SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q.1 - Q.10 carry one mark each.

1. **If $\{x_n\}_{n\geq 1}$ is a sequence of real numbers such thah $\lim_{n\to\infty} \frac{x_n}{n} = 0.001$, then**

(A) $\{x_n\}_{n\geq 1}$ is a bounded sequence
(B) $\{x_n\}_{n\geq 1}$ is an unbounded sequence
(C) $\{x_n\}_{n\geq 1}$ is a convergent sequence
(D) $\{x_n\}_{n\geq 1}$ is a monotonically decreasing sequence

2. **For real constant a and b, let $f(x) = \begin{cases} \frac{a\sin x - 2x}{x}, & x < 0 \\ bx, & x \geq 0 \end{cases}$. If f is a differentiable function then the value of $a + b$ is**

(A) 0
(B) 1
(C) 2
(D) 3

3. **The area of the region bounded by the curves $y_1(x) = x^4 - 2x^2$ and $y_2(x) = 2x^2,\ x \in \mathbb{R}$, is**

(A) $\frac{128}{15}$
(B) $\frac{129}{15}$
(C) $\frac{133}{15}$
(D) $\frac{134}{15}$

4. **Consider the following system of linear equation**

$ax + 2y + z = 0$
$y + 5z = 1$
$by - 5z = -1$

Which one of the following statements is TRUE?

(A) The system has unique solution for $a = 1, b = -1$
(B) The system has unique solution for $a = -1, b = 1$
(C) The system has no solution for $a = 1, b = 0$
(D) The system has infinitely many solutions for $a = 0, b = 0$

5. **Let E and F be two events. Then which one of the following statements is NOT always TRUE?**

(A) $P(E \cap F) \leq \max\{1 - P(E^c) - P(F^c), 0\}$
(B) $P(E \cup F) \geq \max\{P(E),\ P(F)\}$
(C) $P(E \cup F) \leq \min\{P(E) + P(F),\ 1\}$
(D) $P(E \cap F) \leq \min\{P(E),\ P(F)\}$

6. **Let X be a random variable having Poisson (2) distribution. Then $E\left(\frac{1}{1+X}\right)$ equals**

(A) $1 - e^{-2}$
(B) e^{-2}

(C) $\frac{1}{2}(1-e^{-2})$

(D) $\frac{1}{2}(e^{-1})$

7. The mean and the standard deviation of weights of ponies in a large animal shelter are 20 kg and 3 kg, respectively. A pony is selected at random from the shelter. Using Chebyshev's inequality, the value of the lower bound of the probability that the weight of the selected pony is between 14 kg and 26 kg is

(A) $\frac{3}{4}$

(B) $\frac{1}{4}$

(C) 0

(D) 1

8. Let $X_1, X_2, \ldots, X_{10}$ be a random sample from $N(1,2)$ distribution. If $\bar{X} = \frac{1}{10}\sum_{i=1}^{10} X_i$ and $S^2 = \frac{1}{9}\sum_{i=1}^{10}(X_i - \bar{X})^2$, then $\mathrm{Var}(S^2)$ equals

(A) $\frac{2}{5}$

(B) $\frac{4}{9}$

(C) $\frac{11}{9}$

(D) $\frac{8}{9}$

9. Let $\{x_n\}_{n\geq 1}$ be a sequence of i.i.d. random variables such that $E(X_i) = 1$ and $\mathrm{Var}(X_i) = 1, i = 1, 2, \ldots$. Then the approximate distribution of $\frac{1}{\sqrt{n}}\sum_{i=1}^{n} X_i(X_{2i} - X_{2i-1})$, for large n is

(A) $N(0,1)$

(B) $N(0,2)$

(C) $N(0,0.5)$

(D) $N(0,0.25)$

10. Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables having $N(\mu, \sigma^2)$ distribution, where $\mu \in \mathbb{R}$ and $\sigma > 0$. Define $W = \frac{1}{2n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}(X_i - X_j)^2$. Then W, as an estimator of σ^2, is

(A) biased and consistent

(B) unbiased and consistent

(C) biased and inconsistent

(D) unbiased and inconsistent

Q.11 - Q.30 carry two marks each.

