CSIR NET MATHEMATICAL SCIENCE JULY 2025
(IFAS SOLVED PAPER)

PART-A

Q1. Consider the following statements:

Statement I: All Booklets are Manuals.

Statement II: All Manuals are Catalogues.

If Statements I and II are True, which one of the following conclusions can be conclusively drawn?

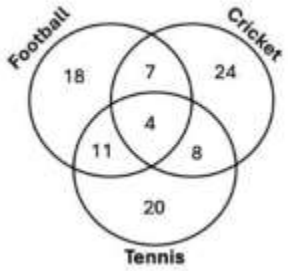
1. All Manuals are Booklets.

2. All Catalogues are Booklets.

3. All Booklets are Catalogues.

4. All Catalogues are Manuals.

Q2. The given Venn diagram shows numbers of players playing one or more than one sport.



The percentage of players who play exactly two sports is closest to_____%.

1. 5

2. 14

3. 28

4. 32

Q3. The value of a company is measured as the total value of its shares owned by different investors. Rakesh owns 2/15 of the shares of a company. He sells 1/3 of his shares for Rs. 75,000/-. What is the total value of the company at that time?

1. Rs. 15, 75, 800

2. Rs. 16, 87, 500

3. Rs. 17, 75, 800

4. Rs. 18, 27, 500

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QUES 4: A car has wheels of diameter 36 cm . If it runs at a speed of 60 km/h, then the rotation per minute (RPM) will be closest to____.

1. 884

2. 898

3. 906

4. 986

Q5. A cylindrical container of radius 20 cm was filled with water up to 25 cm height. A solid spherical ball of radius 7 cm was then immersed in the water. What would be the approximate increase in water level in the container after the ball was fully immersed?

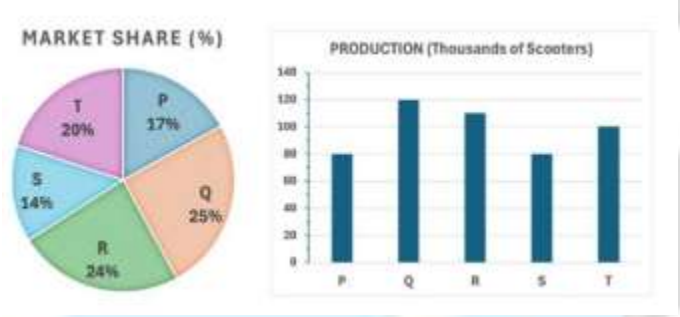
1. 1.14 cm

2. 2.28 cm

3. 5.50 cm

4. 7.00 cm

Q6. The market share (%) and annual production of scooters from five automobile companies P, Q, R, S, and T are shown in graphs.



If the profit of a company is directly proportional to the ratio of market share to production, then which of the following statements is/are CORRECT?

Statement X : Companies T and P have same profit

Statement Y : Company R has the maximum profit

Statement Z : Company S has the minimum profit

1. X and Y

2. X and Z

3. Y and Z

4. Only Z

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Q 7. Rahul and his father started jogging on a circular track of radius 'r'(r>2). Rahul completed one round and stopped. His father got tired half way into the first round and returned to his starting point along a straight line. What is the ratio of the distances covered by Rahul and his father?

1. $\pi r/(\pi + 2)$

2. $2\pi/(\pi + 2)$

3. 1

4. 2

Q 8. Kavita starts from her house and walks 200 m northward, then turns 45^o right and walks 70 m . After that, she turns 90^o right and walks 70 m . Which of the following is the closest value of the shortest distance between Kavita's current location and her house?

1. 296 m

2. 240 m

3. 200 m

4. 223 m

Q9. The initial monthly salaries of employees John, Riya, and Sunil were in the proportion 4:3:5. After an increase of Rs 10000 monthly to all, the new proportion becomes 6 : 5 : 7. What was the initial salary of Sunil?

1. Rs 20000

2. Rs 25000

3. Rs 30000

4. Rs 35000

Q10. Numbers of Rose, Lotus, and Marigold plants in a garden are in the proportion 8:5:7. Later, 75%, 40% and 50% more plants of their respective categories were added. What will be the new proportion of plants, in the same order?

1. 5 : 3 : 4

2. 4 : 2 : 3

3. 5 : 4 : 3

4. 7 : 4 : 5

Q11. What will be the digit at the unit's place of $1^3 + 2^3 + 4^3 + 5^3 + 6^3 + 7^3 + 8^3 + 9^3$?

1. 0

2. 5

3. 7

4. 9

Q12. Suresh asked Ramesh to identify the person in a photo that the latter is holding. Ramesh responds, "I have no brothers or sisters. However, that man's father is my father's son." Who is the person in the photo?

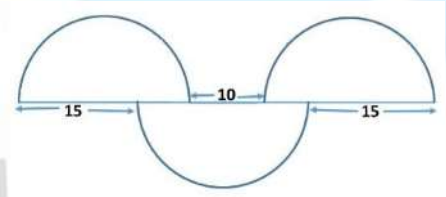
1. Suresh

2. Ramesh

3. Ramesh's son

4. Ramesh's cousin

Q13. Three identical semi-circles are arranged as shown. What is the diameter of the semi-circles?



1. 5π

2. 20

3. $15\pi/2$

4. 25

Q14. A number is mistakenly divided by 2 instead of being multiplied by 2 . What is the change in the result caused by this mistake?

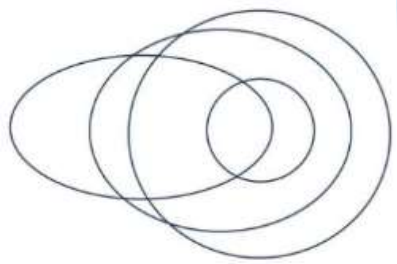
1. 25%

2. 50%

3. 75%

4. 100%

Q 15. The following diagram represents the relationship between four categories.



The categories could be

1. Rivers, water bodies, oceans, sources of evaporation

2. Parliamentarians, celebrities, elected persons, professional politicians

3. Monkeys, four-legged animals, pet animals, land animals

4. Furniture, chairs, seats, wooden objects

Q 16. In a code, the word DELTOID is written as 3152893 . Then LOTION could be written as

1. 582986

2. 582981

3. 198396

4. 198392

Q17. Sum of the digits of a two-digit number 'ab' is subtracted from the number and the result is divided by 9 . Then the result of this will be

1. always a

2. always b

3. neither a nor b

4. either a or b depending on a + b

Q18. A circle of radius 1 unit is divided into four quarters and rejoined as shown below.



