density function

$$f(x;\theta) = \begin{cases} \theta^2 x e^{-\theta x} & x > 0 \\ 0, & otherwise, \end{cases}$$

Where $\theta > 0$ is the unknown parameter.

(A) Y is a complete sufficient statistic for θ

(B) $\frac{2n}{Y}$ is the uniformly minimum variance unbiased estimator of θ

(C) $\frac{2n-1}{Y}$ is the uniformly minimum variance unbiased estimator of θ

(D) $\frac{2n+1}{Y}$ is the uniformly minimum variance unbiased estimator of θ

40. Let $\{X_1, X_2, \dots, X_n\}$ be a random sample from $U(\theta, \theta + 1)$, where $\theta \in \mathbb{R}$ is the unknown parameter.

Let $U = \max\{X_1, X_2, \dots, X_n\}$ and $V = min\{X_1, X_2, \dots, X_n\}$ .Then which of the following statements (s) is (are) true?

(A) U is a consistent estimator of θ

(B) V is a consistent estimator of θ

(C) $2U - V - 2$ is a consistent estimator of θ

(D) $2V - U + 1$ is a consistent estimator of θ

SECTION – C
NUMERICAL ANSWER TYPE (NAT)

Q.41 - Q.50 carry one mark each

41. Let $\{a_n\}_{n\geq 1}$ be a sequence of real numbers such that $a_n = \frac{1+3+5+\cdots+(2n-1)}{n!}$, $n \geq 1$

Then $\sum_{n\geq 1} a_n$ converges to _____

42. Let $S = \{(x, y) \in \mathbb{R}^2 : x, y \geq 0, \sqrt{4-(x-2)^2} \leq y \leq \sqrt{9-(x-3)^2}\}$

Then the area of S equals_____

43. Let $S = \{(x, y) \in \mathbb{R}^2 : |x| + |y| \leq 1\}$. Then the area of S equals_____

44. Let

$$J = \frac{1}{n}\int_0^1 t^{-\frac{1}{2}}(1-t)^{\frac{3}{2}}\,dt$$

Then the value of J equals_____

45. A fair die is rolled three times independently. Given that 6 appeared at least once, the conditional probability that 6 appeared exactly twice equals_____

46. Let X and Y be two positive integer valued random variables with the joint probability mass function

$$P(X=m,Y=n) = \begin{cases} g(mh(n), & m,n \geq 1 \\ 0, & otherwise, \end{cases}$$

Where $g(m) = \left(\frac{1}{2}\right)^{m-1}, m \geq 1$ and $h(n) = \left(\frac{1}{3}\right)^{n}, n \geq 1$. Then $E(XY)$ equals _____

47. Let E, F and G be three events such that
$P(E \cap F \cap G) = 0.1, P(G|F) = 0.3$ and $P(E|F \cap G) = P(E|F)$
Then $P(G|E \cap F)$equals_____

48. Let A_1, A_2 and A_3 be three events such that

$P(A_i) = \frac{1}{3}, i = 1,2,3; P(A_i \cap A_j) = \frac{1}{6}, 1 \leq i \neq j \leq 3$ and $P(A_1 \cap A_2 \cap A_3) = \frac{1}{6}$.
Then the probability that none of the events A_1, A_2, A_3 occurs equals _____

49. Let $X_1, X_2, \ldots, X_n$ be a random sample from the distribution with the probability density function $f(x) = \frac{1}{4}e^{-|x-4|} + \frac{1}{4}e^{-|x-6|}, x \in \mathbb{R}$ Then $\frac{1}{n}\sum_{i=1}^{n} X_i$ converges in probability to _____

50. Let $x_1 = 1.1, x_2 = 2.2$ and $x_3 = 3.3$ be the observed values of a random sample of size three form a distribution with the probability density function

$$f(x;\theta) = \begin{cases} \frac{1}{\theta}e^{-x/\theta} & x > 0 \\ 0, & otherwise, \end{cases}$$

Where $\theta \in \circledast = \{1,2,\ldots\}$ is the unknown parameter. Then the maximum likelihood estimate of θ equals_____

Q.51- Q.60 carry two marks each

51. Let $f: \mathbb{R} \to \mathbb{R}$ be a differentiable function such that f' is continuous on $\mathbb{R}$ with $f'(3) = 18$ Define

$$g_n(x) = n\left(f\left(x+\frac{5}{n}\right) - f\left(x-\frac{2}{n}\right)\right)$$

