IFAS PREVIOUS YEAR QUESTION PAPER – IIT JAM MATHS 2019

SECTION – A
MULTIPLE CHOICE QUESTIONS (MCQ)

1. Let $a_1 = b_1 = 0$, and for each $n \geq 2$, let a_n and b_n be real numbers given by
$a_n = \sum_{m=2}^{n} \frac{(-1)^m m}{(\log(m))^m}$ and $b_n = \sum_{m=2}^{n} \frac{1}{(\log(m))^m}$
Then which one of the following is TRUE about the sequences $\{a_n\}$ and $\{b_n\}$?
(A) Both $\{a_n\}$ and $\{b_n\}$ are divergent
(B) $\{a_n\}$ is convergent and $\{b_n\}$ is divergent
(C) $\{a_n\}$ is divergent and $\{b_n\}$ is convergent
(D) Both $\{a_n\}$ and $\{b_n\}$ are convergent

2. Let $T \in M_{m\times n}(\mathbb{R})$. Let V be the subspace of $M_{n\times p}(\mathbb{R})$ defined by
$V = \{X \in M_{m\times n}(\mathbb{R}): TX = 0\}$
Then the dimension of V is
(A) $pn - rank(T)$
(B) $mn - p\ rank(T)$
(C) $p\big(m - rank(T)\big)$
(D) $p\big(n - rank\ (T)\big)$

3. Let $g: \mathbb{R} \to \mathbb{R}$ be a twice differentiable function. Define $f: \mathbb{R}^3 \to \mathbb{R}$ by
$f(x, y, z) = g(x^2 + y^2 - 2z^2)$
Then $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$ is equal to
(A) $4(x^2 + y^2 - 4z^2)g''(x^2 + y^2 - 2z^2)$
(B) $4(x^2 + y^2 + 4z^2)g''(x^2 + y^2 - 2z^2)$
(C) $4(x^2 + y^2 - 2z^2)g''(x^2 + y^2 - 2z^2)$
(D) $4(x^2 + y^2 + 4z^2)g''(x^2 + y^2 - 2z^2) + 8g'(x^2 + y^2 - 2z^2)$

4. Let $\{a_n\}_{n=0}^{\infty}$ and $\{b_n\}_{n=0}^{\infty}$ be sequences of positive real numbers such that $na_n < b_n < n^2 a_n$ for all $n \geq 2$. If the radius of convergence of the power series $\sum_{n=0}^{\infty} a_n\, x^n$ is 4, then the power series $\sum_{n=0}^{\infty} b_n\, x^n$
(A) converges for all x with $|x| < 2$
(B) converges for all x with $|x| > 2$
(C) does not converge for any x with $|x| > 2$
(D) does not converge for any with $|x| < 2$

5. Let S be the set of all limit points of the set $\left\{\frac{n}{\sqrt{2}} + \frac{\sqrt{2}}{n} : n \in \mathbb{N}\right\}$. Let $\mathbb{Q}_+$ be the set of all positive rational numbers. Then
(A) $\mathbb{Q}_+ \subseteq S$
(B) $S \subseteq \mathbb{Q}_+$
(C) $S \cap (\mathbb{R}/\mathbb{Q}_+) \neq \emptyset$
(D) $S \cap \mathbb{Q}_+ \neq \emptyset$

6. If $x^h y^k$ is an integrating factor of the differential equation
$y(1 + xy)dx + x(1 - xy)dy = 0$,
Then the ordered pair (h, k) is equal to
(A) $(-2, -2)$
(B) $(-2, -1)$
(C) $(-1, -2)$
(D) $(-1, -1)$

7. If $y(x) = \lambda e^{2x} + e^{\beta x}, \beta \neq 2$, is a solution of the differential equation
$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$
Satisfying $\frac{dy}{dx}(0) = 5$, then $y(0)$ is equal to
(A) 1
(B) 4
(C) 5
(D) 9

8. The equation of the tangent plane to the surface $x^2z + \sqrt{8 - x^2 - y^4} = 6$ at the point (2, 0, 1) is
(A) $2x + z = 5$
(B) $3x + 4z = 10$
(C) $3x - z = 10$
(D) $7x - 4z = 10$

9. The value of the integral
$\int_{y=0}^{1} \int_{x=0}^{1-y^2} y \sin(\pi(1-x)^2)\, dx\, dy$ is
(A) $\frac{1}{2\pi}$
(B) 2π
(C) $\frac{\pi}{2}$
(D) $\frac{2}{\pi}$

