IFAS PREVIOUS YEAR QUESTION PAPER – IIT JAM MATHS 2023

Joint Admission Test for Masters 2023_12th Feb Shift 1

Q.1 Let $v_1 \ldots, v_9$, be the column vectors of a non-zero 9×9 real matrix A. Let $a_1, \ldots, a_9 \in \mathbb{R}$, not all zero, be such that $\sum_{i=1}^{9} a_i v_i = 0$. Then the system $Ax = \sum_{i=1}^{9} v_i$ has

1. more than one but only finitely many solutions
2. no solution
3. infinitely many solutions
4. a unique solution

Q.2 Let $a_n = \left(1 + \frac{1}{n}\right)^n$ and $b_n = n \cos\left(\frac{n!\pi}{2^{10}}\right)$ for $n \in \mathbb{N}$. Then

1. (a_n) is NOT convergent and (b_n) is bounded
2. (a_n) is convergent and (b_n) is bounded
3. (a_n) is NOT convergent and (b_n) is unbounded
4. (a_n) is convergent and (b_n) is unbounded

Q.3 Let $f: \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $f(x, y) = (e^x \cos(y), e^x \sin(y))$. Then the number of points in $\mathbb{R}^2$ that do NOT lie in the range of f is

1. 2
2. 0
3. 1
4. infinite

Q.4 Let $p(x) = x^{57} + 3x^{10} - 21x^3 + x^2 + 21$ and
$q(x) = p(x) + \sum_{j=1}^{57} p^{(j)}(x)$ for all $x \in \mathbb{R}$,
where $p^{(j)}(x)$ denotes the j^{th} derivative of $p(x)$. Then the function q admits

1. a global minimum but NOT a global maximum on $\mathbb{R}$
2. NEITHER a global maximum NOR a global minimum on $\mathbb{R}$
3. a global minimum and a global maximum on $\mathbb{R}$
4. a global maximum but NOT a global minimum on $\mathbb{R}$

Q.5 The limit
$\lim_{a \to 0} \left(\frac{\int_0^a \sin(x^2) dx}{\int_0^a (\ln(x+1))^2 dx} \right)$
is

1. $\frac{\pi}{e}$
2. non-existent
3. 0
4. 1

Q.6 Consider the initial value problem
$\frac{dy}{dx} + \alpha y = 0,$
$y(0) = 1,$
Where $\alpha \in \mathbb{R}$.

1. there is an a such that $y(1) = 0$

2. there is NO a such that $y(2) = 1$
3. there is a unique a such that $\lim_{x\to\infty} y(x) = 0$
4. there is a unique α such that $y(1) = 2$

Q.7 Let (a_n) be a sequence of real numbers defined by

$$a_n = \begin{cases} 1 & if\ n\ is\ prime \\ -1 & if\ n\ is\ not\ prime \end{cases}$$

1. (a_n) is NOT convergent but (b_n) is convergent
2. both (a_n) and (b_n) are convergent
3. (a_n) is convergent but (b_n) is NOT convergent
4. both (a_n) and (b_n) are NOT convergent

Q.8 The value of

$$\int_0^1 \int_0^{1-x} \cos(x^3 + y^2)\, dydx - \int_0^1 \int_0^{1-y} \cos(x^3 + y^2)\, dxdy$$

Is

1. $\frac{\cos(1)}{2}$
2. $\frac{\sin(1)}{2}$
3. $\cos\left(\frac{1}{2}\right) - \sin\left(\frac{1}{2}\right)$
4. 0

Q.9 Let G be a finite group. Then G is necessarily a cyclic group if the order of G is

1. 10
2. 6
3. 7
4. 4

Q.10 Which of the following is a subspace of the real vector space $\mathbb{R}^3$?