What is the area of this shape?

1. π

2. 1

3. 2

4. 4

Q19. A stock market trader has lost two thirds of her investment on a day. Next day she recovered one third of the previous day's loss. What fraction of her initial investment is she left with?

1. $\frac{1}{2}$

2. $\frac{2}{3}$

3. $\frac{2}{9}$

4. $\frac{5}{9}$

Q20. Three friends, Mr. Rahman, Mr. George and Mr. Vedant, met after a long time. They were wearing red, green and violet colour shirts. Mr. Rahman and the person wearing violet shirt noticed that none of the three is wearing a colour that starts with same letter as his name. Which one of the following is the correct match of the persons with the colour of their shirts?

1. Rahman-Violet, George-Red, Vedant-Green

2. Rahman-Green, George-Violet, Vedant-Red

3. Rahman-Green, George-Red, Vedant-Violet

4. Rahman-Red, George-Violet, Vedant-Green

PART-B

Q21. Let p, q be non-negative integers. Consider the following statements:
(A) There is an integer $k \geq 1$ such that $p + k = q$.
(B) There is an integer $k \geq 1$ such that $q + k = p$.
Which of the following statements is true?
1. There exist non-negative integers p, q such that both (A) and (B) are true.
2. Both (A) and (B) are false if and only if p = q.
3. For all non-negative integers p and q. (A) or (B) is true.
4. There exists $p \neq q$ such that both (A) and (B) are false.

Q22. Let

$$A = \left\{\frac{\mathrm{p}}{\mathrm{q}} \in (0,1): \mathrm{p} \in \mathbb{N}, \mathrm{q} = 2^{\mathrm{n}} \text{ for some } \mathrm{n} \in \mathbb{N} \cup \{0\}, \gcd(\mathrm{p}, \mathrm{q}) = 1\right\}$$

$$B = \left\{\frac{\mathrm{p}}{\mathrm{q}} \in (0,1): \mathrm{p} \in \mathbb{N}, \mathrm{q} = 2^{\mathrm{n}}5^{\mathrm{m}} \text{ for some } \mathrm{n}, \mathrm{m} \in \mathbb{N} \cup \{0\}, \gcd(\mathrm{p}, \mathrm{q}) = 1\right\}$$

$$C = \left\{\frac{\mathrm{p}}{\mathrm{q}} \in (0,1): \frac{\mathrm{p}}{\mathrm{q}} \text{has terminating decimal expansion}\right\}$$

be subsets of (0, 1). Which of the following statements is true?
1. $A \subsetneq C$ and $B \subsetneq C$
2. $A \subsetneq C \subsetneq B$
3. $A \subsetneq B \subsetneq C$
4. $A \subsetneq B = C$

Q23. Let A, B be non-empty subsets of N with cardinality $|A| \geq 2$. Let
$S_1 = \{f: A \to B | f \text{ is one} - \text{to} - \text{one}\}$ and $S_2 = \{g: B \to A | g \text{ is onto}\}$.
Which of the following statements is true?
1. If $A \nsubseteq B$ and B is finite, then there is a one-to-one map from S_2 to S_1.

2. If $B = \mathbb{N}$, then there exists a one-to-one map from S_2 to B.
3. If $B = \mathbb{N}$ and A is finite, then there exists a one-to-one map from B to S_1.
4. If A is finite, then S_2 is finite for any B.

Q24. Let $f: \mathbb{R}\backslash\mathbb{Q} \to \mathbb{R}\backslash\mathbb{Q}$ be the function defined as

$$f(x) = \frac{3x+2}{4x+3}.$$

Let $x_1 \in \mathbb{R}\backslash\mathbb{Q}$. For $n \geq 1$, define $x_{n+1} = f(x_n)$. Suppose that the sequence $(x_n)_{n\geq 1}$ converges to a real number l. Which of the following statements is true?

1. If l is positive, then $l = \frac{\sqrt{3}}{2}$
2. If l is positive, then $l = \frac{1}{\sqrt{2}}$.
3. If l is negative, then $l = -\frac{\sqrt{3}}{2}$
4. If l is negative, then $l = -\frac{1}{2}$

Q25. Let $f(x) = x \log_e\left(1 + \frac{1}{x}\right)$ for $x \in (0, \infty)$. Which of the following statements is true?

1. f is unbounded.
2. f is increasing.
3. $\lim_{x\to\infty} f(x) = 2$.
4. f is decreasing.

Q26. For each $n \geq 1$, let $f_n: [0, 1] \to \mathbb{R}$ be defined as

$$f_n(x) = \begin{cases} nx & \text{if } x \in \left[0, \frac{1}{n}\right], \\ 2 - nx & \text{if } x \in \left(\frac{1}{n}, \frac{2}{n}\right], \\ 0 & \text{if } x \in \left(\frac{2}{n}, 1\right]. \end{cases}$$

Which of the following statements is true?

1. $(f_n)_{n\geq 1}$ converges uniformly on [0, 1] to a continuous function f.
2. $(f_n)_{n\geq 1}$ converges pointwise on [0, 1] to a discontinuous function f.
3. $(f_n)_{n\geq 1}$ converges pointwise on [0, 1] to a continuous function f.
4. $(f_n)_{n\geq 1}$ does not converge pointwise on [0, 1].

Q27. Which of the following polynomials is the characteristic polynomial of a real 2×2 matrix A such that $trace(A) = 7$ and $trace(A^2) = 29$?

1. $t^2 + 7t + 10$
2. $t^2 - 7t + 29$
3. $t^2 - 7t - 10$
4. $t^2 - 7t + 10$

Q28. Let $\mathbb{F}_5$ denote the field with 5 elements. How many 2×2 matrices with entries in $\mathbb{F}_5$ have rank one?

1. 125
2. 144
3. 145

4. 480

Q29. Let X be the $\mathbb{R}$-vector space of all twice differentiable real valued functions on [0,1]. Consider the linear map $\phi: X \to \mathbb{R}^3$ defined by $\phi(f) = \big(f(1), f'(1), f''(1)\big)$. Which of the following statements is true?

1. The dimension of $X \setminus \ker \phi$ is 3.
2. $\ker \phi$ is finite dimensional.
3. The dimension of $X/\ker \phi$ is 1.
4. X is finite dimensional.

