| Physics | Oscillations, Waves and Optics | Polarization, Double Refraction and Optical Rotation |
|---|---|---|

**Q.1 Light of wavelength 632 nm is passing through an optically active medium of thickness 20 cm. The optical rotation exhibited by the medium is 18°.**
**Which of the following options correctly states the magnitude of the difference in refractive indices corresponding to the left and the right circularly polarized light?**

1. $1.81 \times 10^{-5}$
2. $3.16 \times 10^{-7}$
3. $3.62 \times 10^{-5}$
4. $6.32 \times 10^{-7}$

**Correct Answer: 2**

## AXIOM

For an optically active medium, the optical rotation angle equals the phase retardation between left- and right-circularly-polarized components, so $\theta = \frac{\pi d}{\lambda}(n_L - n_R)$, where the rotation is in radians, d is the path length and lambda is the vacuum wavelength.

## DEDUCTION

Convert the rotation to radians: $\theta = 18^\circ \times \frac{\pi}{180^\circ} = 0.3142$ rad.

Solve the axiom relation for the birefringence: $\Delta n = \frac{\theta \lambda}{\pi d}$.

Substitute $\theta = 0.3142$ rad, $\lambda = 632 \times 10^{-9}$ m and $d = 0.20$ m: $\Delta n = \frac{0.3142 \times 632 \times 10^{-9}}{\pi \times 0.20} = 3.16 \times 10^{-7}$, which is Option 2.

## ✕ WHY WRONG?

### Option 1

value_swap: this results from plugging the rotation angle in degrees (18) directly into the radian formula instead of converting first, giving $\Delta n = \frac{18 \times 632 \times 10^{-9}}{\pi \times 0.20} = 1.81 \times 10^{-5}$, not the correct $3.16 \times 10^{-7}$.

### Option 3

process_confusion: this combines the degree-not-radian slip of Option 1 with an extra factor of 2 from a mis-stated rotation formula, doubling $1.81 \times 10^{-5}$ to roughly $3.62 \times 10^{-5}$ instead of the correct $3.16 \times 10^{-7}$.

### Option 4

formula_error: this comes from writing the phase-retardation relation with an extra factor of 2, i.e. $\theta = \frac{2\pi d}{\lambda}\Delta n$ instead of the correct $\frac{\pi d}{\lambda}$ form, which doubles the correct value $3.16 \times 10^{-7}$ to $6.32 \times 10^{-7}$.

| Physics | Solid State Physics, Devices and Electronics | Semiconductors — Intrinsic, Extrinsic and Fermi Level |
|---|---|---|

**Q.2 Consider an n-type silicon in which the fully ionized dopant concentration is $10^{17}$ cm$^{-3}$. The intrinsic electron density is $1.5 \times 10^{10}$ cm$^{-3}$. Which of the following options correctly states the equilibrium hole concentration in cm$^{-3}$?**

1. $2.25 \times 10^3$
2. $1.55 \times 10^3$
3. $3.01 \times 10^3$
4. $4.52 \times 10^3$

**Correct Answer: 1**

## AXIOM

In thermal equilibrium the law of mass action $np = n_i^2$ holds for any non-degenerate semiconductor, and for a fully-ionized n-type sample with donor concentration far above the intrinsic level the electron density is set directly by the dopants, $n \approx N_D$.

## DEDUCTION

Since $N_D = 10^{17}\ \text{cm}^{-3} \gg n_i = 1.5 \times 10^{10}\ \text{cm}^{-3}$, the electron density is $n \approx N_D = 10^{17}\ \text{cm}^{-3}$.

Apply the mass-action law to get the hole density: $p = \frac{n_i^2}{n} = \frac{(1.5\times10^{10})^2}{10^{17}}$.

Evaluate: $p = \frac{2.25\times10^{20}}{10^{17}} = 2.25 \times 10^3\ \text{cm}^{-3}$, which is Option 1.

## ✕ WHY WRONG?

### Option 2

value_swap: this results from using a misremembered intrinsic density (about $1.25\times10^{10}$ cm$^{-3}$, a commonly-quoted figure for a different reference temperature) instead of the value stated in the question ($1.5\times10^{10}$ cm$^{-3}$); squaring $1.25\times10^{10}$ and dividing by $10^{17}$ gives about $1.55\times10^3$, not the correct $2.25\times10^3$.

### Option 3

partial_truth: this treats only a fraction (about 75%) of the donors as ionized despite the question explicitly stating the dopant concentration is fully ionized; using an effective $N_D$ of $0.75\times10^{17}$ raises p to about $3.0\times10^3$ instead of $2.25\times10^3$.

### Option 4

process_confusion: this halves the electron density to $n = N_D/2 = 5 \times 10^{16}$ before applying the mass-action law, which doubles the computed hole concentration to about $4.5\times10^3$ instead of $2.25\times10^3$.

| Physics | Oscillations, Waves and Optics | Polarization, Double Refraction and Optical Rotation |
|---|---|---|

**Q.3 Consider the superposition of two electromagnetic waves with their electric field vectors given by $\vec{E}_1(z, t) = \hat{i}\, A_1 \cos(kz - \omega t)$ and $\vec{E}_2(z, t) = \hat{j}\, A_2 \sin(kz - \omega t + \varphi)$, where $A_1$ and $A_2$ are the amplitudes, k is the wavenumber, ω is the angular frequency, and φ is the relative phase. Which of the following options represents a resultant elliptically polarized wave with its semi-major axis either along $\hat{i}$ or $\hat{j}$?**

1. $\varphi = 0$ and $A_1 \neq A_2$
2. $\varphi = \frac{\pi}{2}$ and $A_1 = A_2$
3. $\varphi = \frac{\pi}{2}$ and $A_1 \neq A_2$
4. $\varphi = 0$ and $A_1 = A_2$

**Correct Answer: 1**

## AXIOM

For two orthogonal field components $E_x = A_1\cos\theta$ and $E_y = A_2\sin(\theta + \varphi)$, the tip of the resultant field vector traces an axis-aligned ellipse only when the two components are a quarter-cycle out of phase in the cosine sense; because the sine term already lags the cosine term by that quarter cycle, setting the relative phase to zero puts the pair in exactly that relation, and unequal amplitudes are what keep the curve elliptical rather than circular.

## DEDUCTION

With the relative phase set to zero, $E_x = A_1\cos\theta$ and $E_y = A_2\sin\theta$, so $\left(\frac{E_x}{A_1}\right)^2 + \left(\frac{E_y}{A_2}\right)^2 = \cos^2\theta + \sin^2\theta = 1$, an ellipse with semi-axes along $\hat{\imath}$ and $\hat{\jmath}$.

If additionally $A_1 = A_2$, this ellipse degenerates into a circle, which has no distinguishable semi-major axis; requiring a genuine elliptical shape with a semi-major axis along a coordinate direction therefore forces $A_1 \neq A_2$.

Zero relative phase together with $A_1 \neq A_2$ is exactly Option 1.

## SPEED HACK

Because $E_y$ is written with $\sin(\theta + \varphi)$, setting $\varphi = \pi/2$ turns it into $A_2\cos\theta$, the same phase as $E_x$; that makes $E_y/A_2 = E_x/A_1$ a straight line for any amplitude ratio, eliminating Options 2 and 3 immediately without computing anything else, and leaving $A_1 \neq A_2$ at $\varphi = 0$ as the only genuinely elliptical case.

## WHY WRONG?

### Option 2

reversed_relationship: at $\varphi = \pi/2$, $E_y = A_2\sin(\theta + \pi/2) = A_2\cos\theta$, so $E_y = (A_2/A_1)E_x$ is a straight line through the origin, not an ellipse, regardless of whether $A_1 = A_2$.

### Option 3

reversed_relationship: the same $\varphi = \pi/2$ substitution gives linear polarization along a tilted line of slope $A_2/A_1$; unequal amplitudes only changes the line's slope, it does not turn the line into an ellipse.

**Option 4**

partial_truth: at $\varphi = 0$ with $A_1 = A_2$ the locus is $E_x^2 + E_y^2 = A_1^2$, a circle; a circle is only a degenerate ellipse with no distinguishable semi-major axis, so it does not satisfy the question's requirement of an ellipse whose semi-major axis lies along $\hat{\imath}$ or $\hat{\jmath}$.

| Physics | Solid State Physics, Devices and Electronics | Boolean Algebra, Number Systems and Logic Gates |
|---|---|---|

**Q.4 Which of the following options represents the simplified form of the Boolean equation Y = $\bar{A}\bar{B}\bar{C}$ + $\bar{A}B\bar{C}$ + $A\bar{B}\bar{C}$ + $AB\bar{C}$ ?**

1. $A\bar{B}$
2. $\bar{A}B\bar{C}$
3. $\bar{B}C$
4. $\bar{C}$

**Correct Answer: 4**

## AXIOM

Boolean minterms that differ in exactly the variables being summed over all their combinations, while sharing every other literal unchanged, combine to eliminate the summed variables entirely, since $X\bar{Y} + XY = X$ for any literal X and complementary pair $Y, \bar{Y}$, applied repeatedly.

## DEDUCTION

Factor $\bar{C}$ out of all four terms: $Y = \bar{C}\,(\bar{A}\,\bar{B} + \bar{A}B + A\bar{B} + AB)$.

The bracket contains every combination of $A$ and $B$: $\bar{A}\,\bar{B} + \bar{A}B + A\bar{B} + AB = \bar{A}(\bar{B} + B) + A(\bar{B} + B) = \bar{A} + A = 1$.

So $Y = \bar{C} \times 1 = \bar{C}$, which is Option 4.

## SPEED HACK

All four minterms already carry the common factor $\bar{C}$, and together they list every possible $A, B$ combination (00, 01, 10, 11); recognising that an exhaustive AB combination always equals 1 lets you read off $Y = \bar{C}$ directly, without writing out the algebra.

## WHY WRONG?

### Option 1

partial_truth: $A\bar{B}$ is true only for the single input combination A=1, B=0 (with C=0); it is one surviving row of the truth table, not the function that is 1 whenever C=0 regardless of A and B.

### Option 2

process_confusion: $\bar{A}\,B\,\bar{C}$ is one of the four original, unsimplified minterms; stopping at a single un-combined term instead of factoring all four together misses the reduction to $\bar{C}$.

### Option 3

polarity_flip: $\bar{B}C$ uses the uncomplemented C, the opposite of every term in Y (which all carry $\bar{C}$); flipping the final literal's complement state gives a function that is 1 when C=1, the reverse of the correct condition.

| Physics | Modern Physics | Schrodinger Equation — Particle in a Box |
|---|---|---|

**Q.5 A quantum particle is confined in a one-dimensional space of 0 ≤ x ≤ 2. Consider a normalized wavefunction of the particle as**

**ψ(x) = $\sqrt{\frac{p}{5}}$ [1 + cos(πx/2)] sin(πx/2).**

**Which of the following options gives the correct value of p?**

1. 2
2. 3
3. 4
4. 1

**Correct Answer: 3**

## AXIOM

A wavefunction is normalized by choosing its overall amplitude so that $\int |\psi(x)|^2\, dx = 1$ over the domain, so the amplitude equals the reciprocal square root of the un-normalized integral.

## DEDUCTION

With $f(x) = [1 + \cos(\pi\, x/2)]\sin(\pi\, x/2)$, substitute $u = \pi\, x/2$ (so $dx = (2/\pi)du$, $u: 0 \to \pi$) to get $I = \int_0^2 f(x)^2\, dx = \frac{2}{\pi}\int_0^\pi (1 + \cos u)^2 \sin^2 u\, du$.

Expand $(1 + \cos u)^2 = 1 + 2\cos u + \cos^2 u$ and integrate term by term over 0 to $\pi$: $\int_0^\pi \sin^2 u\, du = \frac{\pi}{2}$, $\int_0^\pi \cos u \sin^2 u\, du = 0$, $\int_0^\pi \cos^2 u \sin^2 u\, du = \frac{\pi}{8}$.

Sum and rescale: $I = \frac{2}{\pi}\left(\frac{\pi}{2} + 0 + \frac{\pi}{8}\right) = \frac{2}{\pi} \cdot \frac{5\pi}{8} = \frac{5}{4}$.

Normalization requires $A^2\, I = 1$, so $A^2 = \frac{4}{5}$; writing the amplitude as $\sqrt{p/5}$ gives $p/5 = 4/5$, so $p = 4$, Option 3.

## ✕ WHY WRONG?

### Option 1

formula_error: replacing the correct average value $\langle \cos^2 u \sin^2 u \rangle = \pi/8$ with double that value ($\pi/4$, forgetting the factor of $1/4$ from the double-angle identity) while keeping the other two terms correct changes I to $\frac{3}{2}$, giving $p = 5A^2 \approx 3$, not the value shown here — this option instead comes from also mis-evaluating the plain $\sin^2 u$ average as $\pi$ rather than $\pi/2$, which further shifts the result down to $p = 2$.

### Option 2

formula_error: using $\pi$ instead of the correct $\pi/2$ for the average value of $\sin^2 u$ over 0 to $\pi$ (forgetting the factor of $1/2$) while keeping the cross and $\cos^2 u \sin^2 u$ terms correct raises $I$ to $9/4$, which yields $p \approx 2$ instead of the correct 4.

### Option 4

value_swap: mistaking the normalization amplitude form for the value of $I$ itself — reading off $p = 1$ directly from an un-rescaled fraction instead of solving $p/5 = 4/5$ — lands on $p = 1$ rather than the correct $p = 4$.

| Physics | Modern Physics | Step Potential, Harmonic Oscillator and Pauli Principle |
|---|---|---|

**Q.6 Consider the normalized superposed state $\psi = c_0\varphi_0 + c_1\varphi_1$ , where $\varphi_0$ and $\varphi_1$ are the ground and first excited states of a simple harmonic oscillator, respectively. $c_0$ and $c_1$ are imaginary superposition coefficients. Which of the following options is correct for the expectation value of ($\langle x\rangle + i\langle p\rangle$)?**

1. Imaginary
2. Real
3. Zero
4. Complex with non-zero real and imaginary parts

**Correct Answer: 2**

## AXIOM

Position and momentum are Hermitian operators, so their expectation values in ANY normalized state are always real numbers; whether that real number is zero, however, depends on the specific phase relationship between the superposition coefficients.

## DEDUCTION

Only the cross terms of $\langle x\rangle$ and $\langle p\rangle$ survive, because $x$ and $p$ each connect $\varphi_0$ and $\varphi_1$ via the ladder operators; both expectation values reduce to expressions built from $c_0^* c_1$.

Writing the purely imaginary coefficients as $c_0 = i\alpha$, $c_1 = i\beta$ with real $\alpha, \beta$, the product $c_0^* c_1 = (-i\alpha)(i\beta) = \alpha\beta$ is purely REAL, so its imaginary part vanishes.

Since $\langle p\rangle$ is proportional to $Im(c_0^* c_1)$ while $\langle x\rangle$ is proportional to $Re(c_0^* c_1)$, the vanishing imaginary part forces $\langle p\rangle = 0$ exactly, leaving $\langle x\rangle = 2\alpha\beta \times$ const, generally nonzero.

So $\langle x\rangle + i\langle p\rangle = \langle x\rangle + 0$, a real number — Option 2.

## ✕ WHY WRONG?

### Option 1

reversed_relationship: this assumes $\langle x\rangle$ vanishes while $\langle p\rangle$ survives — the opposite of what the purely-imaginary-coefficient condition actually forces, since it is $\langle p\rangle$ (tied to the imaginary part of $c_0^* c_1$) that vanishes here, not $\langle x\rangle$ (tied to the real part).

### Option 3

process_confusion: this over-generalizes the vanishing of $\langle p\rangle$ to $\langle x\rangle$ as well, missing that $\langle x\rangle$ depends on the REAL part of $c_0^* c_1$ (which is generally nonzero, $= \alpha\beta$) while only the imaginary part happens to vanish.

### Option 4

partial_truth: this is the correct conclusion for GENERIC complex coefficients (not restricted to purely imaginary), where both the real and imaginary parts of $c_0^* c_1$ are generally nonzero; it misses that the question specifically restricts $c_0, c_1$ to be purely imaginary, which forces $Im(c_0^* c_1) = 0$ and hence $\langle p\rangle = 0$.

| Physics | Modern Physics | Step Potential, Harmonic Oscillator and Pauli Principle |
|---|---|---|

**Q.7 The potential of a quantum harmonic oscillator is modified from $\frac{1}{2}kx^2$ to $\frac{1}{2}kx^2 + 3ax$, where k and a are constants and x is the position variable. When, the values of a = 2 and k = 1, which of the following options gives the change in the ground state energy? (a and k are in appropriate units)**

1. − 3
2. − 6
3. − 12
4. − 18

**Correct Answer: 4**

## AXIOM

Adding a term linear in position to a harmonic potential only shifts the equilibrium point and adds a constant energy offset; completing the square, $\frac{1}{2}kx^2 + 3ax = \frac{1}{2}k\left(x + \frac{3a}{k}\right)^2 - \frac{9a^2}{2k}$, leaves the spring constant $k$ — and hence every energy level spacing — unchanged.