11. Let $\{a_n\}_{n\geq 1}$ be a sequence of real numbers such that $a_1 = 1, a_2 = 7$ and $a_{n+1} = \frac{a_n + a_{n-1}}{2}, n \geq 2$. Assuming that $\lim_{n\to\infty} a_n$ exists, the value of $\lim_{n\to\infty} a_n$ is

(A) $\frac{19}{4}$

(B) $\frac{9}{2}$

(C) 5

(D) $\frac{21}{4}$

12. Which one of the following series is convergent?

(A) $\sum_{n=1}^{\infty}\left(\frac{5n+1}{4n+1}\right)^n$

(B) $\sum_{n=1}^{\infty}\left(1-\frac{1}{n}\right)^{n}$

(C) $\sum_{n=1}^{\infty}\frac{\sin\left(\frac{1}{n}\right)}{n^{1/n}}$

(D) $\sum_{n=1}^{\infty}\sqrt{n}\left(1-\cos\left(\frac{1}{n}\right)\right)$

13. Let α and β be two real numbers. If $\lim_{x\to 0}\frac{\tan 2x-2\sin\alpha x}{x(1-\cos 2x)}=\beta$ then $\alpha+\beta$ equals

(A) $\frac{1}{2}$

(B) 1

(C) $\frac{3}{2}$

(D) $\frac{5}{2}$

14. Let $f:\mathbb{R}^2\to\mathbb{R}$ be defined by $f(x,y)=\begin{cases}\frac{2x^3+3y^3}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$. Let $f_x(0,0)$ and $f_y(0,0)$ denote the first order partial derivatives of $f(x,y)$ with respect to x and y, respectively, at the point $(0,0)$. Then which one of the following statements is TRUE?

(A) f is continuous at $(0,0)$ but $f_x(0,0)$ and $f_y(0,0)$ do not exist

(B) f is differentiable at $(0,0)$

(C) f is not differentiable at $(0,0)$

(D) f is not continuous at $(0,0)$ but $f_x(0,0)$ and $f_y(0,0)$ exist

15. If the volume of the region bounded by the paraboloid $z=x^2+y^2$ and the plane $z=2y$ is given by $\int_0^{\alpha}\int_{\beta(y)}^{2y}\int_{-\sqrt{z-y^2}}^{\sqrt{z-y^2}}dx\,dz\,dy$, then

(A) $\alpha=2$ and $\beta(y)=y, y\in[0,2]$

(B) $\alpha=1$ and $\beta(y)=y^2, y\in[0,1]$

(C) $\alpha=2$ and $\beta(y)=y^2, y\in[0,2]$

(D) $\alpha=1$ and $\beta(y)=y, y\in[0,1]$

16. The value of the integral $\int_0^2\int_0^{\sqrt{2x-x^2}}\sqrt{x^2+y^2}\,dy\,dx$ is

(A) $\frac{17}{9}$

(B) $\frac{16}{9}$

(C) $\frac{14}{9}$

(D) $\frac{13}{9}$

17. Let $T:\mathbb{R}^3\to\mathbb{R}^4$ be a linear transformation. If $T(1,1,0)=(2,0,0,0)$, $T(1,0,1)=(2,4,0,0)$ and $T(0,1,1)=(0,0,2,0)$, then $T(1,1,1)$ equals

(A) $(1,1,1,0)$

(B) $(0,1,1,1)$

(C) $(2,2,1,0)$

(D) $(0,0,0,0)$

18. Let M be an $n\times n$ non-zero skew symmetric matrix. Then the matrix $(I_n-M)(I_n+M)^{-1}$ is always

(A) singular
(B) symmetric
(C) orthogonal
(D) idempotent

19. A packet contains 10 distinguishable firecrackers out of which 4 are defective. If three firecrackers are drawn at random (without replacement) from the packet, then the probability that all three firecrackers are defective equals