Then $\lim_{n\to\infty} g_n(3)$ equals _____

52. Let $M = \sum_{i=1}^{4} X_i X_i^T$ where
$X_1^T = [1 \;\; -1 \;\; 1 \;\; 0], X_2^T = [1 \;\; 1 \;\; 0 \;\; 1], X_3^T = [1 \;\; 3 \;\; 1 \;\; 0]$ and $X_4^T = [1 \;\; 1 \;\; 1 \;\; 1 \;\; 0]$.
Then the rank of M equals_____

53. Let $f: \mathbb{R} \to \mathbb{R}$ be a differential function with $\lim_{x\to\infty} f(x) = \infty$ and $\lim_{x\to\infty} f'(x) = 2$. Then

$$\lim_{x\to\infty}\left(1+\frac{f(x)}{x^2}\right)^x$$

Equals _____

54. The value of $\int_0^{\frac{\pi}{2}}\left(\int_0^x e^{\sin y}\sin x\ dy\right)dx$ equals _____

55. Let X be a random variable with the probability density function

$$f(x) = \begin{cases} 4x^k & 0 < x < 1, \\ x - \frac{x^2}{2}, & 1 \le x < 2 \\ 0, & otherwise, \end{cases}$$

Where k is a positive integer. Then $P\left(\frac{1}{2} < x < \frac{3}{2}\right)$ equals _____

56. Let X and Y be two discrete random variables with the joint moment generating function

$$M_{X,Y}(t_1,t_2) = \left(\frac{1}{3}e^{t_1}+\frac{2}{3}\right)^2\left(\frac{2}{3}e^{t_2}+\frac{1}{3}\right)^3, t_1, t_2 \in \mathbb{R}$$

Then $P(2X+3Y>1)$ equals _____

57. Let X_1, X_2, X_3 and X_4 be i.i.d. discrete random variables with the probability mass function

$$P(X_1 = n) = \begin{cases} \frac{3^{n-1}}{4^n} & n = 1, 2, \dots, \\ 0, & otherwise. \end{cases}$$

Then $P(X_1+X_2+X_3+X_4 = 6)$ equals_____

58. Let X be a random variable with the probability mass function $P(X=n) = \begin{cases} \frac{1}{10} & n = 1, 2, .., 10 \\ 0, & otherwise, \end{cases}$

Then $E(\max\{X, 5\})$ equals _____

59. Let X be a sample observation from $U(\theta, \theta^2)$ distribution, where $\theta \in \circledast = \{2, 3\}$ is the known as parameter. For testing

$H_0: \theta = 2$ against $H_1: \theta = 3$,

Let α and β be the size and power, respectively, of the test that rejects H_0 if and only $X \ge 3.5$. Then $\alpha+\beta$ equals _____

60. A fair die is rolled four times independently. For $i = 1, 2, 3, 4$ define

$$Y_i = \begin{cases} 1, & if\ 6\ appears\ in\ the\ ith\ throw, \\ 0, & otherwise, \end{cases}$$

Then $P(\max\{Y_1, Y_2, Y_3, Y_4\} = 1)$ equals _____

ANSWER KEY

SECTION A

1	2	3	4	5	6	7	8	9	10
A	A	B	B	B	C	C	B	C	D
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
B	D	C	C	A	D	C	B	A	A
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
D	B	C	D	A	D	C	B	A	D

SECTION B

31	32	33	34	35	36	37	38	39	40
A;B;C;D	A;C	A;C	A;C	A;C	B;C;D	B;C;D	A	A;C	B;C;D

SECTION C

41	42	43	44	45	46	47	48	49	50
5.40 to 5.50	7.80 to 7.90	1.90 to 2.10	0.35 to 0.40	0.16 to 0.17	2.50 to 3.50	0.25 to 0.35	0.30 to 0.40	4.75 to 5.25	1.90 to 2.10
51	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
125 to 127	3.0 to 3.0	7.25 to 7.50	1.70 to 1.80	0.85 to 0.90	0.97 to 0.99	0.01 to 1.03	6.25 to 6.75	1.10 to 1.20	0.50 to 0.53

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