Q30. Consider the real matrix $A = \begin{pmatrix} 29 & 0 & 55 & 17 \\ 1 & 28 & 46 & 26 \\ 17 & 13 & 33 & 38 \\ 21 & 67 & 0 & 13 \end{pmatrix}$. What is the largest real eigenvalue of A?

1. 101
2. 67
3. 103
4. 113

Q31. Let $C[0,\pi]$ be the real vector space of real-valued continuous functions on the closed interval $[0,\pi]$. For positive integers n, define $f \in C[0,\pi]$ by

$$f_n(x) = \begin{cases} \dfrac{\sin(nx)}{\sin x} & \text{if x} \in (0,\pi), \\ n & \text{if x} = 0, \\ (-1)^{n-1} n & if\ x = \pi. \end{cases}$$

Let V be the real subspace of $C[0,\pi]$ spanned by $\{f_1, f_2, f_3\}$. Consider the inner product on V given by

$$\langle f, g \rangle = \frac{2}{\pi} \int_0^{\pi} f(x)\, g(x)\, sin^2\, x dx.$$

Which of the following statements is true?

1. $f_4 \in V$
2. $\{f_1, f_2, f_3\}$ is an orthonormal basis of V.
3. The dimension of V is 2.
4. $\{f_1, f_2, f_3\}$ is an orthogonal set but not orthonormal.

Q32. Let $V = \{ax^3 + bx^2 + cx | a, b, c \in \mathbb{R}\}$. For $f \in V$, define

$$Q(f) = \int_{-1}^{1} \big(f'(t)\big)^2\, dt,$$

where f' denotes the derivative of f. Which of the following statements is FALSE?

1. Q is a positive definite quadratic form on V.
2. Q takes every positive real value.
3. $Q(x) = 2$.
4. For all $f, g \in V, Q(f+g) = Q(f) + Q(g)$.

Q33. Let $f: \mathbb{C} \to \mathbb{C}$ be a polynomial map. For $R > 0$, let $\gamma_R: [0,1] \to \mathbb{C}$ be the map $t \to Re^{2\pi i t}$. Suppose that there exists $c \in \mathbb{R}$ such that

$\int_0^1 |(f o \gamma_R)(t)\gamma_R'(t)|\, dt \to c \; as \; R \to \infty.$

Which of the following statements is FALSE?

1. The function $zf\left(\frac{1}{z}\right) \to 0$ as $|z| \to \infty$.
2. The function f is constant.
3. $c = 0$.
4. $c > 0$.

Q34. Let X be the image of the interval [0, 1] under the Möbius transformation $f(z) = \frac{z-i}{z+i}$. Which of the following statements is true?

1. X is the line segment joining -1 and $-i$.
2. $X = \left\{e^{i\theta} | \theta \in \left[\pi, \frac{3\pi}{2}\right]\right\}$.
3. X is the line segment joining -1 to 1.
4. $X = \left\{e^{i\theta} | \theta \in \left[-\frac{\pi}{2}, \pi\right]\right\}$.

Q35. Which of the following statements is true?

1. There exists an entire function f such that $f^{(n)}(0) = \frac{n!}{n^n}$ for all positive integers n.
2. There exists an entire function f such that $f^{(n)}(0) = n!\, n^n$ for all positive integers n.
3. There exists an entire function f such that $f^{(n)}(0) = (n-1)!$ for all positive integers n.
4. There exists an entire function f such that $f^{(n)}(0) = n!\, n$ for all positive integers n.

Q36. Let f be an entire function such that $f(\mathbb{C}) \subset \{x + iy | y = x + 1\}$. Which of the following statements is true?

1. $|f(z)| \to \infty \; as \; |z| \to \infty$.
2. $\frac{f(z)}{z} \to 0 \; as \; |z| \to \infty$.
3. $zf(z) \to 0 \; as \; |z| \to \infty$.
4. $f(z) \to 0 \; as \; |z| \to \infty$.

Q37. Which of the following statements is true?

1. $p | 1 + (p-1)!$ for some odd prime p.
2. $p | (1234)^{p-1} - 1$ for all primes $p > 700$.
3. There exist $\alpha \in \mathbb{Z}$ and a prime $p > 11$ such that $p | a^p - a$.
4. $p | \frac{(p^2)!}{(p!)^2}$ for some odd prime p.

Q38. Let A be a subring of the field of rationals $\mathbb{Q}$ such that for any nonzero rational $r \in \mathbb{Q}, r \in A$ or $1/r \in A$. Which of the following statements is FALSE?

1. The set $\left\{\alpha \in A : \frac{1}{\alpha} \notin A\right\} \cup \{0\}$ is an additive subgroup of $\mathbb{Q}$.
2. A has at most one maximal ideal.
3. If $A \neq \mathbb{Q}$, then A has infinitely many prime ideals.
4. For any nonzero $a, b \in A, \alpha$ divides b or b divides a in A.

Q39. Which of the following statements is true?

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1. The ideal $2\mathbb{Z}[i]$ is maximal in $\mathbb{Z}[i]$.
2. The ideal $X\mathbb{C}[X,Y]$ is maximal in $\mathbb{C}[X,Y]$.
3. The set of all polynomials in $\mathbb{C}[X]$ whose coefficients add up to 0 is a maximal ideal in $\mathbb{C}[X]$.
4. The ideal $\left(\sqrt{2}-1\right)\mathbb{Z}\left[\sqrt{2}\right]$ is maximal in $\mathbb{Z}\left[\sqrt{2}\right]$.

Q40. Let $S=\{1,2,3,4,5\}$ be equipped with the topology $\tau=\{\phi,\{1\},S\}$. What is the number of homeomorphisms of S onto itself?