(A) $\frac{1}{10}$

(B) $\frac{1}{20}$

(C) $\frac{1}{30}$

(D) $\frac{1}{40}$

20. Let X_1, X_2, X_3, X_4 be i.i.d. random variables having a continuous distribution. Then $P(X_3 < X_2 < \max(X_1, X_4))$ equals

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{1}{4}$

(D) $\frac{1}{6}$

21. Consider the simple linear regression model $y_i = \alpha + \beta x_i + \epsilon_i, \quad i = 1, 2, \ldots, n$, where ϵ_i' are i.i.d. random variables with mean 0 and variance $\sigma^2 \in (0, \infty)$. Suppose that we have a data set $(x_1, y_1), \ldots, (x_n, y_n)$ with $n = 20, \sum_{i=1}^{n} x_i = 100, \sum_{i=1}^{n} y_i = 50, \sum_{i=1}^{n} x_i^2 = 600, \sum_{i=1}^{n} y_i^2 = 500$ and $\sum_{i=1}^{n} x_i y_i = 400$. Then the least square estimates of α and β are, respectively,

(A) 5 and $\frac{3}{2}$

(B) -5 and $\frac{3}{2}$

(C) 5 and $-\frac{3}{2}$

(D) -5 and $-\frac{3}{2}$

22. Let Z_1 and Z_2 be i.i.d. $N(0, 1)$ random variables. If $Y = Z_1^2 + Z_2^2$, then $P(Y > 4)$ equals

(A) e^{-2}

(B) $1 - e^{-2}$

(C) $\frac{1}{2}e^{-2}$

(D) e^{-4}

23. Consider a sequence of independent Bernoulli trials with probability of success in each trial being $\frac{1}{3}$. Let X denote the number of trials required to get the second success. Then $P(X \geq 5)$ equals

(A) $\frac{3}{7}$

(B) $\frac{16}{27}$

(C) $\frac{16}{21}$

(D) $\frac{9}{13}$

24. Let the joint probability density function of (X, Y) be $f(x, y) = \begin{cases} 2e^{-(x+y)}, & 0 < x < y < \infty \\ 0, & \text{otherwise} \end{cases}$. Then $P\left(X < \frac{Y}{2}\right)$ equals

(A) $\frac{1}{6}$

(B) $\frac{1}{3}$

(C) $\frac{2}{3}$

(D) $\frac{1}{2}$

25. Let X_1, X_2, X_3, X_4, X_5 be a random sample from $N(0,1)$ distribution and let $W = \frac{X_1^2}{X_2^2+X_3^2+X_4^2+X_5^2}$. Then $E(W)$ equals

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{1}{4}$

(D) $\frac{1}{5}$

26. Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\mu_1, \sigma^2)$ distribution and $Y_1, Y_2, \ldots, Y_m$ be a random sample from $N(\mu_2, \sigma^2)$ distributiion, where $\mu_1 \in \mathbb{R}, i = 1, 2$ and $\sigma > 0$. Suppose that the two random samples are independent. Define $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ and $W = \frac{\sqrt{mn}\,(\bar{X}-\mu_1)}{\sqrt{\sum_{i=1}^{m}(Y_i-\mu_2)^2}}$. Then which one of the following statements is TRUE for all positive integers m and n?

(A) $W \sim t_m$

(B) $W \sim t_n$

(C) $W^2 \sim F_{m,1}$

(D) $W^2 \sim F_{m,n}$

27. Let $X_1, X_2, \ldots, X_n$ be a random sample from $U(\theta - 0.5, \theta + 0.5)$ distribution, where $\theta \in \mathbb{R}$. If $X_{(1)} = \min\{X_1, X_2, \ldots, X_n\}$ and $X_{(n)} = \max\{X_1, X_2, \ldots, X_n\}$, then which one of the the following estimators is NOT a maximum likelihood estimator of θ?

