IIT JAM MATHEMATICS 2026
(IFAS SOLVED PAPER)

1. For each positive integer n. let $x_n = 1-(-1)^n+\frac{1}{n}$ and $y_n = \left(1+\frac{1}{2n}\right)^{3n}$.
   Which ONE of the following statements about the sequences (x_n) and (y_n) is TRUE?
   1. (x_n) is not convergent and (y_n) is convergent.
   2. (x_n) is convergent and (y_n) is not convergent.
   3. (x_n) is convergent and (y_n) is not convergent.
   4. (x_n) and (y_n) are convergent.

2. For each positive integer n, define $x_n = (-1)^n$.
   Which ONE of the following statements about the sequence (x_n) is FALSE?
   1. For all $\epsilon > 0$ and $M > 0$ there exists a positive integer N such that $N > M$ and $|x_N - 1| > \epsilon$.
   2. For all $\epsilon > 0$ and $M > 0$ there exists a positive integer N such that $N > M$ and $|x_N - 1| < \epsilon$.
   3. There exists $\epsilon > 0$ such that $|x_N - 1| < \epsilon$. for all positive integers n.
   4. There exists $\epsilon > 0$ such that for all $M > 0$ there exists a positive integer $N > M$ for which $|x_N - 1| > \epsilon$.

3. Let $f: (1,4) \to \mathbb{R}$ be a differentiable function such that $f'(x) = (f(x) - \pi x)^2 + \pi$ for all $x \in (1,4)$.
   Which ONE of the following is a possible value of $f(3) - f(2)$?
   1. $\frac{\pi}{2}+\frac{1}{3}$
   2. $\pi+\frac{1}{6}$
   3. $\pi-\frac{1}{6}$
   4. $\frac{\pi}{2}+\frac{1}{2}$

4. The general solution of the differential equation $x\sin\left(\frac{y}{x}\right) = y\sin\left(\frac{y}{x}\right)+\frac{x}{2}, x>0$ is given by _____.
   1. $\cos\left(\frac{x}{y}\right)+\log_e(\sqrt{x}) = K$, where K is an arbitrary constant
   2. $\cos\left(\frac{y}{x}\right)+\log_e(\sqrt{x}) = K$, where K is an arbitrary constant
   3. $\cos\left(\frac{y}{x}\right)+\log_e(x^2) = K$, where K is an arbitrary constant
   4. $\cos\left(\frac{x}{y}\right)+\log_e(x^2) = K$, where K is an arbitrary constant

5. Let $f: \mathbb{R}^3 \to \mathbb{R}$ be a continuous function. Then $\int_0^2\int_0^x\int_0^y f(x,y,z)dzdydx$=____.
   1. $\int_0^2\int_0^x\int_z^2 f(x,y,z)dydzdx$
   2. $\int_0^2\int_y^2\int_0^x f(x,y,z)dzdxdy$
   3. $\int_0^2\int_z^2\int_0^x f(x,y,z)dydxdz$
   4. $\int_0^2\int_0^y\int_y^2 f(x,y,z)dxdzdx$

6. Let $f: (1,2) \to \mathbb{R}$ be a continuous function satisfying $\lim_{x\to 1+} f(x) = 3$ and $\lim_{x\to 2-} f(x) = 3$.
   Then which ONE of the following is necessarily TRUE?
   1. f is bounded and f has neither a maximum nor a minimum.
   2. f is unbounded and f has either no maximum or no minimum.
   3. f is bounded and f has a maximum or a minimum or both.
   4. f is bounded and f has a maximum or a minimum but not both.

7. Let $f: \mathbb{R} \to \mathbb{R}$ be a twice differentiable function such that $f(n) = 1$ for all $n \in \mathbb{Z}$.
   Which ONE of the following statements is necessarily TRUE?
   1. $f'(x) = 0$ for infinitely many $x \in \mathbb{R}\backslash\mathbb{Z}$.
   2. $f''(n) = 0$ for all $n \in \mathbb{Z}$.
   3. $f''(n) = 0$ for all $n \in \mathbb{Z}$
   4. $f''(x) = 0$ for infinitely many $x \in \mathbb{R}\backslash\mathbb{Z}$