1. 25
2. 120
3. 24
4. 6

Q41. If $\phi(x)=x$ is a solution of the ordinary differential equation (ODE)

$$\frac{d^2y}{dx^2}-\left(\frac{2}{x^2}+\frac{1}{x}\right)\left(x\frac{dy}{dx}-y\right)=0,0<x<\infty,$$

then the general solution of the ODE is given by

1. $(a+be^{-2x})\ x,a,b\in\mathbb{R}$
2. $(a+be^{2x})x,a,b\in\mathbb{R}$
3. $ae^{x}+bx,a,b\in\mathbb{R}$
4. $(a+be^{x})x,a,b\in\mathbb{R}$

Q42. Let $(\lambda_n)_{n\in\mathbb{N}}$ be the sequence of eigenvalues of the Sturm-Liouville problem

$$\frac{d}{dx}\left(x\frac{dy}{dx}\right)+\frac{\lambda}{x}y=0,1<x<e^{2\pi}.$$

$\mathrm{y}(1)=0,\mathrm{y}(\mathrm{e}^{2\pi})=0.$

Then $\sum_{n=1}^{\infty}\frac{1}{\lambda_n}$ is equal to

1. $\frac{\pi^2}{12}$
2. $\frac{2\pi^2}{3}$
3. $\frac{\pi^2}{4}$
4. $\frac{\pi^2}{16}$

Q43. Let $u=u(x,y)$ be the solution to the Cauchy problem

$$(y+u)\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=x-y,x\in\mathbb{R},y>0,$$

$u(x,1)=1+x,x\in\mathbb{R}.$

Then which of the following statements is true?

1. $u(1,1)=2$
2. $u(2,2)=4$
3. $u(3,3)=\frac{3}{2}$
4. $u(4,4)=\frac{2}{3}$

Q44. Let $u=u(x,t)$ be the solution of

$$\frac{\partial^2u}{\partial t^2}-\frac{\partial^2u}{\partial x^2}=0,x\in\mathbb{R},t>0,$$

$u(x,0) = 1 + x^2, x \in \mathbb{R}$,
$\frac{\partial u}{\partial t}(x,0) = x + 1, x \in \mathbb{R}$.
Then the value of u(1, 1) is
1. 2
2. 3
3. 4
4. 5

Q45. If the functions $s: [0,4] \to \mathbb{R}$ defined by

$$s(x) = \begin{cases} a(x-2)^2 + b(x-1)^2, & 0 \le x \le 1, \\ (x-2)^2, & 1 < x \le 3, \\ 2c(x-2)^2 + (x-3)^3, & 3 < x \le 4 \end{cases},$$

is a cubic spline, then the value of $2a + b + 2c$ is
1. 2
2. 3
3. 4
4. 5

Q46. Let y(x) be the extremal of the functional

$$J[y] = \int_0^{\frac{\pi}{4}} ((y')^2 - 4y^2 + 2xy)\, dx$$

subject to $y(0) = 0, y\left(\frac{\pi}{4}\right) = 1$. Then y(x) is equal to
1. $\left(1 - \frac{\pi}{4}\right) sin(2x) + x$
2. $\left(1 - \frac{\pi}{16}\right) sin(2x) + \frac{x}{4}$
3. $\left(1 + \frac{\pi}{4}\right) sin(2x) - x$
4. $\left(1 + \frac{\pi}{16}\right) sin(2x) - \frac{x}{4}$

Q47. If $y(x)$ is the solution of the integral equation

$$y(x) = x^2 + 2\int_0^1 x\, ty(t)dt,$$

then which of the following statements is true?
1. $y(0) + y(1) = \frac{1}{2}$
2. $y(-1) + y(1) = 1$
3. $y'(0) + y'(1) = \frac{3}{2}$
4. $y'(-1) + y'(1) = 3$

Q48. Suppose a dynamical system has the Lagrangian

$L = (\dot{q}_1)^2 + (\dot{q}_2)^2 + (\dot{q}_1)^2 + \dot{q}_1\dot{q}_2$

If p_1 and p_2 are momenta conjugate to q_1 and q_2, respectively, then which of the following statements is true?
1. $\dot{p}_1 = 2q_1, \dot{p}_2 = 0$

2. $\dot{p}_1 = -q_1, \dot{p}_2 = 0$
3. $\dot{p}_1 = -\frac{q_1}{2}, p_2 = q_2$
4. $\dot{p}_1 = q_1, p_2 = -q_2$

Q49. Suppose that we have a data set consisting of $2n + 1$ observations for some $n \in \mathbb{N}$. Value of each observation is either x or $x + r$, where $x \in \mathbb{N}, r \geq 0$. Then, which of the following statements is always true?

1. The mean and median of the data will be different if and only if $r > 0$
2. Variance of the data is positive if and only if $r > 0$
3. Mean and mode of the data will be same if and only if $r = 0$
4. Median and mode of the data will be same for all values of $r \geq 0$

Q50. A biased six-faced die is tossed once. Suppose that the probability of any prime number showing up is twice that of any non-prime number showing up. Then, the probability that an odd number will show up is

1. $\frac{1}{3}$
2. $\frac{2}{3}$
3. $\frac{4}{9}$
4. $\frac{5}{9}$

Q51. Let $Z_1, Z_2, \ldots$ be a sequence of independent and identically distributed random variables having discrete uniform distribution over $\{1,2,\ldots,2024\}$. Let $Y_n = \sum_{i=1}^{n} Z_i, n \geq 2$. Further, let X_n be the remainder when Y_n is divided by 2025. Then, which of the following statements is true?

1. $\lim_{n\to\infty} P(X_n = 0) = \frac{1}{2026}$
2. $\lim_{n\to\infty} P(X_n = 0) = \frac{1}{2025}$
3. $\lim_{n\to\infty} P(X_n = 0) = \frac{1}{2024}$
4. $\lim_{n\to\infty} P(X_n = 0) = \frac{1}{2023}$

Q52. A mobile manufacturing company uses two brands of batteries for its mobiles. The life (in years) of batteries of Brand I follows an exponential distribution with the probability density function

$$f(x) = \begin{cases} e^{-x}, & if\ x > 0, \\ 0, & \text{otherwise}, \end{cases}$$

and that of Brand II follows a gamma distribution with the probability density function

$$g(x) = \begin{cases} \frac{x}{4} e^{-\frac{x}{2}} & \text{if x} > 0, \\ 0, & \text{otherwise}. \end{cases}$$

The company uses the batteries of Brands I and II in proportion of 20% and 80% respectively, in its mobiles. The probability that a randomly selected mobile has the battery life more that 2 years is

1. $\frac{13}{5} e^{-2}$
2. $\frac{1}{5}(e^{-2} + 2e^{-1})$
3. $\frac{1}{5}(e^{-2} + 8e^{-1})$