1. $\{(x, y, z) \in \mathbb{R}^3 : y \in \mathbb{Q}\}$
2. $\{(x, y, z) \in \mathbb{R}^3 : (y + z)^2 + (2x - 3y)^2 = 0\}$
3. $\{(x, y, z) \in \mathbb{R}^3 : x + 2y - 3z + 1 = 0\}$
4. $\{(x, y, z) \in \mathbb{R}^3 : yz = 0\}$

Q.11 Let (a_n) be a sequence of real numbers such that the series $\sum_{n=0}^{\infty} a_n(x-2)^n$ converges at $x = -5$. Then this series also converges at

1. $x = 9$
2. $x = 12$
3. $x = -6$
4. $x = 5$

Q.12 Consider the following statements:

I. Every infinite group has infinitely many subgroups.

II. There are only finitely many non-isomorphic groups of a given finite order.

Then

1. I is FALSE but II is TRUE
2. I is TRUE but II is FALSE
3. both I and II are TRUE
4. both I and II are FALSE

Q.13 Suppose $f:(-1,1)\to\mathbb{R}$ is an infinitely differentiable function such that the series $\sum_{j=0}^{\infty} a_j \frac{x^j}{j!}$ converges to $f(x)$ for each $x \in (-1,1)$, where,

$a_j = \int_0^{\pi/2} \theta^j \cos^j(\tan\theta)\, d\theta + \int_{\pi/2}^{\pi} (\theta-\pi)^j \cos^j(\tan\theta)\, d\theta$

For $j \geq 0$. Then

1. f is a non-constant even function on (-1, 1)
2. $f(x) = 0$ for all $x \in (-1,1)$
3. f is NEITHER an odd function NOR an even function on (-1, 1)
4. f is a non-constant odd function on (-1, 1)

Q.14 Let $a_n = \frac{1+2^{-2}+\cdots+n^{-2}}{n}$ for $n \in \mathbb{N}$. Then

1. both the sequence (a_n) and the series $\sum_{n=1}^{\infty} a_n$ are NOT convergent
2. the sequence (a_n) is NOT convergent but the series $\sum_{n=1}^{\infty} a_n$ is convergent
3. the sequence (a_n) is convergent but the series $\sum_{n=1}^{\infty} a_n$ is NOT convergent
4. both the sequence (a_n) and the series $\sum_{n=1}^{\infty} a_n$ are convergent

Q.15 Let $a_n = \sin\left(\frac{1}{n^3}\right)$ and $b_n = \sin\left(\frac{1}{n}\right)$ for $n \in \mathbb{N}$. Then

1. $\sum_{n=1}^{\infty} a_n$ is NOT convergent but $\sum_{n=1}^{\infty} b_n$ is convergent
2. $\sum_{n=1}^{\infty} a_n$ is convergent but $\sum_{n=1}^{\infty} b_n$ is NOT convergent
3. both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are convergent
4. both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are NOT convergent

Q.16 Let

$$A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 0 & 0 \\ -2 & 2 & 2 \end{pmatrix}$$

and $B = A^5 + A^4 + I_3$. Which of the following is NOT an eigenvalue of B?

1. 3
2. 2
3. 49
4. 1

Q.17 Consider the family of curves $x^2 + y^2 = 2x + 4y + k$ with a real parameter $k > -5$. Then the orthogonal trajectory to this family of curves passing through (2, 3) also passes through

1. $(3,4)$
2. $(3,5)$
3. $(1,0)$
4. $(-1,1)$

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Q.18 Let $y : \mathbb{R} \to \mathbb{R}$ be a twice differentiable function such that y^n is continuous on $[0, 1]$ and $y(0) = y(1) = 0$. Suppose $y''(x) + x^2 < 0$ for all $x = [0, 1]$. Then

1. $y(x) = 0$ has exactly one solution in (0, 1)
2. $y(x) > 0$ for all $x \in (0,1)$
3. $y(x) = 0$ has more tha one solution in (0, 1)
4. $y(x) < 0$ for all $x \in (0,1)$

Q.19 How many group homomorphisms are there from $\mathbb{Z}_2$ to S_5?

