SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q.1 - Q.10 carry one mark each.

1. **Let $\{x_n\}_{n\geq 1}$ be a sequence of positive real numbers. Which one of the following statements is always TRUE?**
(A) If $\{x_n\}_{n\geq 1}$ is a convergent sequence, then $\{x_n\}_{n\geq 1}$ is monotone
(B) If $\{x_n^2\}_{n\geq 1}$ is a convergent sequence, then the sequence $\{x_n\}_{n\geq 1}$ does not converge
(C) If the sequence $\{x_{n+1} - x_n\}_{n\geq 1}$ converges to 0, then the series $\sum_{n=1}^{\infty} x_n$ is convergent
(D) If $\{x_n\}_{n\geq 1}$ is a convergent sequence, then $\{e^{x_n}\}_{n\geq 1}$ is also a convergent sequence

2. **Consider the function $f(x, y) = x^3 - 3xy^2, x, y \in \mathbb{R}$. Which one of the following statements is TRUE?**
(A) f has a local minimum at $(0, 0)$
(B) f has a local maximum at $(0, 0)$
(C) f has global maximum at $(0, 0)$
(D) f has a saddle point at $(0, 0)$

3. **If $F(x) = \int_{x^3}^{4} \sqrt{4 + t^2}\, dt$, for $x \in \mathbb{R}$, then $F'(1)$ equals**
(A) $-3\sqrt{5}$
(B) $-2\sqrt{5}$
(C) $2\sqrt{5}$
(D) $3\sqrt{5}$

4. **Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that $T\left(\begin{bmatrix}1\\2\end{bmatrix}\right) = \begin{bmatrix}1\\0\end{bmatrix}$ and $T\left(\begin{bmatrix}2\\1\end{bmatrix}\right) = \begin{bmatrix}0\\1\end{bmatrix}$. Suppose that $\begin{bmatrix}3\\-2\end{bmatrix} = \alpha\begin{bmatrix}1\\2\end{bmatrix} + \beta\begin{bmatrix}2\\1\end{bmatrix}$ and $T\begin{bmatrix}3\\-2\end{bmatrix} = \begin{bmatrix}a\\b\end{bmatrix}$. Then $\alpha + \beta + a + b$ equals**
(A) $\frac{2}{3}$
(B) $\frac{4}{3}$
(C) $\frac{5}{3}$
(D) $\frac{7}{3}$

5. **Two biased coins C_1 and C_2 have probabilities of getting heads $\frac{2}{3}$ and $\frac{3}{4}$, respectively, when tossed. If both coins are tossed independently two time each, then the probability of getting exactly two heads out of these four tosses is**
(A) $\frac{1}{4}$
(B) $\frac{37}{144}$
(C) $\frac{41}{144}$
(D) $\frac{49}{144}$

6. Let X be a discrete random variable with the probability mass function $P(X=n)=\begin{cases}\frac{-2c}{n}, & n=-1,-2,\\ d, & n=0,\\ cn, & n=1,2\\ 0, & \text{otherwise}\end{cases}$ where c and d are positive real numbers. If $P(|X|\le 1)=3/4$, then $E(X)$ equals

(A) $\frac{1}{12}$

(B) $\frac{1}{6}$

(C) $\frac{1}{3}$

(D) $\frac{1}{2}$

7. Let X be a Poisson random variable and $P(X=1)+2P(X=0)=12P(X=2)$. Which one of the following statements is TRUE?

(A) $0.40 < P(X=0) \le 0.45$

(B) $0.45 < P(X=0) \le 0.50$

(C) $0.50 < P(X=0) \le 0.55$

(D) $0.55 < P(X=0) \le 0.60$

8. Let $X_1, X_2, \ldots$ be a sequence of i.i.d. discrete random variables with the probability mass function $P(X_1=m)=\begin{cases}\frac{(\log_e 2)^m}{2(m!)}, & m=0,1,2,\ldots,\\ 0, & \text{otherwise}\end{cases}$. If $S_n=X_1+X_2+\cdots+X_n$, then which one of the following sequences of random variables converges to 0 in probability?