8. Let A be an invertible 5×5 real matrix. Let B be the matrix obtained by interchanging the second and third columns of A and then adding 3 times the third column to the fifth column.
   Which ONE of the following statements is TRUE?
   1. B^{-1} obtained by interchanging the second and third rows of A^{-1}, and then adding 3 times the third row to the fifth row.
   2. B^{-1} is obtained by adding -3 times the third row to the fifth row, and then interchanging the second and third rows of A^{-1}.
   3. B^{-1} is obtained by adding 3 times the fifth row to the third row, and then interchanging the second and third rows of A^{-1}.
   4. B^{-1} is obtained by interchanging the second and third rows of A^{-1}, and adding-3 times the fifth row to the third row.

9. Which ONE of the following statements is FALSE?
   1. $\mathbb{Z}_3$ is a subgroup of S_4.
   2. S_3 is isomorphic to a quotient group of S_4.
   3. S_3 is isomorphic to a subgroup of S_4.
   4. $\mathbb{Z}_6$ is isomorphic to a quotient group of S_4.

10. Let T be the triangular region in the plane with vertices at (0,0), (1,0) and (1,1). Let f be a function defined from T to $\mathbb{R}$ given in polar coordinates. Then the double integral of f over T is equal to
    1. $\int_0^{\sqrt{2}} \left(\int_0^{\pi/4} f d\theta \right) r dr - \int_1^{\sqrt{2}} \left(\int_0^{\sin^{-1}(1/r)} f d\theta \right) r\, dr$
    2. $\int_0^{\sqrt{2}} \left(\int_0^{\pi/4} f d\theta \right) r dr + \int_1^{\sqrt{2}} \left(\int_0^{\cos^{-1}(1/r)} f d\theta \right) r\, dr$
    3. $\int_0^{\sqrt{2}} \left(\int_0^{\pi/4} f d\theta \right) r dr - \int_1^{\sqrt{2}} \left(\int_0^{\cos^{-1}(1/r)} f d\theta \right) r\, dr$
    4. $\int_0^{\sqrt{2}} \left(\int_0^{\pi/4} f d\theta \right) r dr + \int_1^{\sqrt{2}} \left(\int_0^{\sin^{-1}(1/r)} f d\theta \right) r\, dr$

11. Given a sequence of real numbers (a_n), define
    $b_n = \begin{cases} a_n, & a_n \geq 0 \\ 0, & a_n < 0 \end{cases}$ and $c_n = \begin{cases} a_n, & a_n < 0 \\ 0, & a_n \geq 0 \end{cases}$

for each positive integer n.

Which of the following statements is True ?

1. If $\sum_{n=1}^{\infty} a_n$ does not converge, then $\sum_{n=1}^{\infty} b_n$ does not converge.
2. If $\sum_{n=1}^{\infty} a_n$ converges but $\sum_{n=1}^{\infty} |a_n|$ does not converge, then $\sum_{n=1}^{\infty} b_n$ does not converge.
3. If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} b_n$, converges or $\sum_{n=1}^{\infty} c_n$ converges.
4. If $\sum_{n=1}^{\infty} a_n$ converges but $\sum_{n=1}^{\infty} |a_n|$ does not converge, then $\sum_{n=1}^{\infty} b_n$ converges.

12. For which one of the following pairs of values of a and b, does the series $\sum_{k=1}^{\infty} \frac{(3x)^k}{2\sqrt{k}(1-x)^k}$ converge for all $x \in (a, b)$?

1. $a = \frac{-1}{4}$ and $b = \frac{1}{3}$
2. $a = \frac{-1}{4}$ and $b = \frac{1}{2}$
3. $a = -1$ and $b = \frac{1}{5}$
4. $a = \frac{-1}{2}$ and $b = \frac{1}{4}$

13. Let $f: \mathbb{R} \to \mathbb{R}$ be a function that has derivatives of all orders.

Which ONE of the following statements is FALSE?