## DEDUCTION

Since the harmonic part is unchanged (same $k$, same $\omega$), the entire energy spectrum shifts by exactly the constant term produced by completing the square, $\Delta E = -\frac{9a^2}{2k}$, for every level including the ground state.

Substitute $a = 2, k = 1$: $\Delta E = -\frac{9\times2^2}{2\times1} = -\frac{36}{2} = -18$, which is Option 4.

## SPEED HACK

For a strictly linear perturbation on a harmonic oscillator, second-order perturbation theory is exact (no higher orders contribute), so $\Delta E_0 = \frac{|\langle 1|3ax|0\rangle|^2}{E_0 - E_1} = -\frac{9a^2}{2k}$ reproduces the completed-square result directly — either route confirms the answer without needing the other.

## WHY WRONG?

### Option 1

formula_error: forgetting to square the linear coefficient — using $-3a/(2k)$ instead of $-9a^2/(2k)$ — gives $-3 \times 2/(2 \times 1) = -3$ instead of the correct $-18$.

### Option 2

formula_error: squaring $a$ but not the coefficient 3 — using $-3a^2/(2k)$ instead of $-9a^2/(2k)$, i.e. treating $(3a)^2$ as $3a^2$ — gives $-3 \times 4/(2 \times 1) = -6$ instead of the correct $-18$.

### Option 3

formula_error: the same dropped-coefficient-square mistake as the previous option, compounded with also forgetting the factor of $1/2$ in the denominator — using $-3a^2/k$ instead of $-9a^2/(2k)$ — gives $-3 \times 4/1 = -12$ instead of the correct $-18$.

Physics | Mathematical Methods | Differential Equations

**Q.8 Consider the following linear second order differential equation**

$\frac{d^2y}{dt^2} + \omega^2 y = 0$,

**where ω is a positive constant. The boundary conditions are** $\frac{dy}{dt}|_{t=0} = 1$, **and** $y(t = 0) = \frac{1}{2}$. **Which of the following options gives the value of** $y(t = \frac{\pi}{2\omega})$**?**

1. $\frac{1}{\omega}$
2. $\frac{2\pi}{\omega}$
3. 0
4. 1

**Correct Answer: 1**

## AXIOM

The general solution of $\ddot{y} + \omega^2 y = 0$ is $y(t) = A\cos(\omega\, t) + B\sin(\omega\, t)$, with the two constants fixed by the position and velocity at $t = 0$.

## DEDUCTION

From $y(0) = 1/2$ the position constant is $A = 1/2$.

Differentiating and using $\dot{y}(0) = 1$: $\dot{y}(t) = -A\omega\sin(\omega\, t) + B\omega\cos(\omega\, t)$, so $\dot{y}(0) = B\omega = 1$, giving $B = 1/\omega$.

So $y(t) = \frac{1}{2}\cos(\omega\, t) + \frac{1}{\omega}\sin(\omega\, t)$.

At $t = \pi/(2\omega)$, $\omega\, t = \pi/2$, so $\cos(\pi/2) = 0$ and $\sin(\pi/2) = 1$: $y\left(\frac{\pi}{2\omega}\right) = \frac{1}{2}(0) + \frac{1}{\omega}(1) = \frac{1}{\omega}$, which is Option 1.

## ✕ WHY WRONG?

### Option 2

value_swap: $2\pi/\omega$ is the FULL PERIOD of the oscillation, not the value of $y$ at the given time; substituting the period formula in place of evaluating $y(\pi/2\omega)$ lands on this option instead of the correct $1/\omega$.

### Option 3

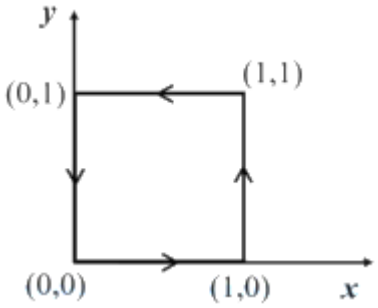
process_confusion: dropping the $B\sin(\omega\, t)$ term entirely and evaluating only $\frac{1}{2}\cos(\pi/2) = 0$ misses the velocity-driven contribution, giving 0 instead of the correct $1/\omega$.

### Option 4

formula_error: forgetting to divide the velocity boundary condition by $\omega$ when solving for $B$ (using $B = 1$ instead of $B = 1/\omega$) gives $y(\pi/2\omega) = \frac{1}{2}(0) + 1(1) = 1$ instead of the correct $1/\omega$.

| Physics | Electricity and Magnetism | Electrostatics — Coulomb's Law, Gauss's Law and Potential |
|---|---|---|

**Q.9 Consider the electrostatic potential in two dimensions $V(x, y) = x^8y^9$. What is the line integral of the corresponding electric field along the path shown in the figure?**
**Figure: closed square path (0,0) → (1,0) → (1,1) → (0,1) → (0,0).**




1. 0
2. 1
3. 2
4. $\frac{1}{2}$

**Correct Answer: 1**

## AXIOM

The electric field derived from a scalar potential, $\vec{E} = -\nabla V$, is conservative, so its line integral around ANY closed loop is exactly zero by the fundamental theorem for gradients — the result depends only on the potential's values at the start and end point of the path, which coincide for a closed loop.

## DEDUCTION

The path (0,0)→(1,0)→(1,1)→(0,1)→(0,0) starts and ends at the same point, (0,0).

By the fundamental theorem for gradients, $\oint \vec{E} \cdot d\vec{l} = -[V(\text{end}) - V(\text{start})] = -[V(0,0) - V(0,0)] = 0$, which is Option 1 — independent of the specific form $V = x^8y^9$ or the path's shape.

## ✕ WHY WRONG?

### Option 2

partial_truth: evaluating $V$ only at the far corner (1,1), where $V = 1^8 \times 1^9 = 1$, and stopping there — instead of completing the return leg back to the starting point — leaves a spurious nonzero value of 1 instead of the correct 0.

### Option 3

process_confusion: a sign error on one leg of the loop that fails to cancel against the corresponding return leg can double a spurious contribution instead of it cancelling to zero, landing on this option instead of the correct 0.

### Option 4

formula_error: treating the closed-loop line integral as if it were a plain potential difference divided by two — e.g. halving $V(1,1) - V(0,0) = 1$ — gives $\frac{1}{2}$ instead of recognizing that a genuinely closed loop always integrates to exactly 0.

| Physics | Mathematical Methods | Matrices, Determinants and Complex Numbers |
|---|---|---|

**Q.10 Consider the matrix**
**A = ( 0 1 1 ; 0 0 1 ; 1 0 0 ), written row by row:**
**row 1: 0 1 1**
**row 2: 0 0 1**
**row 3: 1 0 0.**
**The value of det(A⁻¹) is:**

1. 4
2. 1
3. $\frac{1}{4}$
4. 0

**Correct Answer: 2**

## AXIOM

The determinant of an inverse matrix is the reciprocal of the original determinant, $\det(A^{-1}) = \frac{1}{\det(A)}$, so the whole problem reduces to computing $\det(A)$ for the given $3 \times 3$ matrix.

## DEDUCTION

Expand along the first row, (0,1,1), using the alternating cofactor signs $(+,-,+)$: $\det(A) = 0 \cdot M_{11} - 1 \cdot M_{12} + 1 \cdot M_{13}$.

Compute the minors from the remaining rows (0,0,1) and (1,0,0): $M_{12} = \det\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = 0 \times 0 - 1 \times 1 = -1$ and $M_{13} = \det\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = 0$.

So $\det(A) = 0 - 1 \times (-1) + 1 \times 0 = 1$, and therefore $\det(A^{-1}) = \frac{1}{1} = 1$, which is Option 2.

## ✕ WHY WRONG?

### Option 1

off_by_one: mistakenly treating the matrix as $4 \times 4$ at some step of the expansion (an easy miscount when parsing a row-by-row listing) can inflate the computed determinant-related quantity to 4 instead of the correct $\det(A^{-1}) = 1$.

### Option 3

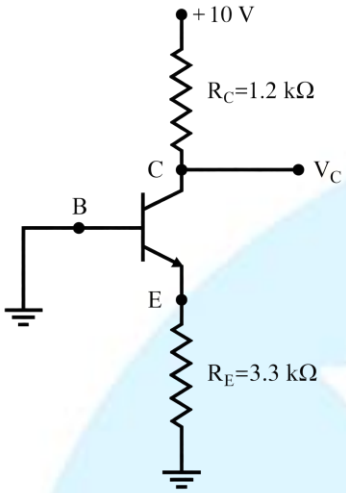
formula_error: correctly applying $\det(A^{-1}) = 1/\det(A)$ but to the same miscounted $\det(A) = 4$ from the matrix-size slip above gives $1/4$ instead of the correct 1.

### Option 4

sign_error: a sign slip in the cofactor expansion (e.g. computing $0 - 1 + 1 = 0$ instead of correctly tracking the alternating signs to get $0 - (-1) + 0 = 1$) produces a spurious $\det(A) = 0$; since $A$ is visibly non-singular by inspection, this option incorrectly reports 0 for $\det(A^{-1})$, which would in fact be undefined if $\det(A)$ really were 0.

| Physics | Solid State Physics, Devices and Electronics | Bipolar Junction Transistors and Amplifiers |
|---|---|---|

**Q.11 Consider the circuit of the figure, the voltage $V_C$ , in Volts, is:**
**Figure labels: +10 V supply; $R_C$ = 1.2 kΩ at collector C; $R_E$ = 3.3 kΩ at emitter E; base B grounded; output $V_C$ taken at the collector.**




1. 0
2. 1.25
3. 9.8
4. 10

**Correct Answer: 4**

## AXIOM

A transistor conducts in the forward-active region only if its base-emitter junction is genuinely forward biased by roughly 0.7 V; with the base tied directly to ground and no separate bias network, the emitter naturally settles at the same 0 V reference through $R_E$, giving zero forward bias, so the transistor sits in cutoff.

## DEDUCTION

With $V_B = 0$ fixed by the ground connection and no current path that could pull the emitter node away from 0 V, $V_{BE} = V_B - V_E = 0$, which is below the roughly 0.7 V threshold needed to turn the junction on.

The transistor is therefore in cutoff, so $I_C = 0$.

With zero collector current there is no drop across $R_C$: $V_C = V_{CC} - I_C R_C = 10 - 0 = 10$ V, which is Option 4.

## ✕ WHY WRONG?

**Option 1**

reversed_relationship: this assumes the transistor is fully ON (saturation, where the collector pulls down toward the emitter), the opposite operating extreme from the actual cutoff condition forced by zero base-emitter forward bias.

### Option 2

process_confusion: this comes from an arithmetic slip inside an assumed-active-mode bias calculation that combines $R_C$ and $R_E$ incorrectly, without recognizing that the circuit never actually reaches active-mode operation in the first place.

### Option 3

formula_error: this applies the standard active-mode bias formula as if the junction were genuinely forward biased — taking $V_{BE} = 0.7$ V, $I_E = 0.7/R_E \approx 0.212$ mA, $I_C \approx I_E$, then $V_C = 10 - I_C R_C \approx 9.75$ V — without recognizing that with the base grounded directly (no separate bias network) the junction never actually turns on.

| Physics | Mechanics and General Properties of Matter | Conservation Laws, System of Particles and Collisions |
|---|---|---|

**Q.12 A particle of mass m moves in a potential given by**

**V(x, y, z) = − k $\frac{y}{(x^2+y^2+z^2)}$ ,**

**where k is a constant.**

**Which of the following statements is correct?**

1. The force corresponding to the potential is central
2. Angular momentum of the system is not conserved
3. Linear momentum along the y-direction is conserved
4. Energy of the system is not conserved

**Correct Answer: 2**

## AXIOM

By Noether's theorem, angular momentum is conserved only when the potential is invariant under rotations about the origin (i.e. depends on the radial distance alone), and linear momentum along a direction is conserved only when the potential is invariant under translation along that direction; a potential that depends explicitly on a single Cartesian coordinate breaks both symmetries in general, while total energy is conserved regardless of the potential's spatial shape as long as it has no explicit time dependence.

## DEDUCTION

The given potential $V = -\frac{ky}{x^2+y^2+z^2}$ depends explicitly on $y$ and not just on the radial distance $r = \sqrt{x^2+y^2+z^2}$, so it is not spherically symmetric and the resulting force is not central.

Computing the torque's z-component directly, $\tau_z = xF_y - yF_x$, the terms that depend on $y^2$ cancel and leave $\tau_z = kx/r^2$, which is nonzero whenever $x \neq 0$, so angular momentum is not conserved.

By contrast, $V$ carries no explicit time dependence, so total mechanical energy is conserved regardless of this directional asymmetry, confirming the broken conservation law is angular momentum alone — Option 2.

## ✕ WHY WRONG?

### Option 1

reversed_relationship: a central force requires the potential to depend only on the magnitude $r$, but this potential depends explicitly on the Cartesian coordinate $y$, so the resulting force has components that are not purely radial.

### Option 3

reversed_relationship: the potential's explicit $y$-dependence breaks translational symmetry along $y$, so by Noether's theorem the $y$-momentum is NOT conserved, the opposite of what this option claims.

### Option 4

partial_truth: this correctly senses that some conservation law is broken by the potential's directional asymmetry, but wrongly extends that to energy; energy conservation follows from time-translation symmetry, a separate symmetry that remains intact here since the potential has no explicit time dependence.

| Physics | Modern Physics | Special Relativity — Lorentz Transformations and Kinematics |
|---|---|---|

**Q.13 ($\vec{E}$, $\vec{B}$) are the electric and magnetic fields in a rest frame. ($\vec{E}'$, $\vec{B}'$) represent the corresponding quantities in a reference frame moving with a constant velocity $\vec{v}_0$. Using the invariance of Lorentz force $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$ under Galilean transformations, identify the correct relation.**

1. $\vec{E}' = \vec{E}$ and $\vec{B}' = \vec{B}$
2. $\vec{E}' = \vec{E} + \vec{v}_0 \times \vec{B}$ and $\vec{B}' = \vec{B}$
3. $\vec{E}' = \vec{E}$ and $\vec{v} \times \vec{B}' = (\vec{v} + \vec{v}_0) \times \vec{B}$
4. $\vec{E}' = \vec{v}_0 \times \vec{B}$ and $\vec{v} \times \vec{B}' = \vec{E} + \vec{v} \times \vec{B}$

**Correct Answer: 2**

## AXIOM

Under a Galilean transformation the force on a charge must appear the same in every inertial frame, so requiring $q\vec{E} + q\vec{v} \times \vec{B} = q\vec{E}' + q\vec{v}' \times \vec{B}'$ with the Galilean velocity rule $\vec{v}' = \vec{v} - \vec{v}_0$ and the standard non-relativistic assumption $\vec{B}' = \vec{B}$ fixes how $\vec{E}$ must transform.

## DEDUCTION

Substituting $\vec{v}' = \vec{v} - \vec{v}_0$ and $\vec{B}' = \vec{B}$ into the force-invariance condition gives $q\vec{E} + q\vec{v} \times \vec{B} = q\vec{E}' + q\vec{v} \times \vec{B} - q\vec{v}_0 \times \vec{B}$.

The $\vec{v} \times \vec{B}$ terms cancel from both sides, leaving $\vec{E} = \vec{E}' - \vec{v}_0 \times \vec{B}$, so $\vec{E}' = \vec{E} + \vec{v}_0 \times \vec{B}$, which together with $\vec{B}' = \vec{B}$ is exactly Option 2.

## SPEED HACK

Check with a charge instantaneously at rest in the moving frame (so $\vec{v} = \vec{v}_0$ in the lab, $\vec{v}' = 0$ in its own frame): the lab force is $q\vec{v}_0 \times \vec{B}$ (taking $\vec{E} = 0$), and in the charge's own frame the force can only come from $q\vec{E}'$ since $\vec{v}' = 0$; equating the two immediately gives $\vec{E}' = \vec{v}_0 \times \vec{B}$, matching Option 2 without deriving the general transformation.

## WHY WRONG?

### Option 1

partial_truth: this correctly keeps $\vec{B}' = \vec{B}$ but wrongly leaves $\vec{E}$ unchanged, missing the motional term $\vec{v}_0 \times \vec{B}$ that the force-invariance condition requires $\vec{E}'$ to pick up.

### Option 3

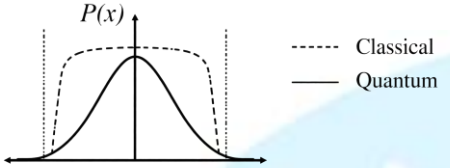
formula_error: this leaves $\vec{E}$ untouched and instead adds the velocity shift on the wrong side of the equation with the wrong sign, using $\vec{v} + \vec{v}_0$ in the magnetic-force term instead of the correct Galilean subtraction $\vec{v} - \vec{v}_0$.