1. 41
2. 26
3. 25
4. 40

Q.20 Let S and T be non-empty subsets of $\mathbb{R}^2$, and W be a non-zero proper subspace of $\mathbb{R}^2$. Consider the following statements:

I. If $span(S) = \mathbb{R}^2$, then $span(S \cap W) = W$.

II. $span(S \cup T) = span\,(S) \cup span(T)$.

Then

1. II is TRUE but I is FALSE
2. both I and II are TRUE
3. I is TRUE but II is FALSE
4. both I and II are FALSE

Q.21 Let $f: \mathbb{R} \to \mathbb{R}$ be an infinitely differentiable function such that f'' has exactly two distinct zeroes. Then

1. f has at most 3 distinct zeroes
2. f has at least 2 distinct zeroes
3. f' has at least 1 zero
4. f' has at most 3 distinct zeroes

Q.22 Let

$f(x, y) = \iint_{(u-x)^2+(v-y)^2 \le 1} e^{-\sqrt{(u-x)^2+(v-y)^2}} dudv$

Then $\lim_{n\to\infty} f(n, n^2)$ is

1. non-existent
2. $\pi(1 - e^{-1})$
3. 0
4. $2\pi(1 - 2e^{-1})$

Q.23 Let (a_n) and (b_n) be sequences of real numbers such that

$|a_n - a_{n+1}| = \frac{1}{2^n}$ and $|b_n - b_{n+1}| = \frac{1}{\sqrt{n}}$ for $n \in \mathbb{N}$

then

1. both (a_n) and (b_n) need NOT be Cauchy sequences
2. (a_n) is a Cauchy sequence but (b_n) need NOT be a Cauchy sequence
3. both (a_n) and (b_n) are Cauchy sequences

4. (a_n) need NOT be a Cauchy sequence but (b_n) is a Cauchy sequence

Q.24 The area of the curved surface
$S = \{(x, y, z) \in \mathbb{R}^3 : z^2 = (x-1)^2 + (y-2)^2\}$
lying between the planes z = 2 and z = 3 is
1. 9π
2. $4\pi\sqrt{2}$
3. $5\pi\sqrt{2}$
4. $9\pi\sqrt{2}$

Q.25 Consider the following statements:
I. There exists a linear transformation from R3 to itself such that its range space and null space are the same.
II. There exists a linear transformation from R^2 to itself such that its range space and null space are the same.
Then
1. II is TRUE but I is FALSE
2. both I and II are FALSE
3. I is TRUE but II is FALSE
4. both I and II are TRUE

Q.26 Let $f(x, y) = ex^2 + y^2$ for $(x, y) \in \mathbb{R}^2$, and a be the determinant of the matrix

$$\begin{pmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} \end{pmatrix}$$

evaluated at the point $\big(cos(n), sin(n)\big)$. Then the limit $\lim_{n\to\infty} a_n$ is
1. $12e^2$
2. 0
3. non-existent
4. $6e^2$

Q.27 From the additive group $\mathbb{Q}$ to which one of the following groups does there exist a non-trivial group homomorphism?
1. $\mathbb{Q}^X$, the multiplicative group of non-zero rational numbers
2. $\mathbb{R}^X$, the multiplicative group of non-zero real numbers
3. $\mathbb{Z}$, the additive group of integers
4. $\mathbb{Z}_2$, the additive group of integers modulo 2

Q.28 The system of linear equations in x_1, x_2, x_3

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & 1 \\ 2 & 3 & \alpha \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ \beta \end{pmatrix}$$

where $\alpha, \beta \in \mathbb{R}$, has

1. NO solution for any α when $\beta \neq 5$
2. at least one solution for any α and β
3. a unique solution for any β when $\alpha \neq 1$
4. infinitely many solutions for any α when $\beta = 5$

29. Let $f(x,y) =; \ln(1+x^2+y^2)$ for $(x,y) \in \mathbb{R}^2$. Define

$$P = \left.\frac{\partial^2 f}{\partial x^2}\right|_{(0,0)} \qquad Q = \left.\frac{\partial^2 f}{\partial x \partial y}\right|_{(0,0)}$$

$$R = \left.\frac{\partial^2 f}{\partial y \partial x}\right|_{(0,0)} \qquad P = \left.\frac{\partial^2 f}{\partial y^2}\right|_{(0,0)}$$