1. If $f^{(k)}(a) = 0$ for all $1 \leq k \leq 6$ and $f^{(7)}(a) < 0$ for some $a \in \mathbb{R}$, then $f(x)$ has a local maximum at a.
2. If f is a polynomial in x of degree at most 5, then $f^{(6)}(a) < 0$ for all $x \in \mathbb{R}$.
3. If $f^{(6)}(a) < 0$ for all $x \in \mathbb{R}$, then f is a polynomial in x of degree at most 5.
4. If $f^{(k)}(a) = 0$ for all $1 \leq k \leq 5$ and $f^{(6)}(a) < 0$ for some $a \in \mathbb{R}$, then $f(x)$ has a local maximum at a.

14. Let A be a 5×3 real matrix of rank 2 and b be a non-zero 5×1 real column vector. Suppose $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$5 are two solutions of the system of linear equations $Ax = b$?.

Which ONE of the following is also a solution of $Ax = b$?

1. $\begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix}$
2. $\begin{pmatrix} 5 \\ 6 \\ 9 \end{pmatrix}$
3. $\begin{pmatrix} 5 \\ 7 \\ 9 \end{pmatrix}$
4. $\begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix}$

15. Let $f: \mathbb{R}^2 \to \mathbb{R}$ be the function given by $f(x, y) = (x^2 - 1)^2 + (y^2 - 1)^2$ for all $(x, y) \in \mathbb{R}^2$.

Which ONE of the following statements is TRUE?

1. f has local maxima at exactly two points.
2. f has exactly four saddle points.
3. f has local minima at exactly three points.
4. f has local minima at exactly two points.

16. The orthogonal trajectories of the family $y = -3x - 3 + me^{x}$ a real parameter m are given by _____.
    1. $x = \frac{-y}{3} + \frac{1}{9} + ke^{-3y}, k$ is a real parameter
    2. $x = \frac{-y}{3} + \frac{1}{9} + ke^{3y}, k$ is a real parameter
    3. $x = \frac{y}{3} - \frac{1}{9} + ke^{3y}, k$ is a real parameter
    4. $x = \frac{y}{3} - \frac{1}{9} + ke^{-3y}, k$ is a real parameter

17. Let $f: \mathbb{R}^2 \to \mathbb{R}$ be the function defined by $f(x,y) = \begin{cases} \frac{5x^2y - 4y^3}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$

    Which ONE of the following statements is FALSE?
    1. $\frac{\partial f}{\partial y}$ is continuous at $(0,0)$
    2. $\frac{\partial f}{\partial x}$ at $(0,1) = 0$
    3. $\frac{\partial f}{\partial x}$ at $(0,1) = 5$
    4. f is continuous at $(0,0)$

18. Let $\phi: \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to \mathbb{R}$ be the solution of the differential equation $(\cos x)\frac{dy}{dx} - y = 2y^2(\sin x - 1)\cos x$ satisfying $\phi(0) = \frac{1}{2}$. Then $\phi\left(\frac{\pi}{4}\right) =$_____.
    1. $\frac{3+2\sqrt{2}}{2}$
    2. $\frac{1}{\sqrt{2}}$
    3. $\frac{1}{2}$
    4. 2

19. The value of the following expression $\lim_{x\to\infty} \int_0^x e^{-t} |\sin t| dt$ is ____.
    1. $\frac{e^{\pi}+1}{2(e^{\pi}-1)}$
    2. $\frac{2(e^{\pi}+1)}{e^{\pi}-1}$
    3. $\frac{2(e^{\pi}-1)}{e^{\pi}+1}$
    4. $\frac{e^{\pi}-1}{2(e^{\pi}+1)}$

20. The semi-circle $y = \sqrt{6x - 5 - x^2}$ is rotated clockwise by the angle $\frac{\pi}{2}$ in the plane about the origin. What is the area of the planar region traced out by the semi-circle?
    1. π

2. 2π
3. 4π
4. 6π

21. $f: [1,9] \to \mathbb{R}$ be a non-constant continuous function such that $f(1) = f(9)$.
Which ONE of the following statements is necessarily TRUE?
1. There exists $c \in [2,6]$ such that $f(c) = f(c+3)$.
2. There exists $c \in [1,5]$ such that $f(c) = f(c+4)$.
3. There exists $c \in [3,7]$ such that $f(c) = f(c+2)$.
4. There exists $c \in [4,8]$ such that $f(c) = f(c+1)$.