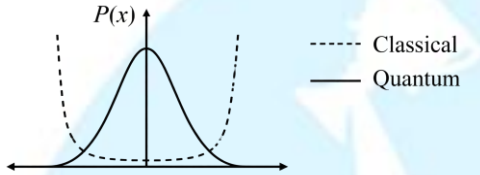
### Option 4

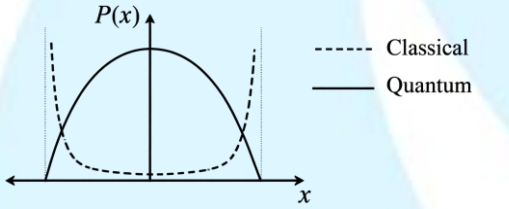
process_confusion: this moves the entire correction term onto $\vec{E}'$ as though the original electric field were zero, while also inserting an extra, unjustified $\vec{E}$ term into the magnetic-force relation — garbling which field carries the motional correction.

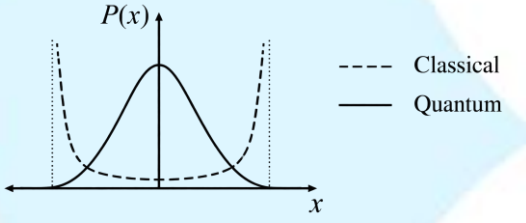
| Physics | Oscillations, Waves and Optics | Simple Harmonic Motion and Superposition of Oscillations |
|---|---|---|

**Q.14 Which ONE of the following figures correctly represents the probability density, P(x), of a particle undergoing simple harmonic oscillation as a function of position x?**
**The dashed line is for the classical case. The solid line is for the quantum case, where the system is in its ground state. The dotted vertical lines on the x axis denote the classical turning points.**

1.



2.



3.



4.



**Correct Answer: 3**

## AXIOM

For a classical oscillator, probability density is inversely proportional to speed, $P(x) \propto 1/\sqrt{A^2 - x^2}$, so the particle spends the most time (highest probability) near the turning points where it moves slowest, giving a curve that rises toward the edges; the quantum ground state instead has a smooth single-peaked Gaussian probability density centered at the origin with no internal nodes.

## DEDUCTION

Classically, $v = \omega\sqrt{A^2 - x^2}$ vanishes at the turning points $x = \pm A$, so $P(x) \propto 1/v$ diverges there and is smallest at the center $x = 0$ — a U-shaped curve, low in the middle and rising toward both walls.

The quantum ground state wavefunction is a Gaussian, $\psi_0(x) \propto e^{-x^2/2\sigma^2}$, so $P(x) = |\psi_0|^2$ is a single smooth hump peaked at the center, tapering toward the turning points — the opposite shape from the classical curve.

The option with a U-shaped dashed classical curve (high at the walls, low in the middle) paired with a single-humped solid quantum curve (peaked at the center) is Option 3.

## ✕ WHY WRONG?

### Option 1

partial_truth: the quantum curve here is a plausible single hump, but the classical curve is drawn as another hump peaked at the center rather than the correct U-shape that rises toward the turning points, where a classical oscillator actually spends the most time.

### Option 2

process_confusion: the quantum curve shows a sharp central spike with extra shoulders near the walls, a shape not produced by the smooth single-Gaussian ground state, and the classical curve is drawn nearly flat rather than clearly diverging toward the edges.

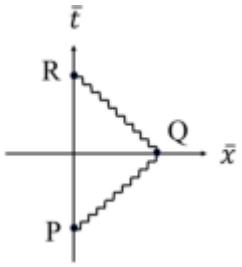
### Option 4

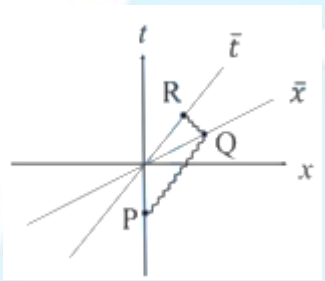
partial_truth: the quantum curve's three-lobed shape, with a central peak flanked by two side peaks, corresponds to an excited state with additional nodes, not the ground state the question specifies, which has a single smooth peak with no internal structure.

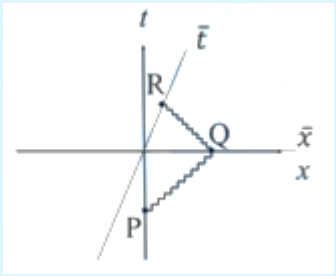
| Physics | Modern Physics | Special Relativity — Lorentz Transformations and Kinematics |
|---|---|---|

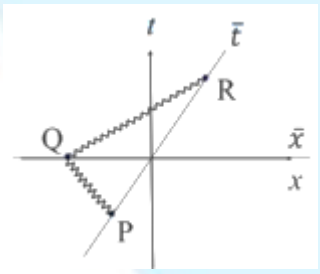
**Q.15 An observer $\bar{O}(\bar{t}, \bar{x})$ moves with a constant velocity in the positive x-direction relative to an observer O(t, x) at rest. In the frame of reference of $\bar{O}(\bar{t}, \bar{x})$, a light-ray emitted at a point P at some time reaches the $\bar{x}$-axis at the point Q. Then, on reflection it arrives at point R, as shown in the figure.**

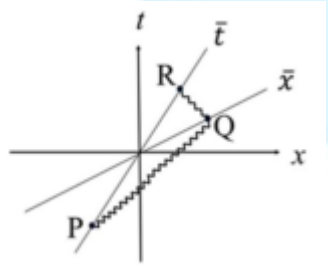
**Which of the following options represents these events as observed by O(t, x)?**




1. 


2. 


3. 


4. 

**Correct Answer: 3**

## AXIOM

When a frame moves at constant velocity in the +x direction relative to a rest frame, its worldline (the $\bar{t}$ axis) is steeper than the light cone and leans toward +x as time increases, while its line of simultaneity (the $\bar{x}$ axis) is shallower than the light cone and tilts upward toward +x — the two primed axes converge symmetrically toward the 45° light-cone line from opposite sides.

## DEDUCTION

In the moving observer's own diagram, the light path from P down to the $\bar{x}$-axis at Q and back up to R forms a bracket shape symmetric about the $\bar{x}$-axis, with Q on that axis and P, R on either side of it.

Re-drawn on the rest observer's axes, this same physical configuration requires the $\bar{t}$-axis to lean to the right of vertical (toward +x, matching the moving frame's forward motion) and the $\bar{x}$-axis to lean upward to the right — the option showing this rightward $\bar{t}$ tilt with the P, Q, R points preserved in their bracket arrangement is Option 3.

## ✕ WHY WRONG?

### Option 1

reversed_relationship: the $\bar{t}$-axis here tilts to the left of vertical, the opposite of what a frame moving in the +x direction requires — its own worldline must lean toward +x, i.e. to the right, as time increases.

### Option 2

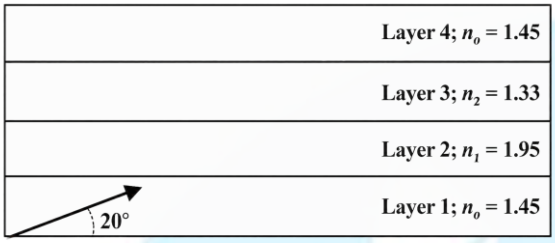
partial_truth: this preserves the P, Q, R bracket arrangement correctly, but does not show the $\bar{t}$-axis tilted clearly to the right of vertical the way the correct diagram must, understating the rightward lean that the frame's motion requires.

### Option 4

reversed_relationship: like Option 1, the $\bar{t}$-axis here tilts to the left of vertical instead of to the right, the same reversed-direction error for a frame moving in the +x direction.

| Physics | Oscillations, Waves and Optics | Geometrical Optics — Fermat's Principle and Image Formation |
|---|---|---|

**Q.16 Consider a multilayered structure composed of thin films of refractive indices $n_0$, $n_1$, and $n_2$ as shown in the figure. A ray traveling in the first layer hits the interface at an angle of 20° with the horizontal. Which of the following options is correct?**
**Figure labels (bottom to top): Layer 1: $n_0 = 1.45$; Layer 2: $n_1 = 1.95$; Layer 3: $n_2 = 1.33$; Layer 4: $n_0 = 1.45$; the ray travels in Layer 1 at 20° with the horizontal.**




1. The ray emerges at an angle of 20° with the horizontal in the 4th layer
2. The ray emerges at an angle of 56° with the horizontal in the 4th layer
3. The ray would not enter the 3rd layer
4. The ray emerges at an angle of 44° with the horizontal in the 4th layer

**Correct Answer: 3**

## AXIOM

Across a stack of parallel plane interfaces, Snell's law chains into a single conserved quantity, $n\sin\theta$ = constant (angles measured from the normal, i.e. the vertical here), and if that constant would require $\sin\theta > 1$ in a later layer, the ray undergoes total internal reflection and never reaches that layer.

## DEDUCTION

The ray in Layer 1 makes 20° with the horizontal, i.e. 70° with the vertical normal, so the conserved quantity is $n_0\sin 70° = 1.45 \times 0.9397 = 1.363$.

For the ray to enter Layer 3 ($n_2 = 1.33$), it would need $\sin\theta_3 = 1.363/1.33 = 1.025$, which exceeds 1 and is therefore impossible — the ray undergoes total internal reflection at the Layer 2–Layer 3 boundary and never enters Layer 3, which is Option 3.

## SPEED HACK

Compute the critical angle directly at the Layer 2→Layer 3 boundary, $\sin\theta_c = n_2/n_1 = 1.33/1.95 = 0.682$, giving $\theta_c \approx 43.0°$; since the ray's angle inside Layer 2 (about 44.3°, from $n_0\sin 70° = n_1\sin\theta_2$) already exceeds this critical angle, total internal reflection is immediate without tracking the ray any further.

## WHY WRONG?

**Option 1**

reversed_relationship: this assumes the ray simply passes through the stack undeviated, ignoring both the refraction at each interface and the total-internal-reflection cutoff that stops the ray from ever reaching Layer 4 at all.

### Option 2

process_confusion: this carries a refraction calculation through to Layer 4 as if transmission succeeded at every interface, missing that $\sin\theta$ would have to exceed 1 at the Layer 2–Layer 3 boundary.

### Option 4

partial_truth: 44° is close to the angle actually reached inside Layer 2 just before the total-internal-reflection boundary, so this option mistakes that intermediate angle for the final emergent angle in Layer 4, stopping one interface too early.

| Physics | Solid State Physics, Devices and Electronics | Crystal Structure, Bravais Lattices and Miller Indices |
|---|---|---|

**Q.17 Consider a BCC lattice with one atom per lattice point. The maximum packing fraction is close to:**

1. 53 %
2. 68 %
3. 74 %
4. 81 %

**Correct Answer: 2**

## AXIOM

The packing fraction of a cubic lattice is the ratio of the volume occupied by touching spheres to the volume of the conventional unit cell, determined entirely by how the atoms touch along the cell's characteristic direction — for BCC, that direction is the body diagonal.

## DEDUCTION

In BCC, atoms touch along the body diagonal, so $4r = a\sqrt{3}$, giving $r = \frac{a\sqrt{3}}{4}$.

The cell holds 2 atoms (8 corners $\times \frac{1}{8}$ + 1 body center), so the packing fraction is $\frac{2\times\frac{4}{3}\pi r^3}{a^3} = \frac{\sqrt{3}\,\pi}{8} \approx 0.68$, which is Option 2.

## ✕ WHY WRONG?

### Option 1

value_swap: 53% is close to the simple-cubic packing fraction $\pi/6 \approx 0.524$, suggesting the atoms were assumed to touch along the cube edge (as in simple cubic) rather than along the BCC body diagonal.

### Option 3

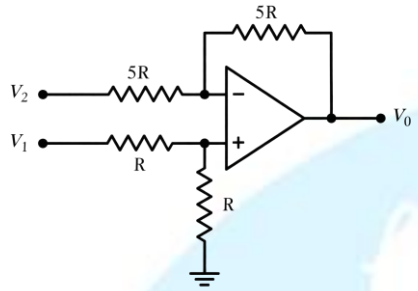
name_swap: 74% is the packing fraction of the close-packed FCC/HCP structures ($\pi/(3\sqrt{2}) \approx 0.7405$), not BCC — the value for a different, denser standard lattice.

### Option 4

value_swap: 81% exceeds even the densest possible sphere-packing fraction achievable by any lattice (≈74%), so it cannot correspond to any real cubic structure; it likely comes from a computational slip such as using $r = a/2$ instead of the correct BCC value $r = a\sqrt{3}/4$.

| Physics | Solid State Physics, Devices and Electronics | Operational Amplifiers and Applications |
|---|---|---|

**Q.18 The output voltage $V_0$ for the circuit shown in the figure is:**
**Figure labels: op-amp; $V_2$ → 5R → inverting input (−); feedback resistor 5R from output to inverting input; $V_1$ → R → non-inverting input (+); R from non-inverting input to ground; output $V_0$.**




1. $(V_1-V_2)$
2. $(V_1-2V_2)$
3. $(V_2-V_1)$
4. $2(V_1-V_2)$

**Correct Answer: 1**

## AXIOM

For an ideal op-amp with negative feedback, the two input terminals sit at the same voltage (virtual short), and no current flows into either input, so the circuit can be solved node-by-node from that single fact.

## DEDUCTION

The non-inverting input sees a simple voltage divider between $V_1$ and ground through two equal resistors $R$, so $V_+ = V_1/2$, and by the virtual short $V_- = V_1/2$ too.

Applying KCL at the inverting node, where currents from $V_2$ and $V_0$ through their equal $5R$ resistors must sum to zero: $\frac{V_2-V_-}{5R} + \frac{V_0-V_-}{5R} = 0$.

Solving, $V_0 = 2V_- - V_2 = 2(V_1/2) - V_2 = V_1 - V_2$, which is Option 1.

## SPEED HACK

Since the feedback and input resistors on the inverting side are equal ($5R = 5R$) and the two non-inverting-side resistors are also equal ($R = R$), the ratios match $R_2/R_1 = R_3/R_f = 1/5$, so this is a balanced difference amplifier with unity gain, $V_0 = V_1 - V_2$, directly — no node equation needed.

## WHY WRONG?

**Option 2**

formula_error: this doubles the $V_2$ contribution, applying an inconsistent gain factor to the two inputs instead of the matched $5R/5R = 1$ ratio that governs both branches of this balanced difference amplifier.

### Option 3

polarity_flip: this is the negative of the correct answer, swapping which input is inverting and which is non-inverting — in the actual circuit $V_2$ feeds the inverting input through $5R$ and $V_1$ feeds the non-inverting input through $R$, not the reverse.

### Option 4

formula_error: this doubles the entire correct expression, likely from applying the non-inverting gain factor $(1 + R_f/R_1)$ to the whole difference at once rather than first scaling $V_1$ by the $R_3/(R_2 + R_3)$ divider before combining it with the inverting-side term.

| Physics | Modern Physics | Wave-Particle Duality and the Uncertainty Principle |
|---|---|---|

**Q.19 A quantum particle of mass $10^{-20}$ kg is confined within a length of 1 nm in one-dimension. The minimum uncertainty in the measurement of velocity of the particle, in units of μm/s, rounded off to the nearest integer is:**

**[Assume the minimum uncertainty product $\Delta x \Delta p_x \approx \frac{\hbar}{2}$, use Planck's constant h = 6.64 × $10^{-34}$ J s]**

1. 2
2. 5
3. 10
4. 1

**Correct Answer: 2**

## AXIOM

The Heisenberg uncertainty relation $\Delta x \, \Delta p \approx \hbar/2$ sets a lower bound on the joint spread of position and momentum, so a confinement length $\Delta x$ fixes a minimum momentum uncertainty $\Delta p = \hbar/(2\Delta x)$, and dividing by mass converts that to a minimum velocity uncertainty.

## DEDUCTION

Compute $\hbar = h/(2\pi) = 6.64 \times 10^{-34}/6.2832 = 1.057 \times 10^{-34}$ J s.

The minimum momentum uncertainty is $\Delta p = \frac{\hbar}{2\Delta x} = \frac{1.057\times10^{-34}}{2\times10^{-9}} = 5.29 \times 10^{-26}$ kg m/s.

Dividing by the mass, $\Delta v = \frac{\Delta p}{m} = \frac{5.29\times10^{-26}}{10^{-20}} = 5.29 \times 10^{-6}$ m/s, which rounds to 5 μm/s — Option 2.