10. The area of the surface generated by rotating the curve $x = y^3, 0 \leq y \leq 1$, about the y – axis, is
(A) $\frac{\pi}{27} 10^{3/2}$
(B) $\frac{4\pi}{3}(10^{3/2} - 1)$
(C) $\frac{\pi}{27}(10^{3/2} - 1)$
(D) $\frac{4\pi}{3} 10^{3/2}$

Q. 11 – Q. 30 carry two marks each.

11. Let H and K be subgroup of $\mathbb{Z}_{144}$. If the order of H is 24 and the order of K is 36, then the order of the subgroup $H \cap K$ is
(A) 3
(B) 4
(C) 6
(D) 12

12. Let P be a 4×4 matrix with entries from the set of rational numbers. If $\sqrt{2} + i$, with $i = \sqrt{-1}$, is a root of the characteristic polynomial of P and I is the 4×4 identity matrix, then
(A) $P^4 = 4P^2 + 9I$
(B) $P^4 = 4P^2 - 9I$
(C) $P^4 = 2P^2 - 9I$
(D) $P^4 = 2P^2 + 9I$

13. The set $\left\{\frac{x}{1+x} : -1 < x < 1\right\}$, as a subset of $\mathbb{R}$, is
(A) connected and compact
(B) connected but not compact
(C) not connected but compact

(D) neither connected nor compact

14. The set $\left\{\frac{1}{m}+\frac{1}{n}: m,n \in \mathbb{N}\right\} \cup \{0\}$, as a subset of $\mathbb{R}$, is
(A) compact and open
(C) not compact but open
(B) compact but not open
(D) neither compact nor open

15. For $-1 < x < 1$, the sum of the power series $1 + \sum_{n=2}^{\infty}(-1)^{n-1} n^2 x^{n-1}$ is
(A) $\frac{1-x}{(1+x)^3}$
(B) $\frac{1-x^2}{(1+x)^4}$
(C) $\frac{1-x}{(1+x)^2}$
(D) $\frac{1-x^2}{(1+x)^3}$

16. Let $f(x) = (\ln x)^2, x > 0$. Then
(A) $\lim_{x\to\infty} \frac{f(x)}{x}$ does not exist
(B) $\lim_{x\to\infty} f'(x) = 2$
(C) $\lim_{x\to\infty} \big(f(x+1) - f(x)\big) = 0$
(D) $\lim_{x\to\infty} \big(f(x+1) - f(x)\big)$ does not exist

17. Let $f: \mathbb{R} \to \mathbb{R}$ be a differentiable function such that $f'(x) > f(x)$ for all $x \in \mathbb{R}$, and $f(0) = 1$
Then $f(1)$ lies in the interval
(A) $(0, e^{-1})$
(B) $\left(e^{-1}, \sqrt{e}\right)$
(C) $\left(\sqrt{e}, e\right)$
(D) (e, ∞)

18. For which one of the following values of k, the equation
$2x^3 + 3x^2 - 12x - k = 0$
Has three distinct real roots?
(A) 16
(B) 20
(C) 26
(D) 31

19. Which of the following series is divergent?
(A) $\sum_{n=1}^{\infty} \frac{1}{n} sin^2 \frac{1}{n}$
(B) $\sum_{n=1}^{\infty} \frac{1}{n} \log n$
(C) $\sum_{n=1}^{\infty} \frac{1}{n^2} \sin \frac{1}{n}$
(D) $\sum_{n=1}^{\infty} \frac{1}{n} \tan \frac{1}{n}$

20. Let S be the family of orthogonal trajectories of the family of curves
$2x^2 + y^2 = k$, for $k \in \mathbb{R}$ and $k > 0$
If $\mathcal{C} \in S$ and $\mathcal{C}$ passes through the point (1,2) then $\mathcal{C}$ also passes through
(A) $\left(4, -\sqrt{2}\right)$

(B) $(2,-4)$
(C) $\left(2,2,\sqrt{2}\right)$
(D) $\left(4,2\sqrt{2}\right)$

21. Let $x, x+e^x$ and $1+x+e^x$ be solutions of a linear second order ordinary differential equation with constant coefficients. If $y(x)$ is the solution of the same equation satisfying $y(0)=3$ and $y'(0)=4$, then $y(1)$ is equal to
(A) $e+1$
(B) $2e+3$
(C) $3e+2$
(D) $3e+1$

22. The function $f(x,y)=x^3+2xy+y^3$ has a saddle point at
(A) $(0,0)$
(B) $\left(-\frac{2}{3},-\frac{2}{3}\right)$
(C) $\left(-\frac{3}{2},-\frac{3}{2}\right)$
(D) $(-1,-1)$