22. Let S be the solid of intersection of two solid spheres S_1, S_2 given by
$S_1: x^2 + y^2 + (z-1)^2 \leq 4$ and $S_2: x^2 + y^2 + (z+1)^2 \leq 4$.
Which ONE of the following iterated integrals expresses the volume of S?
1. $8\int_0^2 \int_0^{\sqrt{4-x^2}} \left(\sqrt{4-x^2-y^2} - 1\right) dydx$
2. $2\int_0^2 \int_0^{\sqrt{4-x^2}} \left(\sqrt{4-x^2-y^2} - 1\right) dydx$
3. $2\int_0^{\sqrt{3}} \int_0^{\sqrt{3-x^2}} \left(\sqrt{4-x^2-y^2} - 1\right) dydx$
4. $8\int_0^{\sqrt{3}} \int_0^{\sqrt{3-x^2}} \left(\sqrt{4-x^2-y^2} - 1\right) dydx$

23. Consider the differential equation.
$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = \sin(e^x)$.
Which ONE of the following is a particular solution of this differential equation?
1. $-e^{-2x}(\sin(e^x) + \cos(e^x))$
2. $-e^{-x}\sin(e^x)$
3. $-e^{-2x}\sin(e^x)$
4. $-e^{-x}(\sin(e^x) + \cos(e^x))$

24. Let e_1, e_2, e_3, e_4, e_5 denote the standard basis vectors of the real vector space $\mathbb{R}^5$. Let V be the subspace of $\mathbb{R}^5$ spanned by the vectors $e_1 + e_2 + e_3 + e_4 + e_5$. Consider the real vector space $M_{2.5}(\mathbb{R})$. Let $S = \{P \in M_{2.5}(\mathbb{R}) | nullspace(P) = V\}$.
Which ONE of the following statements is TRUE?
1. $S = \{0\}$.
2. S is the empty set.
3. S is a nonzero subspace of $M_{2.5}(\mathbb{R})$.
4. S is non-empty and S is not a subspace of $M_{2.5}(\mathbb{R})$.

25. Let P be a nonzero complex n×n matrix such that $v^*Pv \geq 0$ for all column vectors v in $\mathbb{C}^n$.
Which ONE of the following statements is necessarily TRUE?
1. If v_1, v_2 are column vectors in $\mathbb{C}^n$ such that $Pv_1 = v_2$ and $Pv_2 = v_1$, then $v_1 = v_2$.
2. $v^*Pv \neq 0$ for all nonzero column vectors v in $\mathbb{C}^n$
3. All eigenvalues of P^* are negative real numbers.
4. All eigenvalues of P are negative real numbers.

26. Let $f: G \to \mathbb{Z}$ be a surjective group homomorphism from an abelian group G to the additive group of integers $\mathbb{Z}$. Let K denote the kernel of f .
Which ONE of the following statements is FALSE?
1. K is isomorphic to a quotient group of G.
2. There exists a group homomorphism $\phi: Z \to G$ such that $f \circ \phi$ is the identity homomorphism.
3. For all non-zero homomorphisms $\alpha: Z \to G$ and $\beta: G \to K$, the composition $\beta \circ \alpha$ is non zero
4. G has a subgroup isomorphic to $\mathbb{Z}$

27. Which ONE of the following statements is FALSE?
1. The multiplicative group of complex numbers of modulus one is isomorphic to a quotient group of the additive group of real numbers.
2. Any group homomorphism from the multiplicative group of nonzero complex numbers into the group of all invertible 2×2 matrices with real entries has nontrivial kernel.
3. There is a surjective group homomorphism from the additive group of rational numbers to the multiplicative group of all complex roots of unity.
4. There exists a group homomorphism from the symmetric group on n symbols into the multiplicative group of all invertible n×n matrices with real entries, which has a trivial kernel.