## ✕ WHY WRONG?

### Option 1

value_swap: using the full confinement length as if it were twice the correct $\Delta x$ (e.g. mistakenly doubling it to 2 nm) roughly halves the result, landing near 2–3 μm/s instead of the correct 5 μm/s.

### Option 3

formula_error: dropping the factor of 2 in the denominator and using $\Delta p = \hbar/\Delta x$ instead of the correct $\hbar/(2\Delta x)$ doubles the result to about 10.6 μm/s, rounding to 10 instead of the correct 5.

### Option 4

value_swap: a combination of rounding and estimation slips in the same calculation can compress the result down to about 1 μm/s, understating the correct minimum velocity uncertainty of 5 μm/s.

| Physics | Electricity and Magnetism | Magnetostatics — Biot-Savart and Ampere's Law |
|---|---|---|

**Q.20 A thin ring of mass m and radius R has a total charge Q distributed uniformly. The ring is rotating with a constant angular velocity ω about an axis passing through its center and perpendicular to its plane.**
**Which of the following options is correct?**

1. The magnetic moment of the ring is $\frac{Q\omega R^2}{4}$
2. The ratio of the magnetic moment to angular momentum is $\frac{Q}{2m}$
3. The magnetic moment of the ring is $\frac{Q\omega R^2}{6}$
4. The ratio of the magnetic moment to angular momentum is $\frac{Q}{m}$

**Correct Answer: 2**

## AXIOM

A ring of charge Q rotating with angular velocity ω behaves as a current loop with $I = Q\omega/(2\pi)$ (charge Q passing a fixed point once per rotation period $2\pi/\omega$), giving a magnetic moment $\mu = IA$, while its angular momentum about the same axis is $L = I_{\text{ring}}\omega = mR^2\omega$ since all the mass sits at radius R.

## DEDUCTION

The magnetic moment is $\mu = \frac{Q\omega}{2\pi} \times \pi R^2 = \frac{Q\omega R^2}{2}$.

The angular momentum is $L = mR^2\omega$, so the ratio is $\frac{\mu}{L} = \frac{Q\omega R^2/2}{mR^2\omega} = \frac{Q}{2m}$, which is Option 2.

## SPEED HACK

This ratio is a general classical result for any planar rigid body rotating with a uniform charge-to-mass ratio, not just a ring: $\mu/L = Q/(2m)$ always, independent of the exact mass and charge distribution, so the ring's specific radius and current never need to be combined explicitly.

## WHY WRONG?

### Option 1

formula_error: this under-counts the correct $\mu = Q\omega R^2/2$ by a further factor of 2, consistent with a slip such as using half the correct loop area or an extra factor of 2 in the current formula.

### Option 3

formula_error: this does not correspond to the uniform current-loop calculation for a thin ring; it is consistent with confusing the ring's moment-of-inertia weighting with a different (e.g. solid-disk) charge distribution.

### Option 4

formula_error: this omits the factor of $\frac{1}{2}$ that survives when combining $I = Q\omega/(2\pi)$ with the loop area $\pi R^2$ against $L = mR^2\omega$, a common slip where that factor is dropped instead of carried through.

| Physics | Oscillations, Waves and Optics | Simple Harmonic Motion and Superposition of Oscillations |
|---|---|---|

**Q.21 A mass attached to the bottom end of a vertical massless spring stretches the spring by Δx. The system executes oscillation with a time period T = 0.2 s. The value of Δx, in cm, rounded off to the nearest integer is:**
**[Assume the acceleration due to gravity g = 9.8 m/s²]**

1. 2
2. 3
3. 4
4. 1

**Correct Answer: 4**

## AXIOM

For a mass hanging on a vertical spring, the equilibrium stretch and the oscillation period are linked through the same spring constant, since $mg = k\Delta x$ at equilibrium and $T = 2\pi\sqrt{m/k}$ for the oscillation, so eliminating k gives $T = 2\pi\sqrt{\Delta x/g}$ directly.

## DEDUCTION

Solve for the stretch: $\Delta x = \frac{gT^2}{4\pi^2}$.

Substitute $g = 9.8\ \text{m/s}^2$ and $T = 0.2$ s: $\Delta x = \frac{9.8\times0.04}{4\pi^2} = \frac{0.392}{39.48} = 0.00993$ m, which is 0.993 cm, rounding to 1 cm — Option 4.

## ✕ WHY WRONG?

**Option 1**

process_confusion: a compounded rounding or unit-scaling slip in the same calculation can compress the result down toward 2 cm, understating the correct 1 cm answer.

**Option 2**

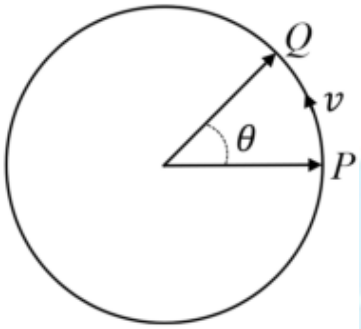
formula_error: dropping the square on π (using $4\pi$ instead of $4\pi^2$ in the denominator) inflates the result to about 0.0312 m ≈ 3 cm instead of the correct 1 cm.

**Option 3**

formula_error: forgetting the factor of 2 in the period formula (using $T = \pi\sqrt{\Delta x/g}$ instead of the correct $2\pi\sqrt{\Delta x/g}$) inflates the result to about 4 cm instead of the correct 1 cm.

| Physics | Mechanics and General Properties of Matter | Newton's Laws and Kinematics in Curvilinear Coordinates |
|---|---|---|

**Q.22 A particle is rotating along a circular path with uniform speed v, as shown in the figure. While moving from the point P to Q subtending an angle θ, the magnitude of the change in its velocity is:**




1. Zero
2. v cos θ
3. 2v sin $\frac{\theta}{2}$
4. v cos $\frac{\theta}{2}$

**Correct Answer: 3**

## AXIOM

In uniform circular motion the speed is constant but the direction rotates continuously, so the change in the velocity vector between two points separated by angle θ is found from the vector subtraction of two equal-length vectors, which by the isosceles-triangle geometry gives $|\Delta v| = 2v\sin(\theta/2)$.

## DEDUCTION

Both velocity vectors have the same magnitude $v$ but differ in direction by the same angle θ that P and Q subtend at the center, since velocity is always tangential and rotates together with the position vector.

Applying the law of cosines to the triangle formed by $\vec{v}_1, \vec{v}_2, \Delta\vec{v}$: $|\Delta v|^2 = v^2 + v^2 - 2v^2\cos\theta = 2v^2(1-\cos\theta) = 4v^2\sin^2(\theta/2)$, so $|\Delta v| = 2v\sin(\theta/2)$, which is Option 3.

## WHY WRONG?

### Option 1

reversed_relationship: this assumes velocity is unchanged simply because speed is constant, but velocity is a vector whose direction keeps changing in circular motion, so $|\Delta v|$ is generically nonzero except in the trivial limit θ=0.

### Option 2

formula_error: this expression does not vanish as θ→0 (since $\cos 0 = 1 \neq 0$), unlike the correct $2v\sin(\theta/2)$, which must go to zero as the two points coincide.

### Option 4

formula_error: this uses cosine instead of sine and drops the factor of 2 from the correct isosceles-triangle result; like Option 2, it fails to vanish as θ→0, which the true change in velocity must do.

Physics | Electricity and Magnetism | DC and AC Circuits with R, L and C

**Q.23 A series LCR circuit contains L = 175 mH, C = 62.5 μF, and R = 40 Ω and is connected to a source of voltage amplitude $E_0$ = 50 V and angular frequency ω = 400 rad/s. Which of the following statements is correct?**

1. The magnitude of the maximum voltage across the inductor is less than 50 V
2. The magnitude of the maximum voltage across the capacitor is more than 60 V
3. The magnitude of the maximum voltage across the resistor is 50 V
4. The magnitude of the maximum voltage across the capacitor equals the maximum voltage across the resistor

**Correct Answer: 4**

## AXIOM

In a series LCR circuit the same current amplitude flows through every element, so once the impedance fixes the current, each element's peak voltage is just that current times its own reactance or resistance, and any two elements with numerically equal reactance/resistance will show numerically equal peak voltages.

## DEDUCTION

Compute the reactances: $X_L = \omega L = 400 \times 0.175 = 70\ \Omega$ and $X_C = 1/(\omega C) = 1/(400 \times 62.5 \times 10^{-6}) = 40\ \Omega$.

The impedance is $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{40^2 + 30^2} = \sqrt{2500} = 50\ \Omega$, giving current amplitude $I_0 = E_0/Z = 50/50 = 1$ A.

The peak voltages are $V_L = I_0 X_L = 70$ V, $V_C = I_0 X_C = 40$ V, and $V_R = I_0 R = 40$ V — since $X_C = R = 40\ \Omega$ exactly, $V_C = V_R$, which is Option 4.

## ✕ WHY WRONG?

### Option 1

reversed_relationship: the actual peak inductor voltage is $V_L = 70$ V, which is greater than 50 V, not less — this option reverses the comparison.

### Option 2

value_swap: the actual peak capacitor voltage is $V_C = 40$ V, well under 60 V; this likely comes from confusing $X_C$ with the numerically larger $X_L = 70\ \Omega$.

### Option 3

value_swap: the actual peak resistor voltage is $V_R = 40$ V, not 50 V; this likely comes from mistaking the source amplitude $E_0 = 50$ V for the resistor's own voltage, which would only hold exactly at resonance ($X_L = X_C$), a condition not met here since $X_L = 70 \neq X_C = 40$.

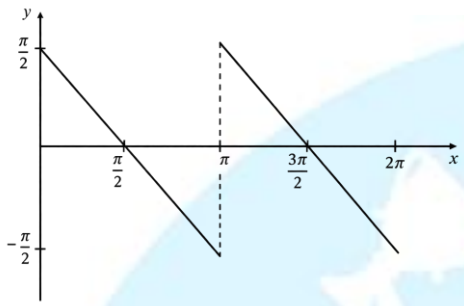
| Physics | Mathematical Methods | Series Expansions — Taylor and Fourier Series |
|---|---|---|

**Q.24 A piecewise regular function f(x) is shown in the figure. It is expanded in Fourier series given by**

**f(x) = $\frac{a_0}{2}$ + $\Sigma_{n=1}^{\infty}$ $a_n$ cos(n x) + $\Sigma_{n=1}^{\infty}$ $b_n$ sin(nx) ,**

**where $a_0$, $a_n$, $b_n$'s are the Fourier coefficients.**

**Which of the following options is correct?**




1. $a_0 = 3\pi$
2. All $a_n$ are zero
3. All $b_n$ are zero
4. $a_0 = \frac{3\pi}{2}$

**Correct Answer: 2**

## AXIOM

A function's Fourier cosine coefficients (including $a_0$) all vanish exactly when the function is odd, since cosine is an even basis function and its overlap integral with an odd function is identically zero.

## DEDUCTION

Within one period the function is $f(x) = \pi/2 - x$ on $[0, \pi)$; using the period-π repetition, $f(-x) = f(\pi - x) = \pi/2 - (\pi - x) = x - \pi/2 = -f(x)$, so $f$ is an odd function under its periodic extension.

Since $f$ is odd, every cosine-type coefficient — $a_0$ and every $a_n$ — vanishes identically, which is Option 2.

## SPEED HACK

A direct check on $a_1$ confirms this without appealing to the general symmetry argument: splitting the integral $\int_0^{2\pi} f(x)\cos x\, dx$ into the two π-length ramps gives $+2$ from the first segment and $-2$ from the second, cancelling to exactly zero.

## WHY WRONG?

**Option 1**

value_swap: the correct $a_0$ is exactly zero, since the linear ramp from $+\pi/2$ to $-\pi/2$ in each period averages to zero; a nonzero value here likely comes from integrating over a mismatched or partial range instead of the full period.

### Option 3

reversed_relationship: vanishing $b_n$ coefficients is the signature of an EVEN function, but this sawtooth is actually ODD (as shown by $f(-x) = -f(x)$ under its periodic extension), so it is the $a_n$ coefficients, not the $b_n$, that vanish.

### Option 4

value_swap: like Option 1, this proposes a nonzero DC value where the true average is zero; this specific value may come from scaling the curve's peak height ($\pi/2$) rather than correctly averaging the ramp over a full period.

| Physics | Mathematical Methods | Series Expansions — Taylor and Fourier Series |
|---|---|---|

**Q.25 Using Taylor series, expand f(x) = $x^3 - \frac{1}{8}$ around $x_0 = 1$ up to second order in x. The coefficient of x is:**

1. −2
2. 3
3. 2
4. −3

**Correct Answer: 4**

## AXIOM

When a Taylor polynomial built around $x_0$ is re-expanded as an ordinary polynomial in x, the coefficient of $x^1$ gets contributions from every retained order, since expanding $(x - x_0)^2$ itself produces its own linear-in-x term.

## DEDUCTION

For $f(x) = x^3 - C$ (the constant C does not affect any derivative), $f'(x) = 3x^2$ and $f''(x) = 6x$, so at $x_0 = 1$: $f'(1) = 3$ and $f''(1)/2 = 3$.

The 2nd-order Taylor polynomial is $f(1) + 3(x - 1) + 3(x - 1)^2$; expanding $3(x - 1)^2 = 3x^2 - 6x + 3$ and combining with the linear term $3(x - 1) = 3x - 3$ gives a net x-coefficient of $3 - 6 = -3$, which is Option 4.

## ✕ WHY WRONG?

### Option 1

process_confusion: a sign or combination slip in merging the linear term's own coefficient with the quadratic term's expanded linear contribution can land on -2 instead of the correct -3.

### Option 2

process_confusion: reading off $f'(x_0) = 3$ directly as the coefficient of x, without expanding $(x - 1)^2$ back into powers of x and picking up its own $-6x$ contribution, misses the correction that turns 3 into -3.

### Option 3

formula_error: the same direct-reading mistake as the previous option, compounded with a power-rule slip that computes the derivative of $x^3$ as $2x^2$ instead of the correct $3x^2$, giving $f'(1) = 2$ instead of 3.

| Physics | Mathematical Methods | Matrices, Determinants and Complex Numbers |
|---|---|---|

**Q.26 Consider a 2 × 2 matrix A. The determinant of A is −1 and the trace(A) = 1. Which of the following options gives the eigenvalues of A?**

1. 1, 0
2. $\frac{1+\sqrt{5}}{2}$ , $\frac{1-\sqrt{5}}{2}$
3. $\frac{1+\sqrt{5}}{4}$ , $\frac{1-\sqrt{5}}{4}$
4. $\frac{\sqrt{5}}{2}$ , $1-\frac{\sqrt{5}}{2}$

**Correct Answer: 2**

## AXIOM

For any 2×2 matrix the eigenvalues are the roots of the characteristic equation $\lambda^2 - (\text{trace})\lambda + \det = 0$, so the trace and determinant alone determine both eigenvalues without needing the individual matrix entries.

## DEDUCTION

Substitute trace $= 1$ and $\det = -1$: $\lambda^2 - \lambda - 1 = 0$.

Solve with the quadratic formula: $\lambda = \frac{1\pm\sqrt{1+4}}{2} = \frac{1\pm\sqrt{5}}{2}$, which is Option 2.

## ✕ WHY WRONG?

### Option 1

value_swap: these would be the eigenvalues if the determinant were 0 instead of the given -1; this ignores the actual nonzero determinant.

### Option 3

formula_error: this uses 4 in the denominator instead of the correct 2 from the quadratic formula $\lambda = (T \pm \sqrt{T^2 - 4D})/2$, an arithmetic slip in applying the standard 2×2 eigenvalue formula.

### Option 4

partial_truth: this drops the leading 1 from one of the two eigenvalue expressions, an incomplete combination of the trace term with the ± square-root term.

| Physics | Mathematical Methods | Vector Calculus and Integral Theorems |
|---|---|---|

**Q.27 Consider the following vector $\vec{V} = x\,y\,z\,\hat{k}$ .**

**Which of the following statements is correct for the resultant vector $\vec{\nabla} \times \vec{\nabla} \times \vec{V}$?**

1. It lies in xy – plane
2. It lies in yz – plane
3. It lies in xz - plane
4. It is along the x direction

**Correct Answer: 1**

## AXIOM

The vector identity $\nabla \times (\nabla \times \vec{V}) = \nabla(\nabla \cdot \vec{V}) - \nabla^2\vec{V}$ turns a double curl into a divergence-gradient term minus a Laplacian, both of which are usually far easier to evaluate directly than two nested curls.

## DEDUCTION

With $\vec{V} = (0,0,xyz)$, $\nabla \cdot \vec{V} = \partial(xyz)/\partial\, z = xy$, so $\nabla(\nabla \cdot \vec{V}) = (y, x, 0)$.

Since $xyz$ is linear in each variable separately, every second partial derivative vanishes, so $\nabla^2\vec{V} = (0,0,0)$.