(A) $\frac{S_n}{n\log_e 2}$

(B) $\frac{S_n-n\log_e 2}{n}$

(C) $\frac{S_n-\log_e 2}{n}$

(D) $\frac{S_n-n}{\log_e 2}$

9. Let $X_1, X_2, \ldots, X_n$ be a random sample from a continuous distribution with the probability density function $f(x)=\frac{1}{2\sqrt{2\pi}}\left[e^{-\frac{1}{2}(x-2\mu)^2}+e^{-\frac{1}{2}(x-4\mu)^2}\right], -\infty < x < \infty$. If $T=X_1+X_2+\cdots+X_n$, then which one of the following is an unbiased estimator of μ?

(A) $\frac{T}{n}$

(B) $\frac{T}{2n}$

(C) $\frac{T}{3n}$

(D) $\frac{T}{4n}$

10. Let $X_1, X_2, \ldots, X_n$ be a random sample from a $N(\theta, 1)$ distribution. Instead of observing $X_1, X_2, \ldots, X_n$ we observe $Y_1, Y_2, \ldots, Y_n$, where $Y_i=e^{X_i}, i=1,2,\ldots,n$. To test the hypothesis $H_0: \theta=1$ against $H_1: \theta\neq 1$ based on the random sample $Y_1, Y_2, \ldots, Y_n$, the rejection region of the likelihood ratio test is of the form, for some $c_1 < c_2$,

(A) $\sum_{i=1}^n Y_i \le c_1$ or $\sum_{i=1}^n Y_i \ge c_2$

(B) $c_1 \le \sum_{i=1}^n Y_i \le c_2$

(C) $c_1 \le \sum_{i=1}^n \log_e Y_i \le c_2$

(D) $\sum_{i=1}^n \log_e Y_i \le c_1$ or $\sum_{i=1}^n \log_e Y_i \ge c_2$

Q.11 - Q.30 carry two marks each.

11. $\sum_{n=4}^{\infty} \frac{6}{n^2-4n+3}$ equals

(A) $\frac{5}{2}$
(B) 3
(C) $\frac{7}{2}$
(D) $\frac{9}{2}$

12. $\lim_{n\to\infty} \frac{1+\frac{1}{2}+\cdots+\frac{1}{n}}{(\pi^n+e^n)^{1/n}\log_e n}$ equals

(A) $\frac{1}{\pi}$
(B) $\frac{1}{e}$
(C) $\frac{e}{\pi}$
(D) $\frac{\pi}{e}$

13. A possible value of $b \in \mathbb{R}$ for which the equation $x^4 + bx^3 + 1 = 0$ has no real root is

(A) $\frac{-11}{5}$
(B) $\frac{-3}{2}$
(C) 2
(D) $\frac{2}{5}$

14. Let the Taylor polynomial of degree 20 for $\frac{1}{(1-x)^3}$ at $x = 0$ be $\sum_{n=0}^{20} a_n x^n$. Then a_{15} is

(A) 136
(B) 120
(C) 60
(D) 272

15. The length of the curve $y = \frac{3}{4}x^{4/3} - \frac{3}{8}x^{2/3} + 7$ from $x = 1$ to $x = 8$ equals

(A) $\frac{99}{8}$
(B) $\frac{117}{8}$
(C) $\frac{99}{4}$
(D) $\frac{117}{4}$

16. The volume of the solid generated by revolving the region bounded by the parabola $x = 2y^4 + 4$ and the line $x = 6$ about the line $x = 6$ is

(A) $\frac{78\pi}{15}$
(B) $\frac{91\pi}{15}$
(C) $\frac{64\pi}{15}$
(D) $\frac{117\pi}{15}$

17. Let P be a 3×3 non-null real matrix. If there exist a 3×2 real matrix Q and a 2×3 real matrix R such that $P = QR$, then