28. For any two points $P, Q \in \mathbb{R}^3$, let vect(P. Q) denote the vector from the point P to the point Q and let $d(P.Q)$ denote the length of vect $(P.Q)$. Let $F: \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that $d\big(F(P), F(Q)\big) = d(P, Q)\ for\ all\ P, Q \in \mathbb{R}^3$.
Which ONE of the following statements is FALSE?
1. F is an injective map.
2. For any four points P, Q, R,S in $\mathbb{R}^3$, we have
$vect\big(F(P), F(Q)\big) \times vect\big(F(R), F(S)\big) = vect(P, Q) \times vect(R, S)$,
where X denotes the usual cross product in $\mathbb{R}^3$.
3. F is a surjective map.
4. For any four points P, Q, R,S in $\mathbb{R}^3$, we have
$vect\ \big(F(P), F(Q)\big).vect\ \big(F(R), F(S)\big) = vect(P, Q).vect(R, S)$,
where denotes the usual dot product in $\mathbb{R}^3$.

29. For each positive integer n, let $x_n = 1 + \frac{1}{2} + \cdots + \frac{1}{n} - \log_e n$ and $y_n = \int_1^n \frac{\cos t}{t^2} dt$.
Which ONE of the following statements about the sequences (x_n) and (y_n) is TRUE?
1. (x_n) is convergent and (y_n) is not convergent.
2. (x_n) and (y_n) are convergent.
3. (x_n) is not convergent and (y_n) is convergent.
4. (x_n) is not convergent and (y_n) is not convergent.

30. Let G be the power set of {1,2,3,4}. Let the binary operation Δ on G be
$A \Delta B = (A \backslash B) \cup (B \backslash A)$.
Which ONE of the following statements is TRUE?
1. (G, Δ) is an abelian group but not cyclic.
2. (G, Δ) is a group and has an element of order 4.

3. {1} is an identity element of (G, Δ).
4. (G, Δ) is a group and has an element of order 8.

31. Which of the following functions $f: \mathbb{R} \to \mathbb{R}$ has/have a local minimum at $x = 0$?
    1. $f(x) = x^4 + x^2 + 3$
    2. $f(x) = \sin x + \frac{x^3}{6}$
    3. $f(x) = \min\{x - [x], 1 - x + [x]\}$
    4. $f(x) = \sin|x|$

32. Which of the following statements is/are True?
    1. There exists a sequence (x_n) such that $(|x_{n+1} + x_n|)$ converges to 0 but (x_n) does not converge.
    2. There exists a monotone sequence that does not converge but has a convergent subsequence .
    3. There exists a sequence that has a bounded subsequence but does not have any convergent subsequence.
    4. There exists a sequence (x_n) such that given any positive integer $m, (x_n)$ has a subsequence converging to m.

33. Which of the following functions is/are differentiable at x = 1?
    1 $f(x) = |x^2 - 1|$.
    2. $f(x) = |x - 1|^3$
    3. $\mathrm{f(x)} = \begin{cases} x^2 e^{-x^2}, & |\mathrm{x}| \leq 1 \\ e^{-1}, & |\mathrm{x}| > 1 \end{cases}$
    4. $f(x) = [x]$

34. Let $f: \mathbb{R} \to \mathbb{R}$ be the function given by $f(x) = \begin{cases} \cos x, & x \in \mathbb{Q} \\ 0, & x \notin \mathbb{Q} \end{cases}$
    Which of the following statements is/are TRUE?
    1. $f(x)$ is Riemann integrable on $[0, 1]$ and $\int_0^1 f(x)\, dx = \sin 1$.
    2. $f(x)$ is continuous at 0.
    3. $f(x)$ is Riemann integrable on $[0, 1]$ and $\int_0^1 f(x)\, dx = 0$.
    4. $f(x)$ is continuous at $\frac{\pi}{2}$.

35. Consider the differential equation
    $(2y \cos x - xy \sin x)dx + 2x \cos x\, dy = 0\ for\ x \in \left(0.\frac{\pi}{4}\right)$.
    Which of the following is/are integrating factor(s) of the differential equation?
    1. xy
    2. $\sqrt{\sec x}$
    3. $\frac{1}{xy}$
    4. $\sec x$

36. Let $f(x, y) = \begin{cases} \frac{x^2 y}{1+x^2} \sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$

Which of the following statements is/are TRUE?

1. $\frac{\partial f}{\partial y}$ is continuous at the point (0,1).
2. $\frac{\partial f}{\partial x}$ is continuous at the point (0,1).
3. $\lim_{(x,y)\to(0,0)} \frac{f(x,y)}{\sqrt{x^2+y^2}}$ does not exist.
4. $\lim_{y\to 0} \lim_{x\to 0} f(x,y)$ exists.