(A) $\frac{1}{2}\left(X_{(1)} + X_{(n)}\right)$

(B) $\frac{1}{4}\left(3\,X_{(1)} + X_{(n)} + 1\right)$

(C) $\frac{1}{4}\left(X_{(1)} + 3X_{(n)} - 1\right)$

(D) $\frac{1}{2}\left(3\,X_{(n)} - X_{(1)} - 2\right)$

28. Let $X_1, X_2, \ldots, X_n$ be a random sample from $\text{Exp}(\theta)$ distribution, where $\theta \in (0, \infty)$. If $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$, then a 95% confidence interval for θ is

(A) $\left(0, \frac{\chi^2_{2n,0.95}}{n\bar{X}}\right]$

(B) $\left[\frac{\chi^2_{2n,0.95}}{n\bar{X}}, \infty\right)$

(C) $\left(0, \frac{\chi^2_{2n,0.05}}{2n\bar{X}}\right]$

(D) $\left[\frac{\chi^2_{2n,0.05}}{2n\bar{X}}, \infty\right)$

29. Let $X_1, X_2, \ldots, X_n$ be a random sample from $U(1,2)$ distribution and let $Y_1, Y_2, \ldots, Y_n$ be a random sample from $U(0,1)$ distribution. Suppose that the two random samples are independent. Define $Z_i = \begin{cases} 1, & \text{if} X_i Y_i < 1 \\ 0, & \text{otherwise} \end{cases}$, $i = 1, 2, \ldots n$. If $\lim_{n\to\infty} P\left(\left|\frac{1}{n}\sum_{i=1}^{n} Z_i - \theta\right| < \epsilon\right) = 1$, for all $\epsilon > 0$, then θ equals

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) $\log_e \frac{3}{2}$

(D) $\log_e 2$

30. Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with probability density function $f_\theta(x) = \begin{cases} \theta(1-x)^{\theta-1}, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$, $\theta > 0$. To test $H_0 : \theta = 1$ against $H_1 : \theta > 1$, the uniformly most powerful test of size $\alpha (0 < \alpha < 1)$ would reject H_0 if

(A) $-\sum_{i=1}^{n} \log_e (1 - X_i)^2 < \chi^2_{2n,1-\alpha}$

(B) $-\sum_{i=1}^{n} \log_e (1 - X_i)^2 < \chi^2_{n,1-\alpha}$

(C) $-\sum_{i=1}^{n} \log_e (1 - X_i)^2 < \chi^2_{2n,\alpha}$

(D) $-\sum_{i=1}^{n} \log_e (1 - X_i)^2 < \chi^2_{n,\alpha}$

SECTION -B

MULTIPLE SELECT QUESTIONS (MSQ)

Q.31 - Q.40 carry two marks each.

31. Let the sequence $\{x_n\}_{n\geq 1}$ be given by $x_n = \sin\frac{n\pi}{6}, n = 1, 2, \ldots$. Then which of the following statements is/are TRUE?

(A) The sequence $\{x_n\}_{n\geq 1}$ has a subsequence that converges to $\frac{1}{2}$

(B) $\lim\limits_{n\to\infty}\sup x_n = 1$

(C) $\lim\limits_{n\to\infty}\inf x_n = -1$

(D) The sequence $\{x_n\}_{n\geq 1}$ has a subsequence that converges to $\frac{1}{\sqrt{2}}$

32. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x, y) = x^2(2 - y) - y^3 + 3y^2 + 9y$, where $(x, y) \in \mathbb{R}^2$. Which of the following is/are saddle point(s) of f?