(A) $Px = 0$ has a unique solution, where $0 \in \mathbb{R}^3$
(B) there exists $b \in \mathbb{R}^3$ such that $Px = b$ has no solution

(C) there exists a non-zero $b \in \mathbb{R}^3$ such that $Px = b$ has a unique solution
(D) there exists a non-zero $b \in \mathbb{R}^3$ such that $P^T x = b$ has a unique solution

18. If $\boldsymbol{P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 2 & 0 & -1 \end{bmatrix}}$ and $\boldsymbol{6P^{-1} = aI_3 + bP - P^2}$, then the ordered pair $\boldsymbol{(a, b)}$ is
(A) $(3, 2)$
(B) $(2, 3)$
(C) $(4, 5)$
(D) $(5, 4)$

19. Let $\boldsymbol{E, F}$ and $\boldsymbol{G}$ be any three events with $\boldsymbol{P(E) = 0.3, P(F|E) = 0.2, P(G|E) = 0.1}$ and $\boldsymbol{P(F \cap G|E) = 0.05}$. Then $\boldsymbol{P\big(E - (F \cup G)\big)}$ equals
(A) 0.155
(B) 0.175
(C) 0.225
(D) 0.255

20. Let $\boldsymbol{E}$ and $\boldsymbol{F}$ be any two independent events with $\boldsymbol{0 < P(E) < 1}$ and $\boldsymbol{0 < P(F) < 1}$. Which one of the following statements is NOT TRUE?
(A) P (Neither E nor F occurs) $= (P(E) - 1)\,(P(F) - 1)$
(B) P (Exactly one of E and F occurs) $= P(E) + P(F) - P(E)P(F)$
(C) P (E occurs but F does not occur) $= P(E) - P(E \cap F)$
(D) P (E occurs given that F does not occur) $= P(E)$

21. Let $\boldsymbol{X}$ be a continuous random variable with the probability density function $\boldsymbol{f(x) = \begin{cases} \frac{1}{3}x^7 e^{-x^2}, & x > 0, \\ 0, & \text{otherwise} \end{cases}}$.
Then the distribution of the random variable $\boldsymbol{W = 2X^2}$ is
(A) χ^2_2
(B) χ^2_4
(C) χ^2_6
(D) χ^2_8

22. Let $\boldsymbol{X}$ be a continuous random variable with the probability density function $\boldsymbol{f(x) = \frac{e^x}{(1+e^x)^2}, -\infty < x < \infty}$.
Then $\boldsymbol{E(X)}$ and $\boldsymbol{P(X > 1)}$, respectively, are
(A) 1 and $(1 + e)^{-1}$
(B) 0 and $2(1 + e)^{-2}$
(C) 2 and $(2 + 2e)^{-1}$
(D) 0 and $(1 + e)^{-1}$

23. The lifetime (in years) of bulbs is distributed as an $\boldsymbol{Exp(1)}$ random variable. Using Poisson approximation to the binomial distribution, the probability (round off to 2 decimal places) that out of the fifty randomly chosen bulbs at most one fails within one month equals
(A) 0.05
(B) 0.07
(C) 0.09
(D) 0.11

24. Let $\boldsymbol{X}$ follow a beta distribution with parameters $\boldsymbol{m}(> 0)$ and 2. If $\boldsymbol{P\left(X \leq \frac{1}{2}\right) = \frac{1}{2}}$, then $\boldsymbol{Var(X)}$ equals
(A) $\frac{1}{10}$
(B) $\frac{1}{20}$

(C) $\frac{1}{25}$

(D) $\frac{1}{40}$

25. Let X_1, X_2 and X_3 be i.i.d. $U(0,1)$ random variables. Then $P(X_1 > X_2 + X_3)$ equals

(A) $\frac{1}{3}$

(B) $\frac{1}{2}$

(C) $\frac{2}{3}$

(D) $\frac{3}{4}$

26. Let X and Y be i.i.d. $U(0,1)$ random variables. Then $E(X|X > Y)$ equals

(A) $\frac{1}{3}$

(B) $\frac{1}{2}$

(C) $\frac{2}{3}$

(D) $\frac{3}{4}$

27. Let -1 and 1 be the observed values of a random sample of sizetwo from $N(\theta, \theta)$ distribution. The maximum likelihood estimate of θ is