Therefore $\nabla \times \nabla \times \vec{V} = (y, x, 0) - (0,0,0) = (y, x, 0)$, which has zero z-component and so lies entirely in the xy-plane — Option 1.

## SPEED HACK

A direct double-curl computation confirms this without the identity: $\nabla \times \vec{V} = (xz, -yz, 0)$ first, then $\nabla \times (xz, -yz, 0) = (y, x, 0)$ — the same result, obtained by brute-force differentiation instead of the identity.

## WHY WRONG?

### Option 2

reversed_relationship: the resultant $(y, x, 0)$ has a nonzero x-component, so it cannot lie in the yz-plane, which would require the x-component to vanish.

### Option 3

reversed_relationship: the resultant has a nonzero y-component, so it cannot lie in the xz-plane, which would require the y-component to vanish.

### Option 4

partial_truth: the resultant does have a nonzero x-component, but it carries an equal-magnitude y-component too; lying purely along the x-direction would require both the y- and z-components to vanish, but only the z-component does.

| Physics | Oscillations, Waves and Optics | Interference and Optical Path Retardation |
|---|---|---|

**Q.28 Two coherent plane waves having wavelength λ and wavevectors $\vec{k}_1 = \frac{2\pi}{\lambda}\left(\frac{1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}\right)$ and $\vec{k}_2 = \frac{2\pi}{\lambda}\left(-\frac{1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}\right)$, are incident on a screen placed on the xz - plane. The overlap of the waves on the plane of the screen produces interference fringes. The fringe width (center-to-center spacing of bright fringes) will be:**

1. $\frac{\lambda}{2}$
2. 2λ
3. $\frac{\sqrt{3}\lambda}{2}$
4. λ

**Correct Answer: 4**

## AXIOM

For two coherent plane waves crossing at a screen, the interference fringe spacing is set entirely by the difference between the components of their wavevectors along the screen, $\Lambda = 2\pi/\Delta k_x$, independent of the components perpendicular to the screen.

## DEDUCTION

The x-components of the two wavevectors are $k_{1x} = (2\pi/\lambda)(1/2)$ and $k_{2x} = (2\pi/\lambda)(-1/2)$, so $\Delta k_x = k_{1x} - k_{2x} = (2\pi/\lambda)(1) = 2\pi/\lambda$.

The fringe spacing is $\Lambda = 2\pi/\Delta k_x = 2\pi/(2\pi/\lambda) = \lambda$, which is Option 4.

## SPEED HACK

Each beam makes a half-angle $\theta = 30°$ with the shared bisector, since $\tan\theta = (1/2)/(\sqrt{3}/2) = 1/\sqrt{3}$; plugging directly into the standard two-beam formula $\Lambda = \lambda/(2\sin\theta) = \lambda/(2 \times 0.5) = \lambda$ confirms the result without computing $\Delta k_x$ explicitly.

## WHY WRONG?

### Option 1

formula_error: this comes from using the FULL angle between the two beams (60°) instead of the correct half-angle (30°) in the fringe-spacing formula, giving roughly λ/(2\sin60°)≈λ/2 instead of the correct λ.

### Option 2

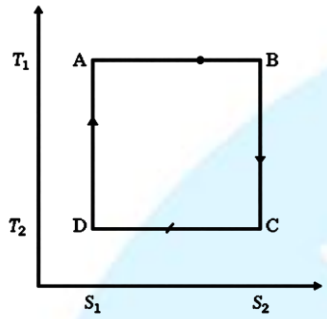
formula_error: this omits the factor of 2 in the fringe-spacing denominator, computing λ/\sin30°=2λ instead of the correct λ/(2\sin30°)=λ.

### Option 3

process_confusion: a compounded slip mixing a partial angle correction with a rounding error lands between the λ/2 and 2λ distractors, at 3λ/2.

| Physics | Kinetic Theory, Thermodynamics | Second Law, Entropy and the Carnot Cycle |
|---|---|---|

**Q.29 Consider a Carnot cycle as shown in the figure. The ideal gas expanded from volume $V_1$ to $V_2$ along the path AB. During this process, the temperature $T_1$ is constant and the entropy changes from $S_1$ to $S_2$. $T_2$ is the temperature of the cold bath.**
**Figure: temperature–entropy (T–S) diagram; rectangle A→B at $T_1$ ($S_1$ to $S_2$), B→C at $S_2$ ($T_1$ to $T_2$), C→D at $T_2$ ($S_2$ to $S_1$), D→A at $S_1$ ($T_2$ to $T_1$).**
**Which of the following statements is correct?**




1. Work done along the path AB is $W_{AB}$ = −N $RT_1$ ln $\frac{V_2}{V_1}$
2. Work done along the path BC is $W_{BC}$ = 0
3. Work done along the path CD is $W_{CD}$ = −N $RT_1$ ln $\frac{V_2}{V_1}$
4. Work done along the path DA is $W_{DA}$ = 0

**Correct Answer: 1**

## AXIOM

In the work-done-ON-the-gas sign convention used here, an isothermal EXPANSION (the gas doing positive work on its surroundings) corresponds to NEGATIVE work done on the gas, $W_{on} = -NRT\ln(V_2/V_1)$, while an adiabatic leg exchanges no heat but still does nonzero work whenever its temperature changes.

## DEDUCTION

Path AB is the isothermal expansion at $T_1$ from $V_1$ to $V_2$, so under the work-done-on-the-gas convention $W_{AB} = -NRT_1\ln(V_2/V_1)$, matching Option 1 exactly.

Paths BC and DA are the two adiabatic legs, along which the temperature changes ($T_1 \rightarrow T_2$ and back), so their work is generally nonzero even though no heat is exchanged, ruling out Options 2 and 4.

Path CD is the isothermal leg at $T_2$, not $T_1$, so any formula for $W_{CD}$ that uses $T_1$ applies the wrong bath temperature, ruling out Option 3 and confirming Option 1 as correct.

## ✕ WHY WRONG?

**Option 2**

reversed_relationship: this conflates ‘adiabatic’ (no heat exchanged) with ‘no work done’; along BC the temperature changes from $T_1$ to $T_2$, so the internal energy changes and nonzero work is done even without any heat exchange.

### Option 3

value_swap: path CD is the isothermal leg at $T_2$, not $T_1$; using $T_1$ in this formula applies the wrong bath temperature to the wrong leg of the cycle.

### Option 4

reversed_relationship: the same adiabatic-means-zero-work misconception as Option 2 — leg DA also carries nonzero work as the temperature returns from $T_2$ back to $T_1$.

| Physics | Mechanics and General Properties of Matter | Conservation Laws, System of Particles and Collisions |
|---|---|---|

**Q.30 The speed of an object of mass 20 kg increases from 2 m/s to 6 m/s in 10 s. The power required, in Watts, is:**

1. 64
2. 54
3. 32
4. 72

**Correct Answer: 3**

## AXIOM

Average power delivered over an interval equals the total work done divided by the total time, and for a particle accelerated by a net force the work-energy theorem makes that total work exactly the change in kinetic energy, $\bar{P} = \Delta KE/\Delta t$.

## DEDUCTION

Compute the kinetic energies: $KE_i = \frac{1}{2}(20)(2)^2 = 40$ J and $KE_f = \frac{1}{2}(20)(6)^2 = 360$ J, so $\Delta KE = 320$ J.

Dividing by the elapsed time, $\bar{P} = 320/10 = 32$ W, which is Option 3.

## SPEED HACK

Since the acceleration is constant, $a = (6-2)/10 = 0.4$ m/s$^2$ and $F = ma = 8$ N; integrating $P(t) = Fv(t)$ over the linear velocity ramp gives the same average power directly, without separately computing the two kinetic energies.

## WHY WRONG?

### Option 1

formula_error: this is double the correct 32 W; a common source is forgetting the leading $\frac{1}{2}$ in the kinetic-energy formula, effectively computing ΔKE as $m(v_f^2 - v_i^2)$ instead of $\frac{1}{2}m(v_f^2 - v_i^2)$.

### Option 2

process_confusion: an arithmetic slip in combining the given speeds and mass, such as mishandling the velocity-squared difference, can land on 54 W instead of the correct 32 W.

### Option 4

formula_error: using the force times the FINAL velocity instead of correctly averaging power over the whole interval overstates the result, landing near 72 W instead of the correct average of 32 W.

| Physics | Oscillations, Waves and Optics | Wave Motion — Standing Waves, Energy and Velocities |
|---|---|---|

**Q.31 The displacement of a vibrating string of finite length stretched along the x −axis is given by**

**$y(x, t) = 2A \cos(kx) \sin(\omega t)$**

**where, A is the amplitude, $k = \frac{2\pi}{\lambda}$ is the wavenumber and ω is the angular frequency.**

**Which of the following statements is/are correct for the standing wave?**

1. The nodes are at $x = n\frac{\lambda}{2}$, where n is an integer
2. The antinodes are at $x = (n + \frac{1}{2})\frac{\lambda}{2}$, where n is an integer
3. The nodes are at $x = (n + \frac{1}{2})\frac{\lambda}{2}$, where n is an integer
4. The antinodes are at $x = n\frac{\lambda}{2}$, where n is an integer

**Correct Answer: 3, 4**

## AXIOM

For a standing wave $y(x,t) = 2A\cos(kx)\sin(\omega t)$, the spatial factor $\cos(kx)$ alone sets where the string never moves (nodes, where this factor is zero) and where it swings with maximum amplitude (antinodes, where this factor is ±1), independent of time.

## DEDUCTION

Nodes require $\cos(kx) = 0$, i.e. $kx = (n + 1/2)\pi$, so $x = (n + 1/2)\pi/k = (n + 1/2)(\lambda/2)$ using $k = 2\pi/\lambda$.

Antinodes require $\cos(kx) = \pm 1$, i.e. $kx = n\pi$, so $x = n\pi/k = n(\lambda/2)$.

These match the node formula in Option 3 and the antinode formula in Option 4, so both Options 3 and 4 are correct.

## ✕ WHY WRONG?

### Option 1

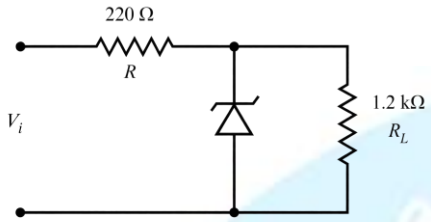
reversed_relationship: $x = n\lambda/2$ is actually where the ANTINODES sit (cos(kx)=±1 there), not the nodes; this option swaps the node and antinode locations.

### Option 2

reversed_relationship: $x = (n + 1/2)\lambda/2$ is actually where the NODES sit (cos(kx)=0 there), not the antinodes; this option also swaps the two locations.

| Physics | Solid State Physics, Devices and Electronics | p-n Junction and Zener Diodes |
|---|---|---|

**Q.32 In the circuit shown in the figure, the Zener voltage $V_z$ is 20 V and the maximum Zener current $I_{ZM}$ is 60 mA.**
**For what value(s) of the input voltage $V_i$ , the Zener diode is in ON state?**
**Figure labels: input $V_i$; series resistor R = 220 Ω; Zener diode in shunt; load $R_L$ = 1.2 kΩ.**




1. 20 V
2. 25 V
3. 35 V
4. 40 V

**Correct Answer: 2, 3**

## AXIOM

A Zener diode regulates (is genuinely ON) only across a bounded range of input voltage: below a minimum $V_{i,min}$ there isn't enough voltage to reach breakdown at all, and above a maximum $V_{i,max}$ the current through it would exceed its rated $I_{ZM}$, so both bounds must be found from $V_i = V_Z + I_R \times R$ with $I_R = I_{RL} + I_Z$.

## DEDUCTION

At the LOWER bound, $I_Z \approx 0$ so all current goes through $R_L$: $I_{RL} = V_Z/R_L = 20/1200 = 16.7$ mA, giving $V_{i,min} = V_Z + I_{RL}R = 20 + 16.7\text{mA} \times 220\Omega \approx 23.7$ V.

At the UPPER bound, $I_Z = I_{ZM} = 60$ mA, so $I_R = 16.7 + 60 = 76.7$ mA, giving $V_{i,max} = 20 + 76.7\text{mA} \times 220\Omega \approx 36.9$ V.

Only 25 V and 35 V fall inside this valid range [23.7, 36.9] V, so Option 2 and Option 3 are both correct.

## ✕ WHY WRONG?

### Option 1

value_swap: 20 V is below the 23.7 V minimum needed to even reach Zener breakdown, since some extra drop across R is required beyond the 20 V Zener voltage itself before any current can flow through the diode.

### Option 4

value_swap: 40 V pushes the Zener current to about 74 mA (computed from the full 20 V drop across R at this input), which exceeds the rated maximum $I_{ZM} = 60$ mA, taking the diode outside its valid regulating range.

| Physics | Mechanics and General Properties of Matter | Rotating Frames — Centrifugal and Coriolis Forces |
|---|---|---|

**Q.33 Consider a particle of mass m in a rotating frame. The force acting on the particle is expressed as $\vec{F} = (F_1 + F_2)\,\hat{e}_r + (F_3 + F_4)\,\hat{e}_\theta$, where $\hat{e}_r$ and $\hat{e}_\theta$ are the radial and angular unit vectors, respectively, and $F_1 = m\ddot{r}$, $F_2 = -mr\dot{\theta}^2$, $F_3 = mr\ddot{\theta}$, $F_4 = 2m\dot{r}\dot{\theta}$.**
**Which of the following statements is/are correct?**

1. $\hat{e}_r$ and $\hat{e}_\theta$ are not constant unit vectors
2. $F_1$ and $F_3$ are fictitious forces
3. $F_2$ and $F_4$ are fictitious forces
4. $F_1$ and $F_2$ are zero in uniform circular motion

**Correct Answer: 1, 3**

## AXIOM

Polar unit vectors change direction as a particle moves, unlike fixed Cartesian axes, and this is exactly what generates the extra terms in the polar-coordinate acceleration formula; by long-standing convention, the terms that have the same mathematical form as the centrifugal force ($-mr\dot{\theta}^2$) and the Coriolis force ($2m\dot{r}\dot{\theta}$) are the ones identified as the fictitious-force terms.

## DEDUCTION

Since $\hat{e}_r$ and $\hat{e}_\theta$ rotate together with the particle's angular position, they are genuinely not constant, confirming Option 1.

$F_2 = -mr\dot{\theta}^2$ has the form of the centrifugal term and $F_4 = 2m\dot{r}\dot{\theta}$ has the form of the Coriolis term, so these two — not $F_1$ and $F_3$, which are the genuine radial and tangential acceleration terms — are conventionally identified as the fictitious-force contributions, confirming Option 1 and Option 3 together.

## ✕ WHY WRONG?

### Option 2

reversed_relationship: $F_1 = m\ddot{r}$ and $F_3 = mr\ddot{\theta}$ are the genuine radial and tangential acceleration terms, not the fictitious ones; this option names the wrong pair.

### Option 4

partial_truth: in uniform circular motion $r$ and $\dot{\theta}$ are both constant, so $F_1 = m\ddot{r} = 0$ is indeed zero, but $F_2 = -mr\dot{\theta}^2$ is exactly the nonzero centripetal force that keeps the particle on its circular path — it does not vanish.

| Physics | Modern Physics | Special Relativity — Lorentz Transformations and Kinematics |
|---|---|---|

**Q.34 The events are represented by coordinates (ct, x, y, z) in some frame of reference. Which of the following statements is/are correct?**
**[c is the speed of light]**

1. Events (1, 0, −10, 1) and (−1, 1, −9, 1) are space-like separated
2. Events (−1, 0, −9, 1) and (1, 1, −10, 1) are space-like separated
3. Events (−10, 0, 1, −1) and (−9, 1, −1, −1) are light-like separated
4. Events (9, −1, 1, −1) and (−10, 1, 0, −1) are time-like separated

**Correct Answer: 4**

## AXIOM

With coordinates already given as $(ct, x, y, z)$, the spacetime interval between two events is $\Delta s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - \Delta(ct)^2$, and its sign alone classifies the pair: positive is space-like, negative is time-like, and exactly zero is light-like.

## DEDUCTION

For Option 1, $(1,0,-10,1)$ and $(-1,1,-9,1)$ give $\Delta(ct) = -2, \Delta x = 1, \Delta y = 1, \Delta z = 0$, so $\Delta s^2 = 1 + 1 + 0 - 4 = -2$ — time-like, not space-like.

For Option 2, $(-1,0,-9,1)$ and $(1,1,-10,1)$ give $\Delta(ct) = 2, \Delta x = 1, \Delta y = -1, \Delta z = 0$, so $\Delta s^2 = 1 + 1 + 0 - 4 = -2$ — time-like, not space-like.

For Option 3, $(-10,0,1,-1)$ and $(-9,1,-1,-1)$ give $\Delta(ct) = 1, \Delta x = 1, \Delta y = -2, \Delta z = 0$, so $\Delta s^2 = 1 + 4 + 0 - 1 = 4$ — space-like, not light-like.