Then

1. PS – QR < 0 and P < 0
2. PS – QR < 0 and P > 0
3. PS – QR > 0 and P > 0
4. PS – QR > 0 and P < 0

Q.30 Let $f(x) = cos(x)$ and $g(x) = 1 - \frac{x^2}{2}$ for $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then

1. $f(x) - g(x)$ changes sign exactly once on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
2. $f(x) > g(x)$ for all $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
3. $f(x) - g(x)$ changes sign more than once on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
4. $f(x) \leq g(x)$ for all $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Q.31 Fro each $t \in (0,1)$, the surface P_t in $\mathbb{R}^3$ is defined by
$P_t = \{(x,y,z) : (x^2+y^2)z = 1, t^2 \leq x^2+y^2 \leq 1\}$.
Let $a_t \in \mathbb{R}$ be surface area of P_t. Then

1. the limit $\lim_{t \to 0^+} a_t$ does NOT exist
2. $a_t = \iint_{t^2 \leq x^2+y^2 \leq 1} \sqrt{1 + \frac{4x^2}{(x^2+y^2)^2} + \frac{4y^2}{(x^2+y^2)^2}}\, dxdy$
3. $a_t = \iint_{t^2 \leq x^2+y^2 \leq 1} \sqrt{1 + \frac{4x^2}{(x^2+y^2)^4} + \frac{4y^2}{(x^2+y^2)^4}}\, dxdy$
4. the limit $\lim_{t \to 0^+} a_t$ exists

Q.32 Which of the following is/are true?

1. Every bijective linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ maps pairs of parallel lines to pairs of parallel lines
2. Every surjective linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ maps lines onto lines
3. Every linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ maps lines onto points or lines
4. Every bijective linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ maps pairs of perpendicular lines to pairs of perpendicular lines

Q.33 Let $f : (-1,1) \to \mathbb{R}$ be a differentiable function satisfying $f(0) = 0$. Suppose there exists an $M > 0$ such that $|f'(x)| \leq M|x|$ for all $x \in (-1,1)$. Then

1. f' is differentiable at x = 0
2. ff' is differentiable at x = 0
3. $(f')^2$ is differentiable at x = 0
4. f' is continuous at x = 0

Q.34 Let R_1 and R_2 be the radii of convergence of the power series $\sum_{n=1}^{\infty}(-1)^n x^{n-1}$ and $\sum_{n=1}^{\infty}(-1)^n \frac{x^{n+1}}{n(n+1)}$, respectively. Then

1. $\sum_{n=1}^{\infty}(-1)^n \frac{x^{n+1}}{n(n+1)}$ converges for all $x \in [-1,1]$
2. $R_1 = R_2$
3. $R_2 > 1$
4. $\sum_{n=1}^{\infty}(-1)^n x^{n-1}$ converges for all $x \in [-1,1]$

Q.35 Let $f: \mathbb{R}^2 \to \mathbb{R}$ be defined as follows :

$$f(x,y) = \begin{cases} \frac{x^4y^3}{x^6+y^6} & if \quad (x,y) \neq (0,0) \\ 0 & if \quad (x,y) = (0,0) \end{cases}$$

Then

1. $\left.\frac{\partial f}{\partial y}\right|_{(0,0)}$ exists and equals 0
2. $\lim_{t\to 0} \frac{f(t,2t)-f(0,0)}{t}$ exists and equals $\frac{1}{3}$
3. $\lim_{t\to 0} \frac{f(t,t)-f(0,0)}{t}$ exists and equals $\frac{1}{2}$
4. $\left.\frac{\partial f}{\partial x}\right|_{(0,0)}$ exists and equals 0

Q.36 Which of the following is/are linear transformations?

1. $T : \mathbb{R} \to \mathbb{R}$ given by $T(x) = \sin(x)$
2. $T : \mathbb{R}^2 \to \mathbb{R}$ given by $T(x,y) = x + y + 1$
3. $T : M_2(\mathbb{R}) \to \mathbb{R}$ given by $T(A) = trace(A)$
4. $T : P_2(\mathbb{R}) \to \mathbb{R}$ given by $T\big(p(x)\big) = p(1)$

Q.37 A subset $S \subseteq \mathbb{R}^2$ is said to be bounded if there is an $M > 0$ such that $|x| \leq M$ and $|y| \leq M$ for all $(x,y) \in S$. Which of the following subsets of $\mathbb{R}^2$ is/are bounded?