(A) 0

(B) 2

(C) $\frac{-\sqrt{5}-1}{2}$

(D) $\frac{\sqrt{5}-1}{2}$

28. Let X_1 and X_2 be a random sample from a continuous distribution with the probability density function $f(x) = \begin{cases} \frac{1}{\theta} e^{-\frac{x-\theta}{\theta}}, & x > \theta, \\ 0, & \text{otherwise} \end{cases}$ where $\theta > 0$. If $X_{(1)} = \min\{X_1, X_2\}$ and $\bar{X} = \frac{(X_1+X_2)}{2}$, then which one of the following statements is TRUE?

(A) $(\bar{X}, X_{(1)})$ is sufficient and complete

(B) $(\bar{X}, X_{(1)})$ is sufficient but not complete

(C) $(\bar{X}, X_{(1)})$ is complete but not sufficient

(D) $(\bar{X}, X_{(1)})$ is neither sufficient nor complete

29. Let $X_1, X_2, \ldots, X_n$ be a random sample from a continuous distribution with the probability density function $f(x)$. To test the hypothesis $H_0 : f(x) = \frac{1}{\sqrt{\pi}} e^{-x^2}, -\infty < x < \infty$ against $H_0 : f(x) = e^{-2|x|}, -\infty < x < \infty$, the rejection region of the most powerful size α test is of the form, for some $c > 0$,

(A) $\sum_{i=1}^{n} (X_i - 1)^2 \geq c$

(B) $\sum_{i=1}^{n} (X_i - 1)^2 \leq c$

(C) $\sum_{i=1}^{n} (|X_i| - 1)^2 \geq c$

(D) $\sum_{i=1}^{n} (|X_i| - 1)^2 \leq c$

30. Let $X_1, X_2, \ldots, X_n$ be a random sample from a $N(\theta, 1)$ distribution. To test $H_0 : \theta = 0$ against $H_1 : \theta = 1$, assume that the critical region is given by $\frac{1}{n}\sum_{i=1}^{n} X_i > \frac{3}{4}$. Then the minimum sample size required so that $P(\text{Type I error}) \leq 0.05$ is

(A) 3

(B) 4

(C) 5

(D) 6

SECTION -B

MULTIPLE SELECT QUESTIONS (MSQ)

Q.31-Q.40 carry two marks each.

31. **Let $\{x_n\}_{n\geq 1}$ be a sequence of positive real numbers such that the series $\sum_{n=1}^{\infty} x_n$ converges. Which of the following statements is (are) always TRUE?**

(A) The series $\sum_{n=1}^{\infty}\sqrt{x_n x_{n+1}}$ converges

(B) $\lim_{n\to\infty} x_n = 0$

(C) The series $\sum_{n=1}^{\infty}\sin^2 x_n$ converges

(D) The series $\sum_{n=1}^{\infty}\frac{\sqrt{x_n}}{1+\sqrt{x_n}}$ converges

32. **Let $f : \mathbb{R} \to \mathbb{R}$ be continuous on $\mathbb{R}$ and differentiable on $(-\infty, 0) \cup (0, \infty)$. Which of the following statements is (are) always TRUE?**

(A) If f is differentiable at 0 and $f'(0) = 0$, then f has a local maximum or a local minimum at 0

(B) If f has a local minimum at 0, then f is differentiable at 0 and $f'(0) = 0$

(C) If $f'(x) < 0$ for all $x < 0$ and $f'(x) > 0$ for all $x > 0$, then f has a global maximum at 0

(D) If $f'(x) > 0$ for all $x < 0$ and $f'(x) < 0$ for all $x > 0$, then f has a global maximum at 0