4. $\frac{1}{5}(4e^{-2}+2e^{-1})$

Q53. Consider a discrete random variable X with the probability mass function

$P(X=0)=\frac{\theta}{3}, P(X=1)=1-\frac{\theta}{2}, P(X=2)=\frac{\theta}{6},$

where $\theta \in (0,1)$ is an unknown parameter. In a random sample of size 90 from this distribution, the observed counts for X = 0,1 and 2 are 20, 60 and 10, respectively. Then, the maximum likelihood estimate of θ is

1. $\frac{1}{3}$
2. $\frac{1}{2}$
3. $\frac{2}{3}$
4. $\frac{3}{4}$

Q54. Let X be a random sample of size 1 from the probability density function

$$f(x|\theta)=\begin{cases}\frac{3}{\theta^3}(\theta-x)^2, & if\ 0<x<\theta,\\ 0, & \text{otherwise.}\end{cases}$$

If $\left(\frac{X}{1-\lambda_1}, \frac{X}{1-\lambda_2}\right)$ is a confidence interval for with confidence coefficient $1-\alpha$, where $\lambda_i \in (0,1), i=1,2, \lambda_1<\lambda_2$, and $\alpha \in (0,1)$, then which of the following statements is true?

1. $\lambda_2^2-\lambda_1^2=1-\alpha$
2. $\lambda_2^3-\lambda_1^3=1-\alpha$
3. $\lambda_2^2-\lambda_1^2=4(1-\alpha)$
4. $\lambda_2^3-\lambda_1^3=9(1-\alpha)$

Q55. Let $X_1, X_2, \ldots, X_n$ be a random sample from a continuous distribution with the common probability density function

$$f(x|\theta)=\begin{cases}\frac{2^\theta\theta}{x^{\theta+1}}, & \text{if x} > 2,\\ 0, & \text{otherwise,}\end{cases}$$

where $\theta(>0)$ is an unknown parameter. Suppose $P\left(Y>X^2_{m,\beta}\right)=\beta$, where $Y \sim X^2_m$. For testing $H_0: \theta=1$ against $H_1: \theta>1$, a uniformly most powerful test of size $\alpha, 0<\alpha<$, will reject H_0 if

1. $\sum_{i=1}^n In\, X_i > \frac{1}{2}X^2_{2n,\alpha}+n\ ln2$
2. $\sum_{i=1}^n In\, X_i < \frac{1}{2}X^2_{2n,1-\alpha}+n\ ln2$
3. $\sum_{i=1}^n In\, X_i > X^2_{n,\alpha}+n\ ln2$
4. $\sum_{i=1}^n In\, X_i < X^2_{n,1-\alpha}+n\ ln2$

Q56. Suppose the distribution of X given θ is normal with mean θ and variance 15. Further, let the prior (improper) distribution of θ be proportional to $1, -\infty<\theta<\infty$. If the observed value of X is 13, then which of the following statements is true?

1. Posterior mean = Maximum likelihood estimate of θ, Posterior variance $= Var(X|\theta)$
2. Posterior mean = Maximum likelihood estimate of θ, Posterior variance $< Var(X|\theta)$
3. Posterior mean > Maximum likelihood estimate of θ, Posterior variance $= Var(X|\theta)$
4. Posterior mean > Maximum likelihood estimate of θ, Posterior variance $< Var(X|\theta)$

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Q57. Consider the multiple linear regression model $y_i = \beta_0 + \beta_1 x_1 + \cdots + \beta_8 x_{8i} + \epsilon_i, i = 1, 2, \ldots, 29$, where $\epsilon_1, \epsilon_2, \ldots, \epsilon_{29}$ are independent and identically normal distributed with mean 0 and variance σ^2. Suppose the model is fitted using the method of least squares. If the calculated value of the F-statistic for testing the significance of regression is 2.50, then the possible values of R^2 and Adjusted R^2 are respectively

1. 0.30 and 0.10
2. 0.50 and 0.30
3. 0.50 and 0.16
4. 0.30 and -0.10

Q58. Suppose $\begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} \sim N_3 \left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right)$ and $\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 3 & 0 & 0 \\ 2 & 2 & 0 \\ 4 & 0 & 4 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix}$. Then the partial correlation coefficient $pyz.x$ is

1. $\frac{1}{2}$
2. $\frac{2}{3}$
3. $\frac{3}{4}$
4. 0

Q59. Suppose we want to estimate the population mean $\hat{Y}$ of a variable for a finite population of size 85, with 34 Statisticians and 51 Biologists. We consider the following sampling scheme:

A stratified random sample with 2 strata of Statisticians (Stratum-1) and Biologists (Stratum- 2), where 12 Statisticians and 15 Biologists are drawn from Stratum-1 and Stratum-2, respectively, using SRSWOR scheme.

Denote $\hat{y}_S, \hat{y}_B$, and $\hat{y}$ as the mean of the variable among the Statistician sample, Biologist sample, and the combined sample, respectively. Which of the following is an unbiased estimator of $\hat{Y}$?

1. $\hat{y}$
2. $\frac{2\hat{y}_S + 3\hat{y}_B}{5}$
3. $\frac{4\hat{y}_S + 5\hat{y}_B}{9}$
4. $\frac{\frac{\hat{y}_S}{12} + \frac{\hat{y}_B}{15}}{\frac{1}{12} + \frac{1}{15}}$

Q60. Solve the following linear programming problem:

maximize z = x + y

subject to

$5x + 3y \leq 30$

$2x + 6y \leq 25$

$2x - y \leq 8$

$x \geq 0, y \geq 0.$

Then the optimal value of the objective function is

1. $\frac{45}{11}$
2. $\frac{74}{11}$

3. $\frac{85}{12}$
4. $\frac{25}{6}$

Q61. Consider the sequences $(S_n)_{n>1}$ and $(t_n)_{n>1}$ defined by

$S_n = \sum_{k=0}^{n} \frac{1}{(k!)^2}$ and $t_n = \sum_{k=0}^{n} \binom{n}{k} \frac{(-1)^k}{n^k}$.

Which of the following statements are true?

1. $\lim\limits_{n\to\infty} sup\ t_n \leq \lim\limits_{n\to\infty} sup\ s_n$
2. $\lim\limits_{n\to\infty} sup\ t_n \leq \boldsymbol{e}$
3. $\lim\limits_{n\to\infty} inf\ S_n \geq e^2$
4. $\lim\limits_{n\to\infty} inf\ t_n \geq e$

Q62. What is the value of the limit $\lim\limits_{n\to\infty} \frac{1}{n}[(n+1)(n+2)\ldots(n+n)]^{1/n}$?