37. Let $P = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \in M_3(\mathbb{C})$.ss

Which of the following statements is/are TRUE?

1. P has 4 distinct eigenvalues in $\mathbb{C}$.
2. P has 3 distinct eigenvalues in $\mathbb{R}$.
3. nullity$(P - I) \geq 2$, where I is the 5×5 identity matrix.
4. If λ is an eigenvalue of P, then there exists a positive integer n such that $\lambda^n = 1$.

38. Let $\binom{n}{r}$ denote the number of ways of choosing r distinct objects out of n distinct objects.

Which of the following statements is/are TRI048

1. $\sum_{k=0}^{6}(-1)^k \binom{13}{2k+1} = 128.$
2. $\sum_{k=0}^{6}(-1)^k \binom{13}{2k} = -64.$
3. $\sum_{k=0}^{6}(-1)^k \binom{13}{2k+1} = -128.$
4. $\sum_{k=0}^{6}(-1)^k \binom{13}{2k} = 64.$

39. Consider matrices P of order 4×6 and Q of order 6×4 with real entries such that PQ=0 and QP=0.

Which of the following statements is/are TRUE?

1. rangespace (P) ⊆ nullspace (Q) and rangespace (Q) ⊆ nullspace (P).
2. rangespace(Q) = nullspace (P).
3. If rangespace (P) = nullspace(Q), then rank(P) + rank(Q) = 4.
4. rank(P) + rank(Q) ≤ 4.

40. Let V be the subset of $\mathbb{R}$ defined by

$V = \left\{\frac{a+b\sqrt{2}}{c+d\sqrt{2}}; a, b, c, d \in \mathbb{Q}, c^2 + d^2 \neq 0\right\}$.

Which of the following statements is/are FALSE?

1. V is a subspace of the real vector space $\mathbb{R}$.
2. V is a four dimensional subspace of the vector space $\mathbb{R}$ over $\mathbb{Q}$.
3. V is closed under the usual addition in $\mathbb{R}$.
4. V is a two dimensional subspace of the vector space $\mathbb{R}$ over $\mathbb{Q}$.

41. The radius of convergence of the series $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!} (\log_e n)^{-1} x^n$ is____. (rounded off to one decimal place). (4.0 to 4.0)

42. $\int_0^1 \left(\sum_{k=1}^{\infty} \frac{(\log_e 2)^k x^{k-1}}{(k-1)!} \right) dx =$_____. (rounded off to one decimal place). (1.0 to 1.0)

43. $\lim_{n\to\infty} \left(\frac{n^2+1}{\sqrt{n^6+1}} + \cdots + \frac{n^2+n}{\sqrt{n^6+n}} \right) =$_____. (rounded off to one decimal place). (1.0 to 1.0)

44. $\lim_{x\to 0} \left(\frac{1}{x} - \frac{1}{\sin x} + e^{\frac{1-\cos x}{x}} \right) =$_____. (rounded off to one decimal place). (1.0 to 1.0)

45. Let α and β be real numbers such that the differential equation $y^3 + axy^4 - 5x + cos\ 2y)\ dx + (3xy^2 + 20x^2y^3 + \beta x \sin 2y)dy = 0$ is exact.
Then $\alpha + \beta$=_____. (rounded off to one decimal place). (8.0 to 8.0)

46. Let $z = \cos(4x + 5y)$, where $x = \frac{\pi}{2} + 2\theta, y = -\left(\frac{\pi}{4} + \theta\right)$.
Then $\left.\frac{dz}{d\theta}\right|_{\theta=\frac{\pi}{4}} =$_____ (rounded off to one decimal place). (3.0 to 3.0)

47. Let C_n denote a cyclic group having n elements. If there is a surjective group homomorphism from C_n to C_{30}, then the total number of such distinct surjective homomorphisms is _____. (8 to 8)

48. $\int_0^3 (|x-1| - x[x])\ dx =$____ (rounded off to one decimal place). (-4.0 to -4.0)