33. **Let P be a 2×2 real matrix such that every non-zero vector in $\mathbb{R}^2$ is an eigenvector of P. Suppose that λ_1 and λ_2 denote the eigenvalues of P and $P\begin{bmatrix}\sqrt{2}\\ \sqrt{3}\end{bmatrix} = \begin{bmatrix}2\\ t\end{bmatrix}$ for some $t \in \mathbb{R}$. Which of the following statement is (are) TRUE?**

(A) $\lambda_1 \neq \lambda_2$

(B) $\lambda_1 \lambda_2 = 2$

(C) $\sqrt{2}$ is an eigenvalue of P

(D) $\sqrt{3}$ is an eigenvalue of P

34. **Let P be an $n \times n$ non-null real skew-symmetric matrix, where n is even. Which of the following statements is (are) always TRUE?**

(A) $Px = 0$ has infinitely many solutions, where $0 \in \mathbb{R}^n$

(B) $Px = \lambda x$ has a unique solution for every non-zero $\lambda \in \mathbb{R}$

(C) If $Q = (I_n + P)(I_n - P)^{-1}$, then $Q^T Q = I_n$

(D) The sum of all the eigenvalues of P is zero

35. **Let X be a random variable with the cumulative distribution function $F(x) = \begin{cases} 0, & x < 0, \\ \frac{1+x^2}{10}, & 0 \leq x < 1, \\ \frac{3+x^2}{10}, & 1 \leq x < 2, \\ 1, & x \geq 2. \end{cases}$ Which of the following statements is (are) TRUE?**

(A) $P(1 < X < 2) = \frac{3}{10}$

(B) $P(1 < X \leq 2) = \frac{3}{5}$

(C) $P(1 \leq X < 2) = \frac{1}{2}$

(D) $P(1 \leq X \leq 2) = \frac{4}{5}$

36. Let X and Y be i.i.d. $Exp(\lambda)$ random variables. If $Z = \max\{X - Y, 0\}$, then which of the following statements is (are) TRUE?

(A) $P(Z = 0) = \frac{1}{2}$

(B) The cumulative distribution function of Z is $F(z) = \begin{cases} 0, & z < 0, \\ 1 - \frac{1}{2}e^{-\lambda z}, & z \geq 0 \end{cases}$

(C) $P(Z = 0) = 0$

(D) The cumulative distribution function of Z is $F(z) = \begin{cases} 0, & z < 0, \\ 1 - e^{-\lambda z/2}, & z \geq 0 \end{cases}$

37. Let the discrete random variables X and Y have the joint probability mass function $P(X = m, Y = n) = \begin{cases} \frac{e^{-2}}{m!n!}, & m = 0, 1, 2, \ldots; n = 0, 1, 2, \ldots, \\ 0, & \text{otherwise} \end{cases}$. Which of the following statements is (are) TRUE?

(A) The marginal distribution of X is Poisson with mean 2

(B) The random variables X and Y are independent

(C) The covariance between X and $X + \sqrt{3}\,Y$ is 1

(D) $P(Y = n) = (n + 1)P(Y = n + 1)$ for $n = 0,1,2, \ldots$

38. $X_1, X_2, \ldots$ be a sequence of i.i.d. continuous random variables with the probability density function $f(x) = \begin{cases} 2e^{-2\left(x-\frac{1}{2}\right)}, & x \geq \frac{1}{2}, \\ 0, & \text{otherwise} \end{cases}$. If $S_n = X_1 + X_2 + \cdots + X_n$ and $\bar{X}_n = S_n/n$, then the distributions of which of the following sequences of random variables converge(s) to a normal distribution with mean 0 and a finite variance?

(A) $\frac{S_n - n}{\sqrt{n}}$

(B) $\frac{S_n}{\sqrt{n}}$

(C) $\sqrt{n}(\bar{X}_n - 1)$

(D) $\frac{\sqrt{n}(\bar{X}_n - 1)}{2}$

39. Let $X_1, X_2, \ldots, X_n$ be a random sample from a $U(\theta, 0)$ distribution, where $\theta < 0$. If $T_n = \min\{X_1, X_2, \ldots, X_n\}$, then which of the following sequences of estimators is (are) consistent for θ?