(A) $(0, -1)$

(B) $(0, 3)$

(C) $(3, 2)$

(D) $(-3, 2)$

33. The arc length of the parabola $y^2 = 2x$ intercepted between the points of intersection of the parabola $y^2 = 2x$ and the straight line $y = 2x$ equals

(A) $\int_0^1 \sqrt{1 + y^2}\, dy$

(B) $\int_0^1 \sqrt{1 + 4y^2}\, dy$

(C) $\int_0^{1/2} \frac{\sqrt{1+2x}}{\sqrt{2x}}\, dx$

(D) $\int_0^{1/2} \frac{\sqrt{1+4x}}{\sqrt{2x}}\, dx$

34. For real constant a and b, let $M = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ a & b \end{bmatrix}$ be an orthogonal matrix. Then which of the following statements is/are always TRUE?

(A) $a + b = 0$

(B) $b = \sqrt{1 - a^2}$

(C) $ab = -\frac{1}{2}$

(D) $M^2 = I_2$

35. Consider a sequence of independent Bernoulli trials with probability of success in each trial being $\frac{1}{5}$. Then which of the following statements is/are TRUE?

(A) Expected number of trials required to get the first success is 5

(B) Expected number of successes in first 50 trials is 10

(C) Expected number of failures preceding the first success is 4

(D) Expected number of trials required to get the second success is 10

36. let (X,Y) have the joint probability mass function $f(x,y) = \begin{cases} \binom{x+1}{y}\binom{16}{x}\binom{1}{6}^{y}\binom{5}{6}^{x+1-y}\binom{1}{2}^{16}, & y=0,1,\dots,x+1;\ x=0,1,\dots,16 \\ 0, & \text{otheriwse} \end{cases}$. Then which of the following statement is/are TRUE?

(A) $E(Y)=\frac{3}{2}$

(B) $\mathrm{Var}(Y)=\frac{49}{36}$

(C) $E(XY)=\frac{37}{3}$

(D) $\mathrm{Var}(X)=3$

37. Let X_1, X_2, X_3 be i.i.d. $N(0,1)$ random variables. Then which of the following statements is/are TRUE?

(A) $\frac{\sqrt{2}\,X_1}{\sqrt{X_2^2+X_3^2}} \sim t_2$

(B) $\frac{\sqrt{2}\,X_1}{|X_2+X_3|} \sim t_1$

(C) $\frac{(X_1-X_2)^2}{(X_1+X_2)^2} \sim F_{1,1}$

(D) $\sum_{i=1}^{3} X_i^2 \sim \chi_2^2$

38. Let $\{X_n\}_{n\geq 1}$ be a sequence of i.i.d. random variables such that $P(X_1=0)=\frac{1}{4}=1-P(X_1=1)$ Define $U_n=\frac{1}{n}\sum_{i=1}^{n}X_i$ and $V_n=\frac{1}{n}\sum_{i=1}^{n}(1-X_i)^2$, $n=1,2,\dots$. Then which of the following statements is/are TRUE?

(A) $\lim_{n\to\infty} P\left(\left|U_n-\frac{3}{4}\right|<\frac{1}{100}\right)=1$

(B) $\lim_{n\to\infty} P\left(\left|U_n-\frac{3}{4}\right|>\frac{1}{100}\right)=1$

(C) $\lim_{n\to\infty} P\left(\sqrt{n}\left(U_n-\frac{3}{4}\right)\leq 1\right)=\Phi(2)$

(D) $\lim_{n\to\infty} P\left(\sqrt{n}\left(V_n-\frac{1}{4}\right)\leq 1\right)=\Phi\left(\frac{4}{\sqrt{3}}\right)$

39. Let $X_1, X_2, \dots, X_n$ be Poisson (λ) random variables, where $\lambda>0$. Define $\bar{X}=\frac{1}{n}\sum_{i=1}^{n}X_i$ and $S^2=\frac{1}{n-1}\sum_{i=1}^{n}(X_i-\bar{X})^2$. Then which of the following statements is/are TRUE?