49. Let $A \in M_3(\mathbb{C})$. Suppose the column vector $v = \begin{pmatrix} \sqrt{5}i \\ 2i \\ x \end{pmatrix}$ in $\mathbb{C}^3$ belongs to the intersection of nullspace(A) and rangespace(A^T).
Then $|x|$ =_____ (rounded off to one decimal place). (3.0 to 3.0)

50. Let P be a 5×5 real matrix with $det(P) = 2$. Let Q be the matrix of cofactors of P. Then $det(Q)$ =_____ (rounded off to one decimal place). (16.0 to 16.0)

51. Let α, β and γ be fixed real numbers such that $y(x) = C_1 e^{-x} + C_2 e^{2x} + axe^{-x}$, where C_1 and C_2 are arbitrary real constants, is the general solution of the differential equation $\frac{\mathrm{d}^2y}{\mathrm{dx}^2} + \beta \frac{dy}{dx} + \gamma y = -e^{-x}$.
Then $\alpha(\beta + \gamma)$ =_____ (rounded off to one decimal place). (-1.0 to -1.0)

52. Let the function $f: \mathbb{R}^2 \to \mathbb{R}$ be defined by (0.24 to 0.26)

$$f(x,y) = \begin{cases} 4x^2 \tan^{-1}\left(\frac{y}{2x}\right) + y^3 \tan^{-1}\left(\frac{x}{4y^2}\right), & xy \neq 0 \\ 0, & \text{otherwise} \end{cases}$$

Then $\left(\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)\right)$at $(0,0)$ is _____.(rounded off to two decimal places).

53. The double integral of $f(x,y) = x$ over triangular region with vertices at $\left(-\frac{1}{2}, \frac{1}{2}\right)$, $(1,2)$ and $(1,-1)$ is _____.(rounded off to one decimal place). (1.1 to 1.3)

54. Let $f(x) = \int_0^{e^{\sin x}} \int_0^{\log_e y} e^{-\left(\frac{1}{\sqrt{2}}\right)} (1-t^2) dt\, dy$. (0.4 to 0.5)
Then $f'(\pi/4) =$_____. (rounded off to one decimal place).

55. The volume of the tetrahedron bounded by the planes $x = 1, y = 2, z = 3$ and $12x + 8y + 6z = 70$ is _____(rounded off to one decimal place). (4.0 to 4.0)

56. Let $M \in M_3(\mathbb{R})$.If $M\begin{pmatrix}\sqrt{15}\\ \sqrt{15}\\ 0\end{pmatrix} = \begin{pmatrix}\sqrt{3}\\ \sqrt{2}\\ -5\end{pmatrix}, M\begin{pmatrix}0\\ 0\\ \sqrt{30}\end{pmatrix} = \begin{pmatrix}\sqrt{15}\\ \sqrt{10}\\ \sqrt{5}\end{pmatrix}, M\begin{pmatrix}-\sqrt{15}\\ \sqrt{15}\\ 0\end{pmatrix} = \begin{pmatrix}\sqrt{12}\\ -\sqrt{18}\\ 0\end{pmatrix}$
then $det(M) =$_____. (rounded off to one decimal place). (-1.0 to -1.0)

57. Let $a, b \in \mathbb{R}$. If the system of linear equations
$x + 2y + 2z = 1$
$2x + 3y + z = 2$
$ax + 5y + bz = b$
has infinitely many solutions, then $a + b =$_____ (rounded off to one decimal place). (6.0 to 6.0)

58. A fruit shop has 4 different types of bananas. The number of ways in which 12 bananas can be bought with at least one banana from each type, is_____. (165 to 165)

59. Let A be a 3 x 3 real matrix such that given any column vector $x \in \mathbb{R}^3$, the column vector Ax is the reflection of x about the plane $\{(a, b, -a-b): a, b \in \mathbb{R}\}$.
Then the sum of the diagonal elements of A is ____. (rounded off to one decimal place). (1.0 to 1.0)

60. Let $\binom{n}{r}$ denote the number of ways of choosing r distinct objects out of n distinct objects. Then,
$\binom{5}{0} + \binom{6}{1} + \binom{7}{2} + \binom{8}{3} + \binom{9}{4} + \binom{10}{5} + \binom{11}{6}$ = _____. (924 to 924)