1. $\frac{2}{e}$
2. $\frac{4}{e}$
3. $\log_e 2 - 1$
4. $2\log_e 2 - 1$

Q63. Let $(f_n)_{n>1}$ be a sequence of real-valued functions on $\mathbb{R}$. Which of the following statements are true?

1. If each f_n is uniformly continuous and $(f_n)_{n\geq 1}$ converges to f uniformly, then f is uniformly continuous.
2. If each f_n is bounded and $(f_n)_{n\geq 1}$ converges to f pointwise, then f is bounded.
3. If each f_n is bounded and continuous, $(f_n)_{n\geq 1}$ converges pointwise to a bounded and continuous function f, then the convergence is uniform.
4. If each f_n is differentiable and $(f_n)_{n\geq 1}$ converges to f uniformly, then f is differentiable.

Q64. Let f be a bounded, twice continuously differentiable real-valued function on $(0,\infty)$ such that $f(x) > 0$ for all $x \in (0,\infty)$. Which of the following statements are true?

1. $f'(x) < 0$ for all $x > 0$.
2. $\lim\limits_{x\to\infty} f'(x) = 0$.
3. $\lim\limits_{x\to\infty} f'(x)$ need not exist.
4. $\lim\limits_{x\to\infty} x\, f'(x) = 0$.

Q65. For each n ≥ 1, let $f_n: \mathbb{R} \to \mathbb{R}$ be a function defined by

$f_n(x) = \frac{e^{-n^2z^2}}{n}$

Which of the following statements are true?

1. $(f_n)_{n\geq 1}$ converges uniformly to 0 on $\mathbb{R}$ and $(f_n')_{n\geq 1}$ converges uniformly to 0 on the interval (-M, M) for some positive real number M.
2. $(f_n)_{n\geq 1}$ converges uniformly to 0 on $\mathbb{R}$ and $(f_n')_{n\geq 1}$ converges pointwise to 0 on $\mathbb{R}$
3. $(f_n)_{n\geq 1}$ converges uniformly to 0 on $\mathbb{R}$ and $(f_n')_{n\geq 1}$ does not converge pointwise to 0 on $\mathbb{R}$
4. $(f_n)_{n\geq 1}$ converges pointwise to 0 on $\mathbb{R}$ but not uniformly on $\mathbb{R}$

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Q66. Let f and g be real-valued Riemann integrable functions on [a, b] such that $g(a, b) \subseteq [a, b]$. Which of the following statements are necessarily true?

1. The composition f o g is Riemann integrable.
2. If $g(x) \neq 0$ for each $x \in [a, b]$, then $\frac{f}{g}$ is Riemann integrable.
3. The positive square root $\sqrt{f^2 + g^2}$ is Riemann integrable.
4. The composition f o g is Riemann integrable, if both f and g are continuous.

Q67. Let μ denote the Lebesgue measure on $\mathbb{R}$. Suppose that f is a non-negative Lebesgue measurable function on $\mathbb{R}$. Let $0 = a_o < a_1 < a_2 < \cdots$ be an unbounded sequence such that $a_{n+1} \leq ca_n$ for some real number c and for all$n \geq 1$. Let $A = \{x \in \mathbb{R} | a_k \leq f(x) < a_{k+1}\}$ for each $k \geq 0$.Which of the following statements are true?

1. If f is Lebesgue integrable on $\mathbb{R}$, then $\sum_{k\geq 0} a_k \, \mu(Ak)$ is finite.
2. If $\sum_{k\geq 0} a_k \, \mu(Ak)$ is finite, then f is Lebesgue integrable on $\mathbb{R}$.
3. If $\sum_{k\geq 0} a_k \, \mu(Ak)$ is finite, and $f(x) > a_1$, for all $x \in \mathbb{R}$, then f is Lebesgue integrable on $\mathbb{R}$.
4. If $\sum_{k\geq 0} a_k \, \mu(Ak)$ is finite and f is bounded, then f is Lebesgue integrable on $\mathbb{R}$.

Q68. Let $f: \mathbb{R}^2 \to \mathbb{R}$ be a twice continuously differentiable non-zero function such that $f(t: x_1, tx_2) = t^3 f(x_1, x_2)$ for all t > 0 and $(x_1, x_2) \in \mathbb{R}^2$. Which of the following statements are necessarily true?

1. $3\frac{\partial f}{\partial x_1}(1,1) + 3\frac{\partial f}{\partial x_2}(1,1) - f(1,1)$
2. $\frac{\partial f}{\partial x_1}(1,-1) - \frac{\partial f}{\partial x_2}(1,-1) - 3f(1,-1)$
3. $x_1^2 \frac{\partial^2 f}{\partial x_1^2}(x_1, x_2) + x_2^2 \frac{\partial^2 f}{\partial x_1^2}(x_1, x_2) + 2x_1 x_2 \frac{\partial^2 f}{\partial x_{1,} \partial x_2}(x_1, x_2) = 6f(x_1, x_2)$
4. $x_1^2 \frac{\partial^2 f}{\partial x_1^2}(x_1, x_2) + x_2^2 \frac{\partial^2 f}{\partial x_1^2}(x_1, x_2) + 2x_1 x_2 \frac{\partial^2 f}{\partial x_{1,} \partial x_2}(x_1, x_2) = 9f(x_1, x_2)$

Q69. Let $M_2(\mathbb{R})$ denote the space of real 2×2 matrices. Let S be the vector subspace of $M_2(\mathbb{R})$ comprising of all symmetric matrices. Let $F: M_2(\mathbb{R}) \to S$ be the map defined by $F(X) = XX^T$. Let $DF_A: M_2(\mathbb{R}) \to S$ be the derivative of F at $A \in M_2(\mathbb{R})$. Which of the following statements are true?

1. If $AA^T = I$, then $DF_A: M_2(\mathbb{R}) \to S$ is surjective.
2. If $AA^T = I$, then $DF_A: M_2(\mathbb{R}) \to S$ need not be surjective.
3. If A is invertible, then $DF_A: M_2(\mathbb{R}) \to S$ is surjective.
4. If A is not invertible, then $DF_A: M_2(\mathbb{R}) \to S$ is surjective.