(A) $\mathrm{Var}(\bar{X}) < \mathrm{Var}(S^2)$

(B) $\mathrm{Var}(\bar{X}) = \mathrm{Var}(S^2)$

(C) $\mathrm{Var}(\bar{X})$ attains the Cramer-Rao lower bound

(D) $E(\bar{X})=E(S^2)$

40. Consider the following two probability density function (pdfs) $f_0(x)=\begin{cases}2x, & \text{if } 0\leq x\leq 1\\ 0, & \text{otherwise}\end{cases}$ and $f_1(x)=\begin{cases}1, & \text{if } 0\leq x\leq 1\\ 0, & \text{otherwise}\end{cases}$. Let X be a random variable having pdf $f\in\mathcal{P}=\{f_0,f_1\}$. Consider testing $H_0: f(x)=f_0(x), \forall\, x\in[0,1]$ against $H_1: f(x)=f_1(x), \forall\, x\in[0,1]$ at $\alpha=0.05$ level of significance. For which of the following observed values of random observation X, the most powerful test would reject H_0?

(A) 0.19

(B) 0.22

(C) 0.25

(D) 0.28

SECTION -C

NUMERICAL ANSWER TYPE (NAT)

Q.41 - Q-50 carry one mark each.

41. $\lim_{n\to\infty} \frac{\left(\sqrt{n^2+1}+n\right)^2}{\sqrt[3]{n^6+1}}$ equals _____

42. The maximum value of the function $y = \frac{x^2}{x^4+4}, x \in \mathbb{R}$ is _____

43. The value of the integral $\int_0^1 \int_{y^2}^1 \frac{e^x}{\sqrt{x}}\, dx\, dy$ equals _____ (round off to two decimal places)

44. The rank of the matrix $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 2 \\ 2 & 5 & 6 & 4 \\ 2 & 6 & 8 & 5 \end{bmatrix}$ is _____

45. Let (X, Y) have the joint probability density function $f(x,y) = \begin{cases} \frac{3}{4}(y-x), & 0 < x < y < 2 \\ 0, & \text{otherwise} \end{cases}$. Then the conditional expectation $E(X \backslash Y = 1)$ equals _____ (round off to two decimal places)

46. Let X be a random variable having the Poisson (4) distribution and let E be an event such that $P(E|X = i) = 1 - 2^{-i}, i = 0, 1, 2, \ldots$. Then $P(E)$ equals _____ (round off to two decimal places)

47. Let X_1, X_2 and X_3 be independent random variables such that $X_1 \sim N(47, 10)$, $X_2 \sim N(55, 15)$ and $X_3 \sim N(60, 14)$. Then $P(X_1 + X_2 \geq 2X_3)$ equals _____ (round off to two decimal places)

48. Let $U \sim F_{5,8}$ and $V \sim F_{8,5}$. If $P[U > 3.69] = 0.05$, then the value of c such that $P[V > c] = 0.95$ equals _____ (round off to two decimal places)

49. Let the sample mean based on a random sample from $\text{Exp}(\lambda)$ distribution be 3.7. Then the maximum likelihood estimate of $1 - e^{-\lambda}$ equals _____ (round off to two decimal places)

50. Let X be a single observation drawn from $U(0, \theta)$ distribution, where $\theta \in \{1, 2\}$. To test $H_0 : \theta = 1$ and $H_1 : \theta = 2$ consider the test procedure that rejects H_0 if and only if $X. 0.75$. If the probabilities of Type-I and Type-II errors are α and β, respectively, then $\alpha + \beta$ equals _____ (round off to two decimal places)

Q.51 - Q.60 carry two marks each.