1. $\{(x,y) \in \mathbb{R}^2 : e^{x^2} + y^2 \leq 4\}$
2. $\{(x,y) \in \mathbb{R}^2 : |x| + |y| \leq 4\}$
3. $\{(x,y) \in \mathbb{R}^2 : x^4 + y^2 \leq 4\}$
4. $\{(x,y) \in \mathbb{R}^2 : e^{x^3} + y^2 \leq 4\}$

Q.38 Which of the following functions is/are Riemann integrable on [0, 1]?

1. $f(x) = \begin{cases} x & if\ x \in [0,1) \\ 0 & if x = 1 \end{cases}$
2. $f(x) = \begin{cases} x\sin(1/x) & if\ x \neq 0 \\ 0 & if\ x = 0 \end{cases}$
3. $f(x) = \begin{cases} 1 & if\ x \in \mathbb{Q} \cap [0,1] \\ -1 & otherwise \end{cases}$

4. $f(x) = \int_0^x \left|\frac{1}{2} - t\right| dt$

Q.39 Let $A \subseteq \mathbb{Z}$ with $0 \in A$. For $r, 8 \in Z$, define
$rA = \{ra: a \in A\}, rA + SA = \{ra + sb: a, b \in A\}$.
Which of the following conditions imply that A is a subgroup of the additive group $\mathbb{Z}$?
1. $2A \subseteq A, A + A = A$
2. $-2A \subseteq A, A + A = A$
3. $A = -A, A + A = A$
4. $A = -A, A + 2A = A$

Q.40 Let $y : (\sqrt{2/3}, \infty) \to \mathbb{R}$ be the solution of
$(2x - y)y' + (2y - x) = 0,$
$y(1) = 3.$
Then
1. y' is bounded on $(1, \infty)$
2. y' is bounded on $(\sqrt{2/3}, 1)$
3. $y(3) = 1$
4. $y(2) = 4 + \sqrt{10}$

Q.41 If y is the solution of
$y'' - 2y' + y = e^x,$
$y(0) = 0, y'(0) = -1/2,$
then $y(1)$ is equal to _____.(rounded off to two decimal places)
Given 0

Q.42 Let $T : P_2(\mathbb{R}) \to P_4(\mathbb{R})$ be the linear transformation given by $T\big(p(x)\big) = p(x^2)$. Then the rank of T is equal to _____.
Given 2

Q.43 The value of

$$\lim_{n\to\infty} \left(n \int_0^1 \frac{x^n}{x+1} dx \right)$$

is equal to _____. (rounded off to two decimal places)
Given 0.5

Q.44 The value of $\lim_{n\to\infty} \left(1 + \frac{1}{2^n} + \frac{1}{3^n} + \cdots + \frac{1}{(2023)^n}\right)^{\frac{1}{n}}$ is equal to _____.
(rounded off to two decimal places)
Given 1

Q.45 The sum of the series $\sum_{n=1}^{\infty} \frac{2n+1}{(n^2+1)(n^2+2n+2)}$ is equal to _____.
(rounded off to two decimal places)
Given 1.00

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Q.46 Let V be the volume of the region $S \subseteq \mathbb{R}^3$ defined by
$S = \{(x, y, z) \in \mathbb{R}^3 : xy \leq x \leq 4, 0 \leq x^2 + y^2 \leq 1\}$.
Then $\frac{V}{\pi}$ is equal to _____. (rounded off to two decimal places)
Given. ____

Q.47 Let $f(x) = \sqrt[3]{x}$ for $x \in (0, \infty)$, and $\infty(h)$ be a function such that

$$f(3 + h) - f(3) = hf'(3 + \theta(h)h)$$

for all $h \in (-1, 1)$. Then $\lim_{h \to \theta} \theta(h)$ is equal to ____.
(rounded off to two decimal places)
Given 0