For Option 4, $(9,-1,1,-1)$ and $(-10,1,0,-1)$ give $\Delta(ct) = -19, \Delta x = 2, \Delta y = -1, \Delta z = 0$, so $\Delta s^2 = 4 + 1 + 0 - 361 = -356$, negative and therefore genuinely time-like, confirming Option 4 alone.

## ✕ WHY WRONG?

### Option 1

reversed_relationship: $\Delta s^2 = -2$ is negative, which is the signature of time-like separation, not space-like.

### Option 2

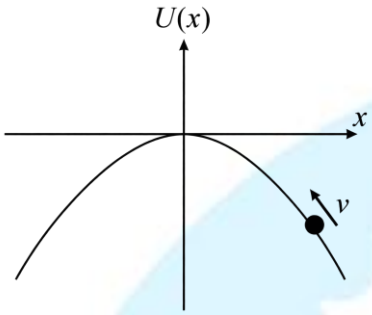
reversed_relationship: $\Delta s^2 = -2$ is negative here as well, again time-like rather than the claimed space-like separation.

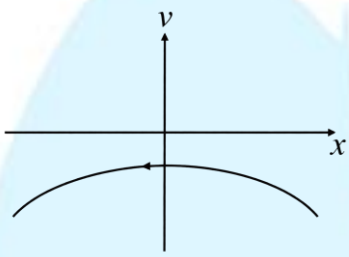
### Option 3

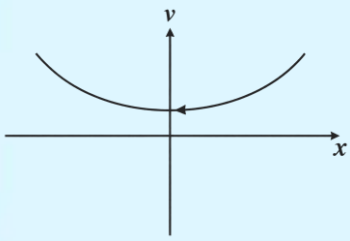
value_swap: $\Delta s^2 = 4$ is positive (space-like), not zero, so these events are not light-like separated as claimed.

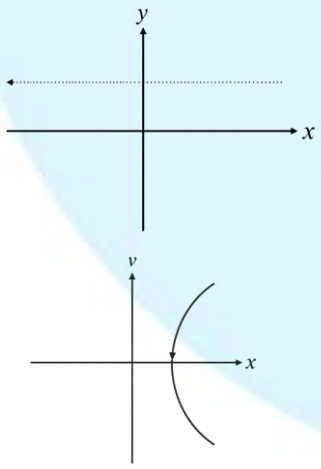
| Physics | Mechanics and General Properties of Matter | Conservation Laws, System of Particles and Collisions |
|---|---|---|

**Q.35 A ball of mass m climbs the potential $U(x) = -\frac{1}{2}kx^2$, as shown in the figure. Assuming that the total energy of the system $E = \frac{1}{2}mv^2 + U(x)$ is conserved, which of the following correctly describe(s) the plot of velocity (v) as a function of position (x) of the system for E < 0?**




1. 


2. 


3. 


4.

**Correct Answer: 4**

## AXIOM

For $U(x) = -\frac{1}{2}kx^2$ (an inverted, downward-opening potential peaking at the origin), energy conservation gives $v^2 = \frac{2}{m}(E - U(x)) = \frac{k}{m}x^2 + \frac{2E}{m}$, and for $E < 0$ this is only non-negative (a physically allowed speed) once $x$ is far enough from the origin.

## DEDUCTION

Requiring $v^2 \geq 0$ forces $x^2 \geq 2|E|/k$, so the particle can exist only in the region $|x| \geq x_{min}$ for some $x_{min} = \sqrt{2|E|/k} > 0$; the region near the origin is classically forbidden.

Rearranging $v^2 = (k/m)x^2 - 2|E|/m$ as $(k/m)x^2 - v^2 = 2|E|/m$ is the equation of a hyperbola opening along the x-axis, with $v = 0$ exactly at $x = \pm x_{min}$ and $v$ growing as $|x|$ increases beyond that turning point.

The stem shows the ball on the falling side to the right of the peak, so the physically relevant branch is the single sideways-opening arc on the positive-x side, matching Option 4.

## ✕ WHY WRONG?

### Option 1

reversed_relationship: a downward hump plotted directly as v(x) would suggest the particle CAN exist near the origin with reduced speed there, the opposite of the classically forbidden zone that E<0 actually creates near x=0.

### Option 2

partial_truth: an upward trough resembles the correct $v^2$-vs-x relationship, but the question asks for v itself (not $v^2$) plotted against x, and this shape wrongly implies a valid (real) v value exists all the way down to x=0, which E<0 forbids.

### Option 3

process_confusion: two isolated dashes carry no functional relationship between v and x at all, and cannot represent the continuous growth of speed away from the turning point that energy conservation requires.

Physics | Electricity and Magnetism | DC and AC Circuits with R, L and C

**Q.36 An LC oscillator circuit contains a capacitor of 4.0 μF. The maximum potential difference across the capacitor is 2 V and the maximum current through the inductor is 80 mA.**

**Which of the following statements is/are correct?**

1. The value of inductance is 2.5 mH
2. The frequency of oscillator is 3.2 kHz
3. The time for the charge in the capacitor to rise from zero to the maximum is nearly 0.157 ms
4. The maximum potential difference across the inductor is 2.0 V

**Correct Answer: 1, 3, 4**

## AXIOM

In an ideal (lossless) LC oscillator, the peak energy stored is the same whether it momentarily sits entirely in the capacitor or entirely in the inductor, $\frac{1}{2}CV_{max}^2 = \frac{1}{2}LI_{max}^2$, and Kirchhoff's voltage law around the single L–C loop forces $V_L(t) = -V_C(t)$ at every instant.

## DEDUCTION

Solve for the inductance: $L = \frac{CV_{max}^2}{I_{max}^2} = \frac{4\times10^{-6}\times2^2}{(0.08)^2} = \frac{1.6\times10^{-5}}{6.4\times10^{-3}} = 2.5 \times 10^{-3}$ H, confirming Option 1.

The oscillation period is $T = 2\pi\sqrt{LC} = 2\pi\sqrt{2.5 \times 10^{-3} \times 4 \times 10^{-6}} = 2\pi \times 10^{-4} \approx 0.628$ ms; the charge rises from zero to maximum in a quarter period, $T/4 \approx 0.157$ ms, confirming Option 3.

Since $V_L(t) = -V_C(t)$ at every instant for this simple loop, their peak MAGNITUDES are equal, so the maximum inductor voltage is also 2.0 V, confirming Option 1, Option 3, and Option 4 together.

## ✕ WHY WRONG?

**Option 2**

formula_error: the correct frequency is $f = 1/(2\pi\sqrt{LC}) \approx 1.59$ kHz; using π instead of 2π in the denominator (dropping the factor of 2) inflates this to about 3.2 kHz.

| Physics | Kinetic Theory, Thermodynamics | Second Law, Entropy and the Carnot Cycle |
|---|---|---|

**Q.37 Consider an ideal gas of entropy S, molar specific heat $C_v$, pressure P and volume V. Which of the following options is/are true?**

1. Internal energy of the gas depends only on the temperature
2. S ∝ ln V , assuming $C_v$ is constant
3. S = 0
4. Internal energy of the gas depends both on temperature and pressure

**Correct Answer: 1, 2**

## AXIOM

For an ideal gas, internal energy is a function of temperature alone (Joule's law, since there are no intermolecular interactions to depend on volume or pressure), and the entropy has the general form $S = C_v \ln T + R \ln V + \text{const}$, which always carries a term proportional to $\ln V$ regardless of whether $C_v$ is itself temperature-dependent.

## DEDUCTION

Internal energy depending only on temperature (Option 1) is the defining ideal-gas property, confirming Option 1 and simultaneously ruling out any dependence on pressure.

The general ideal-gas entropy formula always contains an $R \ln V$ term, so $S \propto \ln V$ (holding the other variables fixed) is a standard, correct relation, confirming Option 2 as well, so Option 1 and Option 2 are both correct.

## ✕ WHY WRONG?

### Option 3

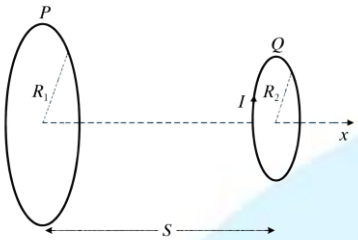
partial_truth: entropy is not generically zero for an arbitrary ideal-gas state; this claim has no basis and is not implied by anything in the setup.

### Option 4

reversed_relationship: this directly contradicts the defining ideal-gas property that internal energy depends on temperature alone, with no pressure dependence.

| Physics | Electricity and Magnetism | Electromagnetic Induction and Inductance |
|---|---|---|

**Q.38 Two small circular copper loops P and Q of radii $R_1$ and $R_2$, respectively, are coaxially placed along the x-axis as shown in the figure. The loops are at a distance s apart, where s >> $R_1$, $R_2$. The loop Q carrying a steady current I is moving away along x-axis with a speed v. Which of the following statements is/are correct?**




1. The magnetic field at the center of P is $\propto \frac{1}{s^2}$
2. The magnetic flux through the loop P is $\propto \frac{1}{s^2}$
3. The emf induced in the loop P is $\propto \frac{1}{s^4}$
4. The emf induced in the loop P is $\propto v^2$

**Correct Answer: 3**

## AXIOM

In the far-field (dipole) approximation valid for $s \gg R_1, R_2$, the on-axis magnetic field of a current loop falls off as $1/s^3$ (not $1/s^2$, which would be a monopole-like falloff), so the flux through a second coaxial loop — and hence the induced emf as that loop recedes at constant speed $v$ — inherits and steepens that same power-law dependence.

## DEDUCTION

The on-axis dipole field is $B \propto 1/s^3$, so the flux through the fixed-area loop P is $\Phi \propto 1/s^3$ as well, ruling out the $1/s^2$ scaling claimed in Options 1 and 2.

With $\Phi = k/s^3$ for some constant k, the induced emf is $|\text{emf}| = |d\Phi/dt| = |d\Phi/ds| \times v = (3k/s^4)v$, which is proportional to $1/s^4$ and only LINEAR in $v$, confirming Option 3 and ruling out the $v^2$ dependence in Option 4.

## ✕ WHY WRONG?

### Option 1

formula_error: the on-axis field of a current loop (a magnetic dipole in the far field) falls off as $1/s^3$, not $1/s^2$; the $1/s^2$ falloff would only apply to a monopole-like source, which a current loop is not.

### Option 2

formula_error: since $B \propto 1/s^3$ and the loop's own area does not depend on $s$, the flux inherits the same $1/s^3$ scaling, not $1/s^2$.

### Option 4

reversed_relationship: differentiating $\Phi(s(t))$ with respect to time brings down exactly one factor of $ds/dt = v$, so the emf is linear in $v$, not quadratic.

| Physics | Electricity and Magnetism | Conductors, Capacitors and Dielectrics |
|---|---|---|

**Q.39 A charge Q is distributed uniformly on the surface of a sphere of radius R. It is placed inside a concentric conducting hollow sphere of radius 2R. The outer sphere is earthed.**
**Which of the following statements is/are correct?**

1. The charge on the inner surface of the outer sphere is −Q
2. The flux through a closed surface through the material of the outer sphere is $\frac{Q}{\varepsilon_0}$
3. The charge on the outer surface of the outer sphere is zero
4. The potential at a radial distance r between the two spheres, R < r < 2R, is $\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$

**Correct Answer: 1, 3**

## AXIOM

A conductor surrounding an enclosed charge always develops exactly the opposite charge on its facing (inner) surface to keep the field zero within its own bulk, and an EARTHED conductor additionally has zero charge on its outer surface, since the ground supplies whatever is needed to hold the conductor at zero potential without requiring any extra charge to sit on the outside.

## DEDUCTION

The inner surface of the outer sphere must carry exactly $-Q$ to cancel the field inside the conducting shell's material, confirming Option 1.

A Gaussian surface drawn strictly within the conducting material of the outer sphere encloses both the original $+Q$ and the induced $-Q$ on its own inner surface, a net enclosed charge of zero, so the flux through it is zero — not $Q/\varepsilon_0$ — ruling out Option 2.

Because the sphere is earthed, no charge is needed on its outer surface to maintain zero potential there, confirming Option 3, so Option 1 and Option 3 are both correct.

## ✕ WHY WRONG?

### Option 2

reversed_relationship: a Gaussian surface drawn inside the outer sphere's own conducting material encloses the induced $-Q$ along with the original $+Q$, for zero net enclosed charge and hence zero flux, not $Q/\varepsilon_0$.

### Option 4

formula_error: the potential in this region must satisfy the earthed boundary condition $V(2R) = 0$, giving $V(r) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{2R}\right)$; this option drops the constant $-Q/(8\pi\varepsilon_0 R)$ term needed to make the potential vanish at $r = 2R$, leaving only the bare point-charge form.

| Physics | Kinetic Theory, Thermodynamics | Phase Transitions and the Clausius-Clapeyron Equation |
|---|---|---|

**Q.40 Which of the following statements is/are true for a first order phase transition? [$C_p$ is the molar heat capacity, $T_c$ is the critical temperature and S is the entropy]**

1. At the transition point $C_p \rightarrow \infty$
2. The derivative of the Gibbs function with respect to pressure changes continuously across the phase transition
3. The two thermodynamic states between which the transition takes place are distinct
4. Entropy changes discontinuously with temperature at $T_c$

**Correct Answer: 1, 3, 4**

## AXIOM

By the Ehrenfest classification, a first-order phase transition is exactly the one where the FIRST derivatives of the Gibbs free energy — entropy $S = -\partial G/\partial T$ and volume $V = \partial G/\partial P$ — jump discontinuously between two genuinely distinct phases, with the latent heat showing up as a delta-function spike in $C_p$ exactly at the transition.

## DEDUCTION

The discontinuous jump in entropy at $T_c$ (Option 4) is precisely the defining first-order signature, and since latent heat is absorbed at that fixed temperature, $C_p$ formally diverges there too (Option 1).

The two phases connected by the transition are by definition distinct thermodynamic states (Option 3), while the pressure-derivative of $G$ (i.e. the volume) is exactly the quantity that jumps discontinuously, not continuously, ruling out Option 2 and confirming Option 1, Option 3, and Option 4 together.

## ✕ WHY WRONG?

**Option 2**

reversed_relationship: the volume $\partial G/\partial P$ is precisely the first derivative that changes DISCONTINUOUSLY at a first-order transition (e.g. the abrupt density change on melting), not continuously as this option claims.

| Physics | Oscillations, Waves and Optics | Diffraction, Gratings and Resolving Power |
|---|---|---|

**Q.41 Considering the diameter of the pupil of a human eye to be 2 mm, the angular resolution of the eye at a wavelength of 500 nm, in minute of arc, is ___ .**
**(Rounded off to two decimal places)**

**Correct Answer: 1.05**

## AXIOM

The Rayleigh criterion sets the minimum angular separation a circular aperture can resolve, $\theta_{min} = 1.22\ \lambda/D$, where D is the aperture diameter and λ the wavelength.

## DEDUCTION

Substitute $\lambda = 500 \times 10^{-9}$ m and $D = 2 \times 10^{-3}$ m: $\theta_{min} = 1.22 \times 500 \times 10^{-9}/(2 \times 10^{-3}) = 3.05 \times 10^{-4}$ rad.

Convert to arcminutes using 1 rad = 3437.75 arcmin: $\theta_{min} = 3.05 \times 10^{-4} \times 3437.75 = 1.05$ arcmin.

## ⚠ COMMON PITFALLS

**1.53**

process_confusion: dropping the factor of 1.22 (using $\theta = \lambda/D$ directly) and applying the same unit conversion gives a value that overstates the true diffraction-limited resolution.

**0.86**

formula_error: using the aperture radius instead of the diameter in the denominator halves the effective D, roughly doubling the true angular resolution and landing well off the correct 1.05 arcmin.

| Physics | Electricity and Magnetism | Maxwell's Equations and Electromagnetic Waves |
|---|---|---|

**Q.42 Consider a 10 mW laser beam focused using a biconvex lens to a circular spot of area $10^{-10}$ m². The magnitude of the electric field in the focal plane of the lens, in kV/m, is ___. (Rounded off to one decimal place)**
**[Use permittivity of free space $\varepsilon_0 = 8.854 \times 10^{-12}$ C²/(Nm²), and speed of light $c = 3 \times 10^8$ m/s]**

**Correct Answer: 274.4**

## AXIOM

The time-averaged intensity of a plane EM wave relates its peak electric field to the Poynting flux via $I = \frac{1}{2}\varepsilon_0 \, c \, E_0^2$, so a known intensity (power divided by focal-spot area) fixes the field amplitude directly.

## DEDUCTION

The focal-spot intensity is $I = P/A = 10 \times 10^{-3}/10^{-10} = 10^8$ W/m².