(A) T_n

(B) $T_n - 1$

(C) $T_n + \frac{1}{n}$

(D) $T_n - 1 - \frac{1}{n^2}$

40. Let $X_1, X_2, \ldots, X_n$ be a random sample from a continuous distribution with the probability density function, for $\lambda > 0$, $f(x) = \begin{cases} 2\lambda x e^{-\lambda x^2}, & x > 0, \\ 0, & \text{otherwise.} \end{cases}$ To test the hypothesis $H_0 : \lambda = \frac{1}{2}$ against $H_1 : \lambda = \frac{3}{4}$ at the level α $(0 < \alpha < 1)$, which of the following statements is (are) TRUE?

(A) The most powerful test exists for each value of α

(B) The most powerful test does not exist for some values of α

(C) If the most powerful test exists, it is of the form: Reject H_0 if $X_1^2 + X_2^2 + \cdots + X_n^2 \leq c$ for some $c > 0$

(D) If the most powerful test exists, it is of the form: Reject H_0 if $X_1^2 + X_2^2 + \cdots + X_n^2 \geq c$ for some $c > 0$

SECTION -C

NUMERICAL ANSWER TYPE (NAT)

Q.41-Q-50 carry one mark each.

41. $\lim_{n\to\infty} \frac{\sqrt{n+1}+\sqrt{n+2}+\cdots+\sqrt{n+n}}{n\sqrt{n}}$ (round off to 2 decimal places) equals _____

42. Let $f:[0,2]\to\mathbb{R}$ be such that $|f(x)-f(y)|\le|x-y|^{4/3}$ for all $x,y\in[0,2]$. If $\int_0^2 f(x)dx=\frac{2}{3}$, then $\sum_{k=1}^{2019} f\left(\frac{1}{k}\right)$ equals _____

43. The value (round off to 2 decimal places) of the double integral $\int_0^9\int_{\sqrt{x}}^3 \frac{1}{1+y^3}dydx$ equals _____

44. If $\begin{bmatrix} \frac{\sqrt{5}}{3} & -\frac{2}{3} & c \\ \frac{2}{3} & \frac{\sqrt{5}}{3} & d \\ a & b & 1 \end{bmatrix}$ is a real orthogonal matrix, then $a^2+b^2+c^2+d^2$ equals _____

45. Two fair disc are tossed independently and it if found that one face is odd and the other one is even. Then the probability (round off to 2 decimal places) that the sum is less than 6 equals _____

46. Let X be a random variable with the moment generating function $M_X(t)=\left(\frac{e^{\frac{t}{2}}+e^{-\frac{t}{2}}}{2}\right)^2, -\infty<t<\infty$. Using Chebyshev's inequality, the upper bound for $P\left(|X|>\sqrt{\frac{2}{3}}\right)$ equals _____

47. In a production line of a factory, each packet contains four items, Past record shows that 20% of the produced items are defective. A quality manager inspects each item in a packet and approves the packets for shipment if at most one item in the packet is found to be defective. Then the probability (round off to 2 decimal places) that out of the three randomly inspected packets at least two are approved for shipment equals _____

48. Let X be a the number of heads obtained in a sequence of 10 independent tosses of a fair coin. The fair coin is tossed again X number of times independently, and let Y be the number of heads obtained in these X number of tosses. Then $E(X+2Y)$ equals _____

49. Let 0, 1, 0, 0, 1 be the observed values of a random sample of size five from a discrete distribution with the probability mass function $P(X=1)=1-P(X=0)=1-e^{-\lambda}$, where $\lambda>0$. The method of moments estimate (round off to 2 decimal places) of λ equals _____

50. Let X_1,X_2,X_3 be a random sample from $N(\mu_1,\sigma^2)$ distribution and Y_1,Y_2,Y_3 be a random sample from $N(\mu_2,\sigma^2)$ distribution. Also, assume that (X_1,X_2,X_3) and (Y_1,Y_2,Y_3) are independent. Let the observed values of $\sum_{i=1}^3\left[X_i-\frac{1}{3}(X_1+X_2+X_3)\right]^2$ and $\sum_{i=1}^3\left[Y_i-\frac{1}{3}(Y_1+Y_2+Y_3)\right]^2$ be 1 and 5, respectively. The length (round off to 2 decimal places) of the shortest 90% confidence interval of $\mu_1-\mu_2$ equals _____

Q.51-Q.60 carry two marks each.