23. The area of the part of the surface of the paraboloid $x^2+y^2+z=8$ lying inside the cylinder $x^2+y^2=4$ is
(A) $\frac{\pi}{2}(17^{3/2}-1)$
(B) $\pi\,(17^{3/2}-1)$
(C) $\frac{\pi}{6}\,(17^{3/2}-1)$
(D) $\frac{\pi}{3}\,(17^{3/2}-1)$

24. Let $\mathcal{C}$ be the circle $(x-1)^2+y^2=1$, oriented counter clockwise. Then the value of the line integral
$\oint_{\mathcal{C}} -\frac{4}{3}xy^3dx+x^4dy$ is
(A) 6π
(B) 8π
(C) 12π
(D) $14\,\pi$

25. Let $\vec{F}(x,y,z)=2y\hat{\imath}+x^2\hat{\jmath}+xy\hat{k}$ and let $\mathcal{C}$ be the curve of intersection of the plane $x+y+z=1$ and the cylinder $x^2+y^2=1$. Then the value of
$\left|\oint_{\mathcal{C}}\vec{F}\cdot d\vec{r}\right|$ is
(A) π
(B) $\frac{3\pi}{2}$
(C) 2π
(D) 3π

26. The tangent line to the curve of intersection of the surface $x^2+y^2-z=0$ and the plane $x+z=3$ at the point $(1,1,2)$ passes through
(A) $(-1,-2,4)$
(B) $(-1,4,4)$
(C) $(3,4,4)$
(D) $(-1,4,0)$

27. The set of eigenvalues of which one of the following matrices is NOT equal to the set of eigenvalues of $\begin{pmatrix}1 & 2\\ 4 & 3\end{pmatrix}$?

(A) $\begin{pmatrix}1 & 4\\ 2 & 3\end{pmatrix}$

(B) $\begin{pmatrix}3 & 2\\ 4 & 1\end{pmatrix}$

(C) $\begin{pmatrix}3 & 4\\ 2 & 1\end{pmatrix}$

(D) $\begin{pmatrix}2 & 3\\ 1 & 4\end{pmatrix}$

28. Let $\{a_n\}$ be a sequence of positive real numbers. The series $\sum_{n=1}^{\infty} a_n$ converges if the series

(A) $\sum_{n=1}^{\infty} a_n^2$ converges

(B) $\sum_{n=1}^{\infty} \frac{a_n}{2^n}$ converges

(C) $\sum_{n=1}^{\infty} \frac{a_{n+1}}{a_n}$ converges

(D) $\sum_{n=1}^{\infty} \frac{a_n}{a_{n+1}}$ converges

29. For $\beta \in \mathbb{R}$, define

$$f(x,y) = \begin{cases} \frac{x^2|x|^\beta y}{x^4+y^2}, x \neq 0 \\ 0, \quad x = 0 \end{cases}$$

Then, at $(0,0)$, the function f is

(A) continuous for $\beta = 0$

(B) continuous for $\beta > 0$

(C) not differentiable for any β

(D) continuous for $\beta < 0$

30. Let $\{a_n\}$ be a sequence of positive real number such that $a_1 = 1, a_{n+1}^2 - 2a_n a_{n+1} - a_n = 0$ for all $n \geq 1$

Then the sum of the series $\sum_{n=1}^{\infty} \frac{a_n}{3^n}$ lies in the interval

(A) $(1,2]$

(B) $(2,3]$

(C) $(3,4]$

(D) $(4,5]$

SECTION – B

MULTIPLE SELECT QUESTION (MSQ)

Q. 31 – Q. 40 carry two marks each.

31. Let G be a noncyclic group of order 4. Consider the statements I and II:

I. There is NO injective (one - one) homomorphism from G to $\mathbb{Z}_8$

II. There is NO Surjective (onto) homomorphism from $\mathbb{Z}_8$ to G

Then

(A) I is true

(B) I is false

(C) II is true

(D) II is false

32. Let G be a nonabelian group, $y \in G$, and let the maps f, g, h from G to itself be defined by $f(x) = yxy^{-1}, g(x) = x^{-1}$ and $h = g \circ g$

Then
(A) g and h are homomorphism and f is not a homomorphism
(B) h is a homomorphism and g is not a homomorphism
(C) f is a homomorphism and g is not a homomorphism
(D) f, g and h are homomorphism