51. Let $f : [-1, 3] \to \mathbb{R}$ be a continuous function such that f is differentiable on $(-1, 3)$, $|f'(x)| \leq \frac{3}{2}$, $\forall\, x \in (-1, 3), f(-1) = 1$ and $f(3) = 7$. Then $f(1)$ equals _____

52. Let α be the real number such that the coefficient of x^{125} in Maclaurin's series of $\left(x + \alpha^3\right) e^x$ is $\frac{28}{124!}$. Then α equals _____

53. Consider the matrix $M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 2 \\ 0 & 1 & 4 \end{bmatrix}$. Let P be a non-singular matrix such that $P^{-1}MP$ is a diagonal matrix. Then the trace of the matrix $P^{-1}M^3P$ equals _____

54. Let P be a 3×3 matrix having characteristic roots $\lambda_1 = -\frac{2}{3}, \lambda_2 = 0$ and $\lambda_3 = 1$. Define $Q = 3P^3 - P^2 - P + I_3$ and $R = 3P^3 - 2P$. If $\alpha = \det(Q)$ and $\beta = \text{trace}\,(R)$, then $\alpha + \beta$ equals _____ (round off to two decimal places)

55. Let X and Y be independent random variables with respective moment generating functions $M_X(t) = \frac{(8+e^t)^2}{81}$ and $M_Y(t) = \frac{(1+3e^t)^3}{64}, -\infty < t < \infty$. Then $P(X + Y = 1)$ equals _____ (round off to two decimal places)

56. Let X be a random variable having $U(0, 10)$ distribution and $Y = X - [X]$, where $[X]$ denotes the greatest integer less than or equal to X. Then $P(Y > 0.25)$ equals _____

57. A computer lab has two printers handling certain types of printing jobs. Printer-I and Printer-II handle 40% and 60% of the jobs, respectively. For a typical printing job, priting time (in minutes) of Printer-I follows $N(10, 4)$ distribution and that of Printer-II follows $U(1, 21)$ distribution. If a randomly selected printing job is found to have been completed in less than 10 minutes, then the conditional probability that it was handled by the Printer-II equals _____ (round off to two decimal places)

58. Let $x_1 = 1, x_2 = 0, x_3 = 0, x_4 = 1, x_5 = 0, x_6 = 1$ be the data on a random sample of size 6 from Bin $(1, \theta)$ distribution, where $\theta \in (0, 1)$. Then the uniformly minimum variance unbiased estimate of $\theta(1 + \theta)$ equals _____

59. Let $x_1 = 1, x_2 = 4$ be the data on a random sample of size 2 from a Poisson (θ) distribution, where $\theta \in (0, \infty)$. Let $\hat{\psi}$ be the uniformly minimum variance unbiased estimate of $\psi(\theta) = \sum_{k=4}^{\infty} \frac{e^{-\theta}\theta^k}{k!}$ based on the given data. Then $\hat{\psi}$ equals _____ (round off to two decimal places)

60. Let X be a random variable having $N(\theta, 1)$ distribution, where $\theta \in \mathbb{R}$. Consider testing $H_0 : \theta = 0$ against $H_1 : \theta \neq 0$ at $\alpha = 0.617$ level of significance. The power of the likelihood ratio test at $\theta = 1$ equals _____ (round off to two decimal places)

IFAS Online Classes : https://www.ifasonline.com/MSC-ENTRANCE-EXAM/mathematics/msc-entrance-math-Exam.jsp

ifas contest : https://ifasonline.com/MSC-ENTRANCE-EXAM/mathematics/iit-jam-LearnPlayWin.jsp

ifas test series : https://ifasonline.com/MSC-ENTRANCE-EXAM/mathematics/iit-jam-math-Test-Series.jsp

IFAS YouTube : http://youtube.com/c/IFASMaths

IFAS Facebook Page : https://www.facebook.com/IFASMaths/

IFAS Facebook Group : https://www.facebook.com/groups/IFASMaths

IFAS Telegram Channel : https://t.me/IFASMaths

IFAS Instagram : https://instagram.com/ifas_mathematics

For more query and information please contact on this no. 9172266888/7248990393

Download free IFAS app from Play Store :
https://play.google.com/store/apps/details?id=com.ifasapp

Register For Free Live Lectures : https://ifasonline.com/register.jsp