Q70. Let $C[0, 1]$ be the $\mathbb{R}$-vector space of real valued continuous functions equipped with the Norm $\|f\| = \sup_{x\in[0,1]} |f(x)|$

Let $T: C[0, 1] \to C[0, 1]$ be defined as $T(f)(x) = \int_0^z f(t)dt, for\ x \in [0,1]$.

Let $T^n = ToTo \ldots oT$ (n times). Which of the following statements are true?

1. There exists $a \in (0,1)$ such that for all $f, g \in C[0,1], ||T(f) - T(g)|| \leq a\|f - g\|$
2. There exists $a \in (0,1)$ such that for all $f, g \in C[0,1], ||T^2(f) - T^2(g)|| \leq a\|f - g\|$
3. The set $\{f \in C[0,1] T(f) = f\}$ is a singleton set.
4. $\|T^n\| \to \infty \ as\ n \to \infty$.

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Q71. Let V be the $\mathbb{R}$ -vector space of all polynomials with real coefficients. Let $f(x) = x^2 + x + 1$. Which of the following subsets of V are linearly independent?
1. $\{f'(x), f(x) - f(x-1), 1\}$
2. $\{f(x+1) - f(x), f(x) - f(x-1), 1\}$
3. $\{f(x), f'(x), 1\}$
4. $\{f(x+1), f(x-1), f(x)\}$

Q72. Consider the field $\mathbb{F}_3$ consisting of 3 elements. Let V be an $\mathbb{F}_3$-vector space of dimension 3 and W an $\mathbb{F}_3$-vector space of dimension 2. Which of the following statements are true?
1. The number of two dimensional subspaces of V is 13.
2. The number of surjective linear transformations from V to W is 624.
3. The number of one dimensional subspaces of V is 13.
4. The number of linear transformations from V to W is 3^6.

Q73. Let A, B be distinct 2×2 real matrices. Which of the following statements are true?
1. If A is invertible, then AB and BA have the same minimal polynomial.
2. If 0 is an eigenvalue of A, then 0 is an eigenvalue of AB.
3. If 0 is the only eigenvalue of A and of B, then 0) is the only eigenvalue of AB.
4. If AB and BA have the same minimal polynomial, then either A or B is invertible.

Q74. Let $S, T: \mathbb{C}^2 \to \mathbb{C}^2$ be $\mathbb{C}$ -linear transformations and I denote the identity transformation on $\mathbb{C}^2$. Which of the following statements are necessarily true?
1. $(ST - TS)^2 - \lambda I$ AI for some $\lambda \in \mathbb{C}$
2. The characteristic polynomial of $(ST - TS)^2$ is $(x - \lambda)^2$ for some $\lambda \in \mathbb{C}$
3. If ST - TS has only one eigenvalue, then $ST - TS = \lambda I$ for some $\lambda \in \mathbb{C}$
4. If ST - TS has only one eigenvalue, then $(ST - TS)^2$ is the zero transformation.

Q75. Let $T: \mathbb{C} \to \mathbb{C}$ be a C-linear operator with eigenvalues 2, 3 and 5. Consider the subspace $W = \{v \in \mathbb{C}^T (T - 5I)^K v = 0$ for some integer $k > 0\}$ of $\mathbb{C}^T$. Suppose that $((T - 2I)^2 (T - 3I)^2 (T - 5I)^2 = 0$. Which of the following statements are necessarily true?
1. T has at least four linearly independent eigenvectors.
2. $\dim W \geq 2$.
3. $\ker((T - 2I)^{2025}) = \ker((T - 2I)^{2026})$
4. $(T - 2I)(T - 3I)$ is a nilpotent operator.

Q76. Which of the following matrices are similar over $\mathbb{R}$ to the matrix $A = \begin{pmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$?

1. $\begin{pmatrix} 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

2. $\begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

3. $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

4. $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

Q77. Let V be a finite dimensional complex inner product space. For a linear map $\mathrm{T}: \mathrm{V} \to \mathrm{V}$, let T denote its adjoint. Which of the following statements are true?

1. If trace of $\mathrm{T} = \mathrm{T}^*$ is zero, then T = 0.
2. Let $\mathrm{u} \in \mathrm{V}$ be such that $\mathrm{T}^*\mathrm{T}(\mathrm{v}) = 0$. Then $\mathrm{T}(\mathrm{v}) = 0$.
3. Suppose $\mathrm{T} = \mathrm{T}^*$ and $\mathrm{N} > 1$ be an integer. Let $\mathrm{v} \in \mathrm{V}$ be such that $\mathrm{T}^{x^N}(\mathrm{v}) = 0$. Then $\mathrm{T}(\mathrm{v}) = 0$.
4. Suppose that $\mathrm{TT}^* = \mathrm{T}^*\mathrm{T}$ and N > 1 be an integer. Let $\mathrm{u} \in \mathrm{V}$ be such that $\mathrm{T}^N(\mathrm{u}) = 0$. Then $\mathrm{T}(\mathrm{v}) = 0$.

Q78. Let $B(v, w)$ be a nondegenerate symmetric bilinear form on $\mathbb{R}^2$ and let $q(v) = B(v, v)$ be the corresponding quadratic form. Suppose there exist vectors $v, w \in \mathbb{R}^2$ such that $B(u, v) = 0$ and $B(v, w) \neq 0$. Which of the following statements are necessarily true?

1. $B(w, w) = 0$
2. There exists an $\alpha \in \mathbb{R}$ such that $q(\alpha v + w) = 0$.
3. There are infinitely many $\alpha \in \mathbb{R}$ such that $q(\alpha v + w) = 0$.
4. q is equivalent to the quadratic form
$Q(x, y) = x^2 - y^2 \; for \; all \; (x, y) \in \alpha \in \mathbb{R}^2$.

Q79. Let f be an entire function which is not a polynomial. Let $\mathrm{A} = \{\alpha \in \mathbb{C} | \mathrm{f}^{(\mathrm{n})}(\alpha) \neq 0 \text{ for all } \mathrm{n} \geq 0\}$ Which of the following statements are true?

1. A is nonempty.
2. A is finite.
3. A is infinite.
4. A is uncountable.

Q80. Let f be an entire function. Which of the following statements are true?