33. Let S and T be linear transformation from a finite dimensional vector space V to itself such that $S(T(v)) = 0$ for all $v \in V$. Then
(A) $rank\ (T) \geq nullity\ (S)$
(B) $rank(S) \geq nullity\ (T)$
(C) $rank(T) \leq nullity\ (S)$
(D) $rank\ (S) \leq nullity\ (T)$

34. Let $\vec{F}$ and $\vec{G}$ be differentiable vector fields and let g be a differentiable scalar function. Then
(A) $\nabla \cdot (\vec{F} \times \vec{G}) = \vec{G} \cdot \nabla \times \vec{F} - \vec{F} \cdot \nabla \times \vec{G}$
(B) $\nabla \cdot (\vec{F} \times \vec{G}) = \vec{G} \cdot \nabla \times \vec{F} + \vec{F} \cdot \nabla \times \vec{G}$
(C) $\nabla \cdot (g\vec{F}) = g\nabla \cdot \vec{F} - \nabla \mathrm{g} \cdot \vec{F}$
(D) $\nabla \cdot (g\vec{F}) = g\nabla \cdot \vec{F} + \nabla \mathrm{g} \cdot \vec{F}$

35. Consider the intervals $S = (0,2]$ and $T = [1,3)$. Let $S°$ and $T°$ be the sets of interior points of S and T, respectively. Then the set of interior points of $S \backslash T$ is equal to
(A) $S \backslash T°$
(B) $S \backslash T$
(C) $S° \backslash T°$
(D) $S° \backslash T$

36. Let $\{a_n\}$ be the sequence given by
$a_n = \max\left\{\sin\left(\frac{n\pi}{3}\right), \cos\left(\frac{n\pi}{3}\right)\right\}, n \geq 1$
Then which of the following statements is/are TRUE about the subsequences $\{a_{6n-1}\}$ and $\{a_{6n+4}\}$?
(A) Both the subsequence are convergent
(B) Only one of the subsequences is convergent
(C) $\{a_{6n-1}\}$ converges to $-\frac{1}{2}$
(D) $\{a_{6n+4}\}$ converges to $\frac{1}{2}$

37. Let
$f(x) = \cos(|\pi - x|) + (x - \pi)\sin|x|$ and $g(x) = x^2$ for $x \in \mathbb{R}$
If $h(x) = f(g(x))$, then
(A) h is not differentiable at $x = 0$
(B) $h'(\sqrt{\pi}) = 0$
(C) $h''(x) = 0$ has a solution in $(-\pi, \pi)$
(D) there exists $x_0 \in (-\pi, \pi)$ such that $h(x_0) = x_0$

38. Let $f: \left(0, \frac{\pi}{2}\right) \to \mathbb{R}$ be given by
$f(x) = (\sin x)^{\pi} - \pi \sin x + \pi$
Then which of the following statements is/are TRUE?
(A) f is an increasing function
(B) f is a decreasing function
(C) $f(x) > 0$ for all $x \in \left(0, \frac{\pi}{2}\right)$

(D) $f(x) < 0$ for some $x \in \left(0, \frac{\pi}{2}\right)$

39. Let

$$f(x,y) = \begin{cases} \frac{|x|}{|x|+|y|}\sqrt{x^4+y^2}(x,y) \neq (0,0) \\ 0, \quad (x,y) = (0,0) \end{cases}$$

Then at $(0,0)$,

(A) f is continuous

(B) $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y}$ does not exist

(C) $\frac{\partial f}{\partial x}$ does not exist and $\frac{\partial f}{\partial y} = 0$

(D) $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y} = 0$

40. Let $\{a_n\}$ be the sequence of real numbers such that

$a_1 = 1$ and $a_{n+1} = a_n + a_n^2$ for all $n \geq 1$

Then

(A) $a_4 = a_1(1+a_1)(1+a_2)(1+a_3)$

(B) $\lim_{n\to\infty} \frac{1}{a_n} = 0$

(C) $\lim_{n\to\infty} \frac{1}{a_n} = 1$

(D) $\lim_{n\to\infty} a_n = 0$

SECTION – C
NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

41. Let x be the 100 – cycle (1 2 3 … 100) and let y be the transposition $(49 \quad 50)$ in the permutation group S_{100}. They the order of xy is _____

42. Let W_1 and W_2 be subspaces of the real vector space $\mathbb{R}^{100}$ defined by

$W_1 = \{(x_1, x_2, \dots, x_{100}) : x_i = 0 \; if \; i \; is \; divisible \; by \; 4\}$

$W_2 = \{(x_1, x_2, \dots, x_{100}) : x_i = 0 \; if \; i \; divisible \; by \; 5\}$