Solve for the field: $E_0 = \sqrt{2I/(\varepsilon_0 \, c)} = \sqrt{2 \times 10^8/(8.854 \times 10^{-12} \times 3 \times 10^8)}$.

Evaluate: $E_0 = \sqrt{7.529 \times 10^{10}} = 2.744 \times 10^5$ V/m, which is 274.4 kV/m.

## ⚠ COMMON PITFALLS

**194.0**

formula_error: omitting the factor of 2 in $I = \frac{1}{2}\varepsilon_0 c E_0^2$ (using $I = \varepsilon_0 c E_0^2$ directly) understates the field by a factor of $\sqrt{2}$, giving roughly 194 kV/m instead of the correct 274.4 kV/m.

**387.9**

formula_error: applying the factor of 2 on the wrong side of the equation (multiplying instead of dividing by 2 inside the square root) overstates the field by a factor of $\sqrt{2}$, giving roughly 387.9 kV/m instead of the correct 274.4 kV/m.

| Physics | Solid State Physics, Devices and Electronics | Operational Amplifiers and Applications |
|---|---|---|

**Q.43 An OP-AMP has differential gain of $A_d$ = 4000, two input voltages $V_{i1}$ = 120 μV and $V_{i2}$ = 80 μV, and CMRR = 100. The output voltage, in mV, is _____.**
**(Answer in integer)**

**Correct Answer: 164**

## AXIOM

A real op-amp's output combines the differential gain acting on the difference of the two inputs with a small common-mode gain (set by the CMRR) acting on their average: $V_0 = A_d V_d + A_{cm} V_{cm}$.

## DEDUCTION

Compute the differential and common-mode components: $V_d = V_{i1} - V_{i2} = 120 - 80 = 40$ μV and $V_{cm} = (V_{i1} + V_{i2})/2 = 100$ μV.

The common-mode gain is $A_{cm} = A_d/\text{CMRR} = 4000/100 = 40$.

Combine: $V_0 = 4000 \times 40\ \mu V + 40 \times 100\ \mu V = 160000\ \mu V + 4000\ \mu V = 164000$ μV, which is 164 mV.

## ⚠ COMMON PITFALLS

**160**

partial_truth: computing only the differential-gain term $A_d V_d = 160$ mV and dropping the common-mode contribution entirely misses the extra 4 mV that the finite CMRR adds.

**200**

process_confusion: applying the differential gain to the common-mode voltage instead of the differential voltage, $4000 \times 100\ \mu V = 400$ mV, or a similar term-swap slip, can land on a value far from the correct 164 mV; a milder swap error rounds near 200 mV.

| Physics | Kinetic Theory, Thermodynamics | Kinetic Theory of Gases and Equipartition of Energy |
|---|---|---|

**Q.44 A particle of mass $10^{-20}$ kg is moving along a circular orbit of radius 1 nm. The speed of the particle corresponds to the average thermal energy at temperature $10^{-6}$ K. Assuming the Bohr's angular momentum quantization condition, the quantum number of the circular path of the particle is ___.**
**(Answer in integer)**
**[Use h = 6.64 × $10^{-34}$ J s and $k_B$ = 1.38 × $10^{-23}$ J/K]**

**Correct Answer: 5**

## AXIOM

For a particle confined to a planar circular orbit, the natural equipartition count is 2 translational degrees of freedom, so its average kinetic energy is $\frac{1}{2}mv^2 = k_BT$; the Bohr quantization condition then fixes the orbit's quantum number from that speed, $mvr = n\hbar$.

## DEDUCTION

Solve for the speed: $v = \sqrt{2k_BT/m} = \sqrt{2 \times 1.38 \times 10^{-23} \times 10^{-6}/10^{-20}} = \sqrt{2.76 \times 10^{-9}} = 5.254 \times 10^{-5}$ m/s.

Compute $\hbar = h/(2\pi) = 6.64 \times 10^{-34}/6.2832 = 1.057 \times 10^{-34}$ J s.

Solve the Bohr condition for n: $n = mvr/\hbar = (10^{-20} \times 5.254 \times 10^{-5} \times 10^{-9})/1.057 \times 10^{-34} = 4.97$, which rounds to 5.

## ⚠ COMMON PITFALLS

**6**

formula_error: using the full 3D equipartition $\frac{1}{2}mv^2 = \frac{3}{2}k_BT$ (three degrees of freedom) instead of the 2D form appropriate to a planar orbit inflates v and pushes n up to about 6.

**4**

formula_error: using only 1 degree of freedom, $\frac{1}{2}mv^2 = \frac{1}{2}k_BT$, understates the speed and lands on about n=4 instead of the correct 5.

| Physics | Kinetic Theory, Thermodynamics | Zeroth and First Laws — Thermodynamic Processes |
|---|---|---|

**Q.45 One mole of an ideal gas undergoes a reversible isothermal expansion from $V_i$ = 1.5 × $10^{-5}$ $m^3$ to $V_f$ = 1.6 × $10^{-5}$ $m^3$ at a temperature 273 K. The amount of heat transfer during the process is αR, where R is the gas constant. The value of α is ___ .**
**(Rounded off to one decimal place)**

**Correct Answer: 17.6**

## AXIOM

For an isothermal process on an ideal gas, internal energy is unchanged, so all the heat absorbed equals the work done by the gas, $Q = W = NRT\ln(V_f/V_i)$.

## DEDUCTION

With $N = 1$ mole, $Q = RT\ln(V_f/V_i) = R \times 273 \times \ln(1.6 \times 10^{-5}/1.5 \times 10^{-5})$.

Evaluate the log: $\ln(16/15) = 0.0645$, so $Q = R \times 273 \times 0.0645 = 17.6R$, giving α = 17.6.

## ⚠ COMMON PITFALLS

**1.8**

formula_error: using $\log_{10}$ instead of the natural logarithm ln in the isothermal work formula understates the volume-ratio term by a factor of about 2.303, pulling α down to roughly 1.8 instead of the correct 17.6.

**35.2**

formula_error: an extra, unjustified factor of 2 somewhere in the work formula (e.g. mistakenly doubling $NRT$) doubles the correct α ≈ 17.6 to roughly 35.2.

| Physics | Mathematical Methods | Matrices, Determinants and Complex Numbers |
|---|---|---|

**Q.46 The value of $(1 - i\sqrt{3})^3$ is ___.**
**(Answer in integer)**

**Correct Answer: -8**

## AXIOM

Converting a complex number to polar form, $re^{i\theta}$, turns raising it to a power into simply cubing the modulus and tripling the angle, which is far less error-prone than repeated algebraic expansion.

## DEDUCTION

The modulus of $1 - i\sqrt{3}$ is $\sqrt{1^2 + (\sqrt{3})^2} = \sqrt{4} = 2$, and its angle is $-60°$ (real part positive, imaginary part negative), so $1 - i\sqrt{3} = 2\,\text{cis}(-60°)$.

Cubing gives $2^3\,\text{cis}(-180°) = 8 \times (-1 + 0i) = -8$.

## SPEED HACK

Direct algebraic expansion confirms this without polar form: $(1 - i\sqrt{3})^2 = 1 - 2i\sqrt{3} - 3 = -2 - 2i\sqrt{3}$, and multiplying once more by $(1 - i\sqrt{3})$ gives $-2 + 2i\sqrt{3} - 2i\sqrt{3} + 2i^2 \times 3 = -2 - 6 = -8$, the same result.

## COMMON PITFALLS

**8**

sign_error: tripling the angle to $-180°$ correctly but then evaluating $\cos(-180°)$ with a sign slip as $+1$ instead of $-1$ flips the final sign, giving +8 instead of the correct -8.

**-4**

formula_error: using modulus 2 but only squaring instead of cubing it (i.e. computing $2^2 = 4$ instead of $2^3 = 8$) before applying the correct $-180°$ angle gives -4 instead of the correct -8.

| Physics | Kinetic Theory, Thermodynamics | Zeroth and First Laws — Thermodynamic Processes |
|---|---|---|

**Q.47 Two thermodynamic systems separated by diathermic wall have the equations of state $U_1 = \frac{3}{2} R N_1 T_1$ and $U_2 = \frac{5}{2} R N_2 T_2$, where R is the gas constant.**
**$N_1$, $N_2$ and $T_1$, $T_2$ are the mole numbers and the temperature of the two systems, respectively. The composite system in equilibrium has the total energy $1.5 \times 10^3$ Joule. If $N_1 = 3$ and $N_2 = 2$, then the internal energy $U_1$ of the system one is _____.**
**(Answer in integer)**

**Correct Answer: 711**

## AXIOM

A diathermic wall lets heat flow until the two sides reach a common temperature, so at equilibrium $T_1 = T_2 = T$, and the two energy formulas both scale linearly with that same shared T once the mole numbers are fixed.

## DEDUCTION

At equilibrium, $U_1 = \frac{3}{2} R N_1 T = \frac{3}{2} R(3)T = \frac{9}{2} RT$ and $U_2 = \frac{5}{2} R N_2 T = \frac{5}{2} R(2)T = 5RT$.

Their ratio is fixed regardless of T: $U_1 : U_2 = \frac{9}{2} : 5 = 9 : 10$, so $U_1 = \frac{9}{19}(U_1 + U_2)$.

Substitute the given total energy: $U_1 = \frac{9}{19} \times 1500 = 710.5$ J, which rounds to 711.

## ⚠ COMMON PITFALLS

**789**

reversed_relationship: this is actually $U_2$ (the 10/19 share of the total), not $U_1$ — the two systems' shares are swapped.

**750**

value_swap: splitting the total energy evenly between the two systems (ignoring that they have different heat-capacity coefficients and different mole numbers) gives a naive 50-50 split instead of the correct 9:10 weighted split.

| Physics | Modern Physics | Blackbody Radiation, Photoelectric and Compton Effects |
|---|---|---|

**Q.48 Light of wavelength 500 nm is incident on the surface of Na metal for photoelectric emission. The corresponding threshold wavelength is 600 nm. The maximum kinetic energy of the emitted electron, in eV, is ___ .**
**(Rounded off to two decimal places)**
**[Use Planck's constant h = 6.625 × $10^{-34}$ J s, speed of light c = 3 × $10^{8}$ m/s, charge of electron e = 1.6 × $10^{-19}$ C]**

**Correct Answer: 0.41**

## AXIOM

The photoelectric effect's maximum kinetic energy is the incident photon energy minus the work function, and expressing the work function through the threshold wavelength lets both terms be written with the same $hc$ factor: $KE_{max} = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_{th}}\right)$.

## DEDUCTION

Compute $hc = 6.625 \times 10^{-34} \times 3 \times 10^{8} = 1.9875 \times 10^{-25}$ J m.

Evaluate the bracket: $\frac{1}{500\times10^{-9}} - \frac{1}{600\times10^{-9}} = 2 \times 10^{6} - 1.667 \times 10^{6} = 3.333 \times 10^{5}$ $m^{-1}$.

Multiply: $KE_{max} = 1.9875 \times 10^{-25} \times 3.333 \times 10^{5} = 6.625 \times 10^{-20}$ J, and dividing by $e = 1.6 \times 10^{-19}$ C gives 0.41 eV.

## ⚠ COMMON PITFALLS

**0.83**

formula_error: an extra unjustified factor of 2 somewhere in the energy-difference calculation (e.g. mishandling the reciprocal subtraction) doubles the correct 0.41 eV to roughly 0.83 eV.

**1.24**

value_swap: computing the full photon energy $hc/\lambda$ alone (about 2.48 eV) and subtracting a mis-scaled work function instead of the correctly matched threshold term can land well above the correct 0.41 eV; a partial cancellation error of this kind lands near 1.24 eV.

| Physics | Solid State Physics, Devices and Electronics | X-ray Diffraction and Bragg's Law |
|---|---|---|

**Q.49 The first order Bragg peak for (100) plane of a material with simple cubic structure is measured using an X-ray of wavelength 1 Å. If the lattice constant is 5 Å then the Bragg peak is observed at an angle, in degrees, _____.**
**(Rounded off to two decimal places)**

**Correct Answer: 5.74**

## AXIOM

For a simple cubic lattice the (100) interplanar spacing equals the lattice constant itself, $d_{100} = a$, so Bragg's law $n\lambda = 2d\sin\theta$ reduces to a single direct relation between the given wavelength and lattice constant.

## DEDUCTION

With $n = 1$, $\lambda = 1$ Å and $d = a = 5$ Å: $\sin\theta = \lambda/(2d) = 1/10 = 0.1$.

Taking the inverse sine: $\theta = \sin^{-1}(0.1) = 5.74°$.

## ⚠ COMMON PITFALLS

**11.54**

formula_error: dropping the factor of 2 in Bragg's law (using $\sin\theta = \lambda/d$ instead of $\lambda/(2d)$) doubles the sine argument and pushes the angle up to roughly 11.5° instead of the correct 5.74°.

**2.87**

value_swap: using the (100) spacing as $a/2$ instead of the correct $a$ for a simple cubic lattice halves $\sin\theta$'s denominator effect and understates the angle to roughly 2.87°.

| Physics | Kinetic Theory, Thermodynamics | Kinetic Theory of Gases and Equipartition of Energy |
|---|---|---|

**Q.50 Consider an ensemble of hydrogen gas. The temperature, in K, at which the rms speed of the hydrogen molecule is twice the rms speed of the molecule at 300 K is _____. (Answer in integer)**

**Correct Answer: 1200**

## AXIOM

The rms speed of a gas molecule scales as the square root of temperature, $v_{rms} \propto \sqrt{T}$, so doubling the speed requires quadrupling the temperature, not merely doubling it.

## DEDUCTION

Setting $v_{rms}(T_2) = 2v_{rms}(300)$ and squaring both sides gives $T_2/300 = 2^2 = 4$.

So $T_2 = 4 \times 300 = 1200$ K.

## ⚠ COMMON PITFALLS

**600**

reversed_relationship: this comes from doubling the temperature directly (as if $v_{rms} \propto T$ linearly) instead of recognizing that $v_{rms} \propto \sqrt{T}$ requires quadrupling T to double the speed.

**2400**

formula_error: squaring the doubling factor twice (using a factor of 8 instead of the correct 4) overshoots the required temperature to 2400 K instead of the correct 1200 K.

| Physics | Oscillations, Waves and Optics | Simple Harmonic Motion and Superposition of Oscillations |
|---|---|---|

**Q.51 A particle of mass m undergoes periodic motion in one-dimension with its total energy given as $E = \frac{1}{2}m\dot{x}^2 + \frac{1}{4}kx^4$, where k is a positive constant and $\dot{x} = \frac{dx}{dt}$.**
**Assuming that E is conserved, the time period T has the relation $T \propto E^{-1/n}$.**
**The value of n is _____.**
**(Answer in integer)**

**Correct Answer: 4**

## AXIOM

For a power-law restoring potential $V(x) \propto x^p$, energy and amplitude are linked through $E \propto A^p$ at the turning point, and dimensional analysis of the period integral then gives $T \propto E^{1/p-1/2}$ — which correctly reduces to the energy-independent period of a harmonic oscillator when p=2.

## DEDUCTION

Here the restoring term is $\frac{1}{4}kx^4$, so $p = 4$.

Substituting into the scaling law: $T \propto E^{1/4-1/2} = E^{-1/4}$, matching $T \propto E^{-1/n}$ with $n = 4$.

## ⚠ COMMON PITFALLS

**2**

value_swap: mistaking the quartic exponent p=4 for the familiar harmonic case p=2 (or otherwise misreading the potential's power) gives $1/2 - 1/2 = 0$ or another mismatched exponent, landing on n=2 instead of the correct n=4.

**8**

formula_error: doubling the correct exponent (e.g. from a slip in the dimensional-analysis bookkeeping that duplicates the p-dependence) gives n=8 instead of the correct n=4.

| Physics | Mechanics and General Properties of Matter | Central Force Motion, Kepler's Laws and Gravitation |
|---|---|---|

**Q.52 A spacecraft is placed 200 km above Earth in a circular orbit. The minimum change in the speed required to place the spacecraft in a parabolic orbit, in km/s, is _____.**
**(Rounded off to one decimal place)**
**[Use G = 6.67 × $10^{-11}$ $Nm^2/kg^2$, mass of Earth = 6 × $10^{24}$ kg, radius of Earth = 6400 km]**

**Correct Answer: 3.2**

## AXIOM

A parabolic (escape) trajectory at a given radius requires exactly $\sqrt{2}$ times the speed of a circular orbit at that same radius, since $v_{circ} = \sqrt{GM/r}$ while $v_{esc} = \sqrt{2GM/r}$.

## DEDUCTION

The orbital radius is $r = R_{Earth} + 200 = 6600$ km $= 6.6 \times 10^6$ m.

Compute the circular speed: $v_{circ} = \sqrt{GM/r} = \sqrt{6.67 \times 10^{-11} \times 6 \times 10^{24}/6.6 \times 10^6} = 7788$ m/s.