Q.48 For $\sigma \in S_8$, let $o(\sigma)$ denote the order of σ. Then $\max\{o(\sigma) : \sigma \in S_8\}$ is equal to _____.
Given 40320

Q.49 For $g \in \mathbb{Z}$, let $\overline{g} \in \mathbb{Z}_8$ denote the residue class of g modulo 8. Consider the group $\mathbb{Z}_8^X = \{\overline{x} \in \mathbb{Z}_8 : 1 \leq x \leq 7, ged(x, 8) = 1\}$ with respect to multiplication modulo 8. The number of group isomorphisms from $\mathbb{Z}_8^X$ onto itself is equal to _____.
Given 4

Q.50 Let $f : \mathbb{R}^2 \to \mathbb{R}$ be the function defined as follows:

$$f(x, y) = \begin{cases} (x^2 - 1)^2 \cos^2\left(\frac{y^2}{(x^2 - 1)^2}\right) & if\ x \neq \pm 1 \\ 0 & if\ x = \pm 1 \end{cases}$$

The number of points of discontinuity of $f(x, y)$ is equal to
Given 0

Q.51 Let $y : (1, \infty) \to \mathbb{R}$ be the solution of the differential equation

$$y'' - \frac{2y}{(1 - x)^2} = 0$$

satisfying $y(2) = 1$ and $\lim_{x \to \infty} y(x) = 0$. Then $y(3)$ is equal to _____.
(rounded off to two decimal places)
Given -0.5

Q.52 Let $f : \mathbb{R}^3 \to \mathbb{R}$ be defined as $f(x, y, z) = x^3 + y^3 + z^3$, and let $L : \mathbb{R}^3 \to \mathbb{R}$ be the linear map satisfying

$$\lim_{(x,y,z) \to (0,0,0)} \frac{f(1 + x, 1 + y, 1 + z) - f(1,1,1) - L(x, y, z)}{\sqrt{x^2 + y^2 + z^2}} = 0$$

Then $L(1, 2, 4)$ is equal to _____. (rounded off to two decimal places)
Given 157

Q.53 The number of permutations in S_4 that have exactly two cycles in their cycle decompo-sitions is equal to _____.
Given 9

Q.54 Let S be the triangular region whose vertices are $(0,0), \left(0, \frac{\pi}{2}\right)$, and $\left(\frac{\pi}{2}, 0\right)$. The value of $\iint_S \sin(x)\cos(y)\,dxdy$ is equal to _____.
(rounded off to two decimal places)
Given-1

Q.55 The maximum number of linearly independent eigenvectors of the matrix

$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 1 & 3 \end{pmatrix}$$

Is equal to _____.
Given 3

Q.56 Let W be the subspace of $M_3(\mathbb{R})$ consisting of all matrices with the property that the sum of the entries in each row is zero and the sum of the entries in each column is zero. Then the dimension of W is equal to _____.
Given 3

Q.57 The global minimum value of

$$f(x) = |x-1| + |x-2|^2$$

on $\mathbb{R}$ is equal to _____. (rounded off to two decimal places)
Given ____

Q.58 Let

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 3 \\ 1 & 1 & 4 & 4 & 4 \end{pmatrix}$$

and B be a 5×5 real matrix such that AB is the zero matrix. Then the maximum possible rank of B is equal to _____.
Given 3

Q.59 Let $f : \mathbb{R} \to \mathbb{R}$ be a bijective function such that for all $x \in \mathbb{R}, f(x) = \sum_{n=1}^{\infty} a_n x^n$ and $f^{-1}(x) \sum_{n=1}^{\infty} b_n x^n$, where f^{-1} is the inverse function of f. If $a_1 = 2$ and $a_2 = 4$, then b_1 is equal to _____.
Given ___

Q.60 Let S be the set of all real numbers a such that the solution y of the initial value problem

$$\frac{dy}{dx} = y(2-y)$$
$$y(0) = \alpha$$

exists on $[0, \infty)$. Then the minimum of the set S is equal to _____.
(rounded off to two decimal places)
Given 0