Then the dimension of $W_1 \cap W_2$ is _____

43. Consider the following system of three linear equation is four unknowns x_1, x_2, x_3 and x_4

$x_1 + x_2 + x_3 + x_4 = 4,$

$x_1 + 2x_2 + 3x_3 + 4x_4 = 5,$

$x_1 + 3x_2 + 5x_3 + kx_4 = 5$

If the system has no solutions, then $k =$ _____

44. Let $\vec{F}(x,y) = -y\hat{\imath} + x\hat{\jmath}$ and let $\mathcal{C}$ be the ellipse

$\frac{x^2}{16} + \frac{y^2}{9} = 1$

Oriented counter clockwise. Then the value of $\oint_{\mathcal{C}} \vec{F} \cdot d\vec{r}$ (round off to 2 decimal places) is _____

45. The coefficient of $\left(x - \frac{\pi}{2}\right)$ in the Taylor series expansion of the function

$$f(x) = \begin{cases} \frac{4(1-\sin x)}{2x-\pi}, & x \neq \frac{\pi}{2} \\ 0, & x = \frac{\pi}{2} \end{cases}$$

About $x = \frac{\pi}{2}$, is _____

46. Let $f: [0,1] \to \mathbb{R}$ be given by

$$f(x) = \frac{\left(1+x^{\frac{1}{3}}\right)^3 + \left(1-x^{\frac{1}{3}}\right)^3}{8(1+x)}$$

Then

$\max\{f(x): x \in [0,1]\} - \min\{f(x): x \in [0,1]\}$ is _____

47. If

$g(x) = \int_{x(x-2)}^{4x-5} f(t)dt$, where $f(x) = \sqrt{1+3x^4}$ for $x \in \mathbb{R}$

Then $g'(1) =$_____

48. Let

$$f(x,y) = \begin{cases} \frac{x^3+y^3}{x^2-y^2}, & x^2 - y^2 \neq 0 \\ 0, & x^2 - y^2 = 0 \end{cases}$$

Then the directional derivative of f at $(0,0)$ in the direction of $\frac{4}{5}\hat{\imath} + \frac{3}{5}\hat{\jmath}$ is _____

49. The value of the integral

$\int_{-1}^{1}\int_{-1}^{1}|x+y|\,dx\,dy$

(round off to 2 decimal places) is _____

50. The volume of the solid bounded by the surfaces $x = 1 - y^2$ and $x = y^2 - 1$, and the planes $z = 0$ and $z = 2$ (round off to 2 decimal places) is _____

Q. 51 - Q. 60 carry two marks each.

51. The volume of the solid of revolution of the loop of the curve $y^2 = x^4(x+2)$ about the x-axis (round off to 2 decimal places) is _____

52. The greatest lower bound of the set

$\left\{(e^n + 2^n)^{\frac{1}{n}}: n \in \mathbb{N}\right\}$,

(round off to 2 decimal places) is _____

53. Let $G = \{n \in \mathbb{N}: n \leq 55, \gcd(n,55) = 1\}$ be the group under multiplication modulo 55. Let $x \in G$ be such that $x^2 = 26$ and $x > 30$. Then x is equal to _____

54. The number of critical points of the function

$f(x,y) = (x^2 + 3y^2)e^{-(x^2+y^2)}$ is _____

55. The number of elements in the set $\{x \in S_3: x^4 = e\}$, where e is the identity element of the permutation group S_3, is _____

56. If $\begin{pmatrix} 2 \\ y \\ z \end{pmatrix}, y, z \in \mathbb{R}$, is an eigenvector corresponding to a real eigenvalue of the matrix $\begin{pmatrix} 0 & 0 & 2 \\ 1 & 0 & -4 \\ 0 & 1 & 3 \end{pmatrix}$ then $z - y$ is equal to _____

Q.57 Let M and N be any two 4×4 matrices with integer entries satisfying

$$MN = 2\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Then the maximum value of $det(M) + det(N)$ is _____

58. Let M be a 3×3 matrix with real entries such that $M^2 = M + 2I$ where I denotes the 3×3 identity matrix. If α, β and γ are eigenvalues of M such that $\alpha\beta\gamma = -4$, then $\alpha + \beta + \gamma$ is equal to _____

59. Let $y(x) = xv(x)$ be a solution of the differential equation

$x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 3y = 0$

If $v = (0) = 0$ and $v(1) = 1$, then $v(-2)$ is equal to _____

60. If $y(x)$ is the solution of the initial value problem

$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0, y(0) = 2, \frac{dy}{dx}(0) = 0$

Then $y(\ln 2)$ is (round off to 2 decimal places) equal to _____