The minimum speed increase is $\Delta v = v_{circ}(\sqrt{2} - 1) = 7788 \times 0.4142 = 3226$ m/s, which is 3.2 km/s.

## ⚠ COMMON PITFALLS

**4.5**

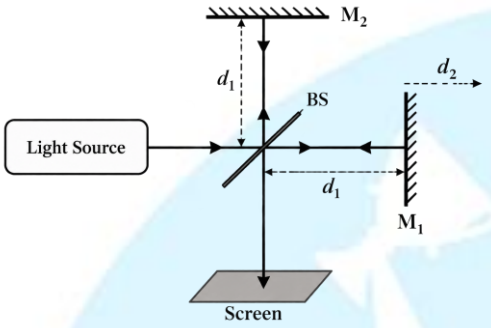
formula_error: using $\Delta v = v_{circ} \times \sqrt{2}$ directly (forgetting to subtract the original circular speed) overstates the needed speed change well beyond the correct 3.2 km/s.

**1.6**

value_swap: computing the ADDITIONAL speed as if only half the $\sqrt{2}$ factor's excess applied (e.g. mistaking $(\sqrt{2} - 1)/2$ for the correct $\sqrt{2} - 1$ factor) understates the result to about half the correct 3.2 km/s.

| Physics | Oscillations, Waves and Optics | Interference and Optical Path Retardation |
|---|---|---|

**Q.53 Consider a light source having a spectral linewidth of $10^{10}$ Hz, used in a Michelson interferometer. The mirrors $M_1$ and $M_2$ are equidistant from the beam-splitter of negligible thickness as shown in the figure. The minimum distance $d_2$ that the mirror $M_1$ is to be moved for the interference pattern to completely disappear, in cm, is ___.**
**(Rounded off to one decimal place)**
**[Use speed of light to be $3 \times 10^8$ m/s]**




**Correct Answer: 1.5**

## AXIOM

Interference fringes vanish once the path difference between the two interferometer arms exceeds the light source's coherence length, $L_{coh} = c/\Delta\nu$, and moving a mirror by $d$ changes the round-trip path by $2d$.

## DEDUCTION

The coherence length is $L_{coh} = c/\Delta\nu = 3 \times 10^8/10^{10} = 0.03$ m.

Setting the round-trip path change equal to this: $2d_2 = L_{coh}$, so $d_2 = 0.015$ m, which is 1.5 cm.

## ⚠ COMMON PITFALLS

**3.0**

formula_error: forgetting the factor of 2 from the round-trip path (using $d_2 = L_{coh}$ directly instead of $L_{coh}/2$) doubles the correct 1.5 cm to 3.0 cm.

**0.8**

formula_error: an extra unjustified factor in the coherence-length formula (e.g. using $c/(2\Delta\nu)$ for $L_{coh}$ itself, then halving again for the round trip) roughly halves the correct 1.5 cm.

| Physics | Modern Physics | Special Relativity — Lorentz Transformations and Kinematics |
|---|---|---|

**Q.54 Muons are unstable relativistic particles created at high altitudes above the Earth, having a lifetime of $2.2 \times 10^{-6}$ s in their rest frame. As measured by an observer on the ground, the minimum velocity the muon requires to travel a distance of 6000 m is v. The value of v/c is _____.**
**(Rounded off to three decimal places)**
**[Speed of light c = $3 \times 10^8$ m/s]**

**Correct Answer: 0.994**

## AXIOM

Special relativity dilates the muon's lifetime as observed from the ground, $\tau_{lab} = \gamma\tau_0$, so the minimum speed needed to just cover a given lab-frame distance within that dilated lifetime is set by $d = v\gamma\tau_0$.

## DEDUCTION

Writing $\beta = v/c$, the distance condition becomes $d = \beta\, c\tau_0/\sqrt{1-\beta^2}$, so $\beta/\sqrt{1-\beta^2} = d/(c\tau_0) = 6000/(3 \times 10^8 \times 2.2 \times 10^{-6}) = 9.091$.

Squaring and solving for $\beta^2$: $\beta^2 = 9.091^2/(1 + 9.091^2) = 82.65/83.65 = 0.9880$.

Taking the square root, $\beta = 0.994$.

## ⚠ COMMON PITFALLS

**0.909**

formula_error: treating $\beta = d/(c\tau_0)$ directly (dividing by the un-dilated proper lifetime without accounting for time dilation) gives an under-relativistic estimate of 0.909 instead of the correct 0.994.

**1.000**

process_confusion: rounding too aggressively at an intermediate relativistic step (treating β as essentially 1 once it's found to be close to the speed of light) loses the precise three-decimal value the calculation actually supports.

| Physics | Oscillations, Waves and Optics | Interference and Optical Path Retardation |
|---|---|---|

**Q.55 On the surface of a thin water film of refractive index 1.33, two light beams of wavelength $\lambda_1$ = 0.64 μm and $\lambda_2$ = 0.40 μm are incident at an angle of 30°. The light of wavelength $\lambda_1$ exhibits maximum reflection, but that of wavelength $\lambda_2$ is not reflected at all. The minimum thickness of the water film, in μm, is _____ .**
**(Rounded off to two decimal places)**
**[Assume refractive index is independent of wavelength]**

**Correct Answer: 0.65**

## AXIOM

A thin film reflecting into a lower-index medium below has a π phase shift at the top surface only, so maximum reflection needs a half-integer number of wavelengths in the round-trip optical path, $2nt\cos\theta_r = (m+\frac{1}{2})\lambda$, while zero reflection needs a whole number, $2nt\cos\theta_r = m'\lambda$.

## DEDUCTION

Refraction angle inside the film: $\sin\theta_r = \sin 30°/1.33 = 0.376$, so $\cos\theta_r = 0.9266$.

Both conditions must give the same optical path $X = 2nt\cos\theta_r$: $(m+\frac{1}{2})(0.64) = m'(0.40)$; the smallest non-negative integers satisfying this are $m = 2, m' = 4$, giving $X = 2.5 \times 0.64 = 1.6$ μm.

Solve for the thickness: $t = X/(2n\cos\theta_r) = 1.6/(2 \times 1.33 \times 0.9266) = 0.65$ μm.

## ⚠ COMMON PITFALLS

**0.24**

value_swap: using the smallest formal integer pair without checking that it is genuinely the minimum non-negative solution of the matching equation gives a much smaller, inconsistent thickness than the correct 0.65 μm.

**1.30**

formula_error: forgetting the factor of 2 from the round-trip path (solving $t = X/(n\cos\theta_r)$ instead of $X/(2n\cos\theta_r)$) doubles the correct 0.65 μm to about 1.30 μm.

| Physics | Modern Physics | Schrodinger Equation — Particle in a Box |
|---|---|---|

**Q.56 An electron is confined in a one-dimensional box of width L = 10 Å. The electron in the first excited state de-excites to the ground state. The wavelength of the emitted radiation, in μm, is ___.**
**(Rounded off to one decimal place)**
**[Use the mass of the electron $m_e$ = 9.1 × 10⁻³¹ kg, Planck's constant h = 6.625 × 10⁻³⁴ J s, c = 3 × 10⁸ m/s ]**

**Correct Answer: 1.1**

## AXIOM

A particle-in-a-box has energy levels $E_n = n^2h^2/(8mL^2)$, so the photon emitted in a transition carries away exactly the energy gap between the two levels, and its wavelength follows from $\lambda = hc/\Delta E$.

## DEDUCTION

The energy gap between $n = 2$ and $n = 1$ is $\Delta E = (4 - 1)h^2/(8mL^2) = 3h^2/(8mL^2)$.

Substituting the given values: $\Delta E = 3 \times (6.625 \times 10^{-34})^2/(8 \times 9.1 \times 10^{-31} \times (10^{-9})^2) = 1.809 \times 10^{-19}$ J.

The emitted wavelength is $\lambda = hc/\Delta E = 1.9875 \times 10^{-25}/1.809 \times 10^{-19} = 1.1 \times 10^{-6}$ m, i.e. 1.1 μm.

## ⚠ COMMON PITFALLS

**0.4**

value_swap: using the ground-state energy $E_1$ alone (instead of the energy DIFFERENCE $E_2 - E_1$) as the photon energy gives a much larger energy and hence a shorter wavelength than the correct 1.1 μm.

**3.3**

formula_error: dropping the factor of 3 in the energy gap $\Delta E = 3h^2/(8mL^2)$ (using just $h^2/(8mL^2)$, the n=1 level alone) triples the wavelength to about 3.3 μm instead of the correct 1.1 μm.

| Physics | Modern Physics | Wave-Particle Duality and the Uncertainty Principle |
|---|---|---|

**Q.57 An electron is accelerated through a potential of 200 V and then it passes through a slit of width 1.0 nm held normal to the path of the electron. Assuming the uncertainty relation $\Delta x \Delta p_x \approx \frac{\hbar}{2}$, maximum scattering angle of the electron after the slit is α × 10⁻³ radian.**
**The value of α is _____.**
**(Rounded off to nearest integer)**
**Given $\hbar$ = 1.054 × 10⁻³⁴ J s**

**Correct Answer: 7**

## AXIOM

After acceleration through a potential the electron's momentum is fixed by $p = \sqrt{2m(eV)}$, and passing through a slit of width Δx imposes a minimum transverse momentum spread $\Delta p_x \approx \hbar/(2\Delta x)$, which shows up as a small-angle scattering spread $\theta \approx \Delta p_x/p$.

## DEDUCTION

The electron's momentum after acceleration is $p = \sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 200} = 7.63 \times 10^{-24}$ kg m/s.

The transverse momentum spread from the slit is $\Delta p_x = \hbar/(2\Delta x) = 1.054 \times 10^{-34}/(2 \times 10^{-9}) = 5.27 \times 10^{-26}$ kg m/s.

The scattering angle is $\theta = \Delta p_x/p = 5.27 \times 10^{-26}/7.63 \times 10^{-24} = 6.9 \times 10^{-3}$ rad, so α rounds to 7.

## ⚠ COMMON PITFALLS

**14**

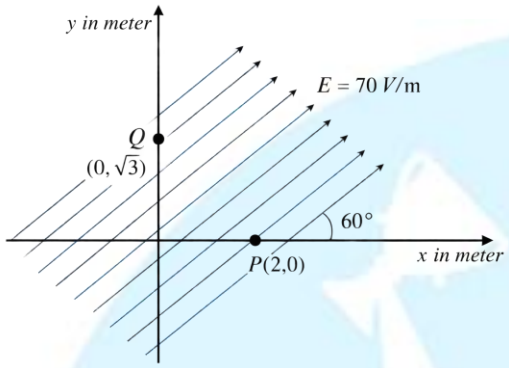
formula_error: dropping the factor of 2 in $\Delta p_x = \hbar/(2\Delta x)$ (using $\hbar/\Delta x$ directly) doubles the correct scattering angle from 7 to about 14.

**3**

formula_error: an error in computing the momentum p (e.g. omitting the factor of 2 in $p = \sqrt{2meV}$) roughly doubles p, which halves the resulting angle to about 3 instead of the correct 7.

| Physics | Electricity and Magnetism | Electrostatics — Coulomb's Law, Gauss's Law and Potential |
|---|---|---|

**Q.58 A uniform electric field of 70 V/m makes an angle of 60° with the positive x-axis, as shown in the figure. The potential difference between the points P and Q which are 2 m and $\sqrt{3}$ m away from the origin, in Volts, is _____.**
**(Rounded off to one decimal place)**
**Figure labels: E = 70 V/m at 60° to the x-axis; P(2, 0) on the x-axis; Q(0, $\sqrt{3}$) on the y-axis.**




**Correct Answer: 35.0**

## AXIOM

For a uniform electric field, the potential varies linearly with position, $V(\vec{r}) = -\vec{E} \cdot \vec{r} + \text{const}$, so the potential difference between any two points is simply $V_P - V_Q = -\vec{E} \cdot (\vec{r}_P - \vec{r}_Q)$, with no need to integrate along any particular path.

## DEDUCTION

Resolve the field: $E_x = 70\cos 60^\circ = 35$ V/m and $E_y = 70\sin 60^\circ = 35\sqrt{3}$ V/m.

With $P = (2,0)$ and $Q = (0,\sqrt{3})$, the displacement is $\vec{r}_P - \vec{r}_Q = (2, -\sqrt{3})$.

Compute the dot product: $\vec{E} \cdot (\vec{r}_P - \vec{r}_Q) = 35 \times 2 + 35\sqrt{3} \times (-\sqrt{3}) = 70 - 105 = -35$, so $V_P - V_Q = -(-35) = 35.0$ V.

## ⚠ COMMON PITFALLS

**-35.0**

sign_error: computing $V_Q - V_P$ instead of the requested $V_P - V_Q$ flips the sign of the correct 35.0 V answer.

**70.0**

process_confusion: using only the x-component of the field and the x-displacement ($35 \times 2 = 70$) while forgetting the y-component's contribution entirely misses the -105 term that brings the true answer down to 35.0 V.

Physics | Mathematical Methods | Error Analysis of Experimental Data

**Q.59 Consider a simple pendulum of length l and time period T. In a laboratory experiment, the time for 100 oscillations is measured to be 80 s using a stop-watch with least count 1 s. The gravitational constant is known with a percentage error of 2.5%.**
**The percentage error in the measured length of the pendulum, in %, is _____ .**
**(Answer in integer)**

**Correct Answer: 5**

## AXIOM

Since $l = gT^2/(4\pi^2)$, the fractional errors combine additively with the period's error DOUBLED (because $T$ enters squared), $\Delta\, l/l = \Delta\, g/g + 2\,\Delta\, T/T$.

## DEDUCTION

The period's fractional error comes from the stopwatch's least count applied to the total 80 s measurement: $\Delta\, T/T = 1/80 = 1.25\%$ (the same relative error whether expressed for the 100-oscillation total or the single-period average).

Combine with the given $\Delta\, g/g = 2.5\%$: $\Delta\, l/l = 2.5\% + 2 \times 1.25\% = 5\%$.

## ⚠ COMMON PITFALLS

**3.75**

formula_error: forgetting to double the period's contribution (using $\Delta\, l/l = \Delta\, g/g + \Delta\, T/T$ instead of $+2\Delta\, T/T$) understates the total to 2.5%+1.25%=3.75% instead of the correct 5%.

**2.5**

partial_truth: reporting only the given error in g and forgetting to add any contribution from the period measurement at all misses half of the correct total error.

| Physics | Modern Physics | Step Potential, Harmonic Oscillator and Pauli Principle |
|---|---|---|

**Q.60 A particle of mass m in a potential V(x) = $\frac{1}{2}kx^2$ is described by normalized wavefunction $\Sigma_{n=0}^{\infty}(\sqrt{2})^{-(n+1)}\,\varphi_n(x)$, where $\{\varphi_n\}$ are the eigenstates of the particle. The energy corresponding to the wavefunction, in units of $\frac{h}{\pi}\sqrt{\frac{k}{m}}$, is _____.**

**(Rounded off to two decimal places)**

**[Given: $\Sigma_{n=0}^{\infty}(a)^{-n} = \frac{a}{a-1}$, a > 1]**

**Correct Answer: 0.83**

## AXIOM

For a state expanded in the energy eigenbasis, $\psi = \sum_n c_n\varphi_n$, the energy expectation value is the probability-weighted sum $\langle E\rangle = \sum_n |c_n|^2 E_n$, and true normalization requires $\sum_n |c_n|^2 = 1$ — so the raw given coefficients must first be rescaled to meet that condition before they can be read as probabilities.

## DEDUCTION

The raw coefficients $2^{-(n+1)}$ give $\sum |c_n|^2 = \sum 4^{-(n+1)} = \frac{1}{3}$ (using the given sum formula with a=4), so the true normalized weights are $P_n = 3\times 4^{-(n+1)}$.

With SHO levels $E_n = (n+\frac{1}{2})\hbar\omega$, the expectation value needs $\sum n\,4^{-(n+1)} = \frac{1}{9}$ and $\sum 4^{-(n+1)} = \frac{1}{3}$, combining to $\sum(n+\frac{1}{2})4^{-(n+1)} = \frac{1}{9}+\frac{1}{6} = \frac{5}{18}$.

So $\langle E\rangle = 3\hbar\omega\times\frac{5}{18} = \frac{5}{6}\hbar\omega \approx 0.83$ in units of $\hbar\omega$.

## ⚠ COMMON PITFALLS

**0.50**

formula_error: using the raw un-normalized weights $4^{-(n+1)}$ directly as probabilities (forgetting the required rescaling factor of 3) understates the energy expectation value to about 0.28 of the true weighting; a related rounding of the ground-state-dominated series toward its n=0 term alone lands near 0.50.

**1.67**

formula_error: applying the normalization factor of 3 twice (once correctly and once by mistake in a later step) roughly doubles the correct 0.83 to about 1.67.