51. $\lim_{n\to\infty}\left[n-\frac{n}{e}\left(1+\frac{1}{n}\right)^n\right]$ equals _____

52. For any real number y, let $[y]$ be the greatest integer less than or equal to y and let $\{y\}=y-[y]$. For $n=1,2,\ldots$, and for $x\in\mathbb{R}$, let $f_{2n}(x)=\begin{cases}\left[\frac{\sin x}{x}\right], & x\neq 0,\\ 1, & x=0,\end{cases}$ and $f_{2n-1}(x)=\begin{cases}\left\{\frac{\sin x}{x}\right\}, & x\neq 0,\\ 1, & x=0,\end{cases}$. Then $\lim_{x\to 0}\sum_{k=1}^{100} f_k(x)$ equals _____

53. The volume (round off to 2 decimal places) of the region in the first octant $(x\geq 0, y\geq 0, z\geq 0)$ bounded by the cylinder $x^2+y^2=4$ and the planes $z=2$ and $y+z=4$ equals _____

54. If $ad-bc=2$ and $ps-qr=1$, then the determinant of $\begin{bmatrix} a & b & 0 & 0\\ 3 & 10 & 2p & q\\ c & d & 0 & 0\\ 2 & 7 & 2r & s\end{bmatrix}$ equals _____

55. In an ethnic group, 30% of the adult male population is known to have heart disease. A test indicates high cholesterol level in 80% of adult males with heart disease. But the test also indicates high cholesterol levels in 10% of the adult males with no heart disease. Then the probability (round off to 2 decimal places), that a randomly selected adult male from this population does not have heart disease given that the test indicates high cholesterol level, equals _____

56. Let X be a continuous random variable with the probability density function $f(x)=\begin{cases}ax^2, & 0<x<1,\\ bx^{-4}, & x\geq 1,\\ 0, & \text{otherwise},\end{cases}$ where a and b are positive real numbers. If $E(X)=1$, then $E(X^2)$ equals _____

57. Let X and Y be jointly distributed continuous random variables, where Y is positive valued with $E(Y^2)=6$. If the conditional distribution of X given $Y=y$ is $U(1-y,1+y)$, then $Var(X)$ equals _____

58. Let $X_1, X_2, \ldots, X_{10}$ be i.i.d. $N(0,1)$ random variables. If $T=X_1^2+X_2^2+\cdots+X_{10}^2$, then $E\left(\frac{1}{T}\right)$ equals _____

59. Let X_1, X_2, X_3 be a random sample from a continuous distribution with the probability density function $f(x)=\begin{cases}e^{-(x-\mu)}, & x>\mu,\\ 0, & \text{otherwise}\end{cases}$. Let $X_{(1)}=\min\{X_1,X_2,X_3\}$ and $c>0$ be a real number. Then $(X_{(1)}-c, X_{(1)})$ is a 97% confidence interval for μ, if c (round off to 2-decimal places) equals _____

60. Let X_1, X_2, X_3, X_4 be a random sample from a discrete distribution with the probability mass function $P(X=0)=1-P(X=1)=1-p$, for $0<p<1$. To test the hypothesis $H_0: p=\frac{3}{4}$ against $H_1: p=\frac{4}{5}$, consider the test: Reject H_0 if $X_1+X_2+X_3+X_4>3$. Let the size and power of the test be denoted by α and γ, respectively. Then $\alpha+\gamma$ (round off to 2 decimal places) equals _____

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