1. If $f(z) = f(z + 1)$ for all $z \in \mathbb{C}$ then f is a constant function.
2. If $f(z) = f(z + 1) = f(z + i)$ for all $z \in \mathbb{C}$ then f is a constant function.
3. If $f\left(\frac{1}{z}\right)$ has a removable singularity at 0 then f is a constant function.
4. If f is a non-constant function then $f\left(\frac{1}{z}\right)$ has a pole at 0.

Q81. Let $\gamma: [0, 1] \to \mathbb{C}$ be the function $\mathrm{t} \to \mathrm{e}^{2\pi \mathrm{i} \mathrm{t}}$ and $\mathrm{I} = \int_\gamma \mathrm{e}^{\mathrm{z}} \, \mathrm{e}^{\frac{1}{\mathrm{z}}} \mathrm{dz}$ Which of the following statements are true?

1. $\mathrm{I} = 0$

2. $\frac{1}{2\pi i} I \in \{4n: n \in Z, n \geq 1\}$
3. $I = 2\pi i \sum_{n=0}^{\infty} \frac{1}{n!}$
4. $I = 2\pi i \sum_{n=0}^{\infty} \frac{1}{n!(n+1)!}$

Q82. Let $\mathbb{D}^x = \{z \in \mathbb{C}: 0 < |z| < 1\}$ be the punctured unit disk and f be a bijective holomorphic map of $\mathbb{D}^x$ onto itself. Which of the following statements are true?

1. $\lim_{z \to 0} f(z)$ does not exist.
2. $\lim_{z \to 0} f(z)$ exists and has absolute value ≤ 1
3. $\lim_{z \to 0} f(z)=0$
4. There exists $\theta \in \mathbb{R}$ such that $f(z) = e^{\theta} z$ for all $z \in \mathbb{D}^x$

Q83. Let G be a group, H a subgroup of G, and $T = \{gH | g \in G\}$, the set of all left cosets of H in G. Let S_T be the set of all permutations of T and $\pi: G \to S_T$ be the map defined by $\pi(g)(g_1 H) = g g_1 H$. For a prime number p, let $\mathbb{F}_p$ denote the field with p elements. In which of the following cases is $ker\ \pi$ trivial?

1. $G = GL_2(\mathbb{F}_p)$ and H is a subgroup of order p.
2. $G = SL_2(\mathbb{F}_p)$ and H is a subgroup of order p.
3. $p = 3(mod\ 4), G = GL_2(\mathbb{F}_p)/SL_2(\mathbb{F}_p)$ and H is a subgroup of order 2.
4. $p = 1(mod\ 4), G = GL_2(\mathbb{F}_p)/SL_2(\mathbb{F}_p)$ and H is a subgroup of order 2.

Q84. For a group G, let Aut(G) denote the group (under composition) of all bijective group homomorphisms from G onto itself. Which of the following statements are true?

1. If G_1, G_2 are two groups such that $Aut(G_1)$ is isomorphic to $Aut(G_2)$, then G_1 is isomorphic to G_2,
2. If $|G| = 2$, then $Aut(G \times G)$ is abelian.
3. If G is the group of complex numbers under addition, then $Aut(G)$ is abelian.
4. If G is finite, then $Aut(G)$ is finite.

Q85. Let G_1 and G_2 be subgroups of a group G. Which of the following statements are true?

1. If G_1 is normal in G, then $(G_2 G_1)/G_1 G_2/(G_2 \cap G_1)$.
2. If H_1 and H_2 are normal subgroups of G_1 and G_2, respectively, then $(G_1 \times G_2)/(H_1 \times H_2) \approx (G_1/H_1) \times (G_2/H_2)$.
3. If G_1 is normal in G_2 and G_2 is normal in G, then G_1 is normal in G.
4. Every subgroup of prime index in G is normal.

Q86. Let $f: \mathbb{Q}[X] \to \mathbb{Q}[X]$ be a ring homomorphism with $f(1) = 1$. For $n \geq 1$, let $f^n = \underbrace{fo \ldots of}_{\text{n times}}$. Which of the following statements are true?

1. If f is onto, then so is f^n for all n ≥ 1.
2. $\ker f^{n+1} = \ker f^n$ for some n > 1.
3. If f is onto, then f is one-to-one.
4. If f is one-to-one, then f is onto.

Q87. Let $f(x) = X^5 + X + 1 \in Q[X]$ and $g(X) = X^5 - X + 1 \in Q[X]$. Which of the following statements are true?

1. $f(X)$ is irreducible in $\mathbb{Q}[X]$, but $g(X)$ is not.
2. $g(X)$ is irreducible in $\mathbb{Q}[X]$,, but $f(X)$ is not.

3. Both $f(X)$ and $g(X)$ are irreducible in $\mathbb{Q}[X]$,.
4. Neither $f(X)$ nor $g(X)$ is irreducible in $\mathbb{Q}[X]$,

Q88. Let $p > 2$ be a prime number. Let $\mathbb{F}_p$ denote the field with p elements and $\overline{\mathbb{F}}_p$ an algebraic closure of $\mathbb{F}_p$. Which of the following statements are true?

1. Let $f(X) \in \mathbb{F}_p[X]$ and α be a root of f in $\overline{\mathbb{F}}_p$. Then $\mathbb{F}_p(\alpha)$ is the splitting field of f in $\overline{\mathbb{F}}_p$.
2. Let $f, g \in \overline{\mathbb{F}}_p[X]$ be irreducible polynomials of same degree and a be a root of f in $\overline{\mathbb{F}}_p$. Then $\mathbb{F}_p(\alpha)$ is the splitting field of g in $\overline{\mathbb{F}}_p$.
3. $\mathbb{F}_p[X]$ has infinitely many irreducible polynomials.
4. The set $\{a + b | a, b \in \mathbb{F}_p\}$ is contained in $\{a^2 + b^2 | a, b \in \mathbb{F}_p\}$.

Q89. Let $p: \mathbb{R} \to \mathbb{R}$ be a nonconstant polynomial. Which of the following statements are true?

1. The preimage of a compact set under p is a compact set.
2. The preimage of a connected set under p is a connected set.
3. Every point $x \in \mathbb{R}$ has an open neighborhood U_x such that the restriction p|v, is a homeomorphism onto an open set in $\mathbb{R}$.
4. The image of a bounded set under p is a bounded set.

Q90. Consider $\mathbb{R}$ with the usual topology and $S = \{a + b\sqrt{2} \mid a, b \in \mathbb{Q}\} \subset \mathbb{R}$ with the subspace topology. Which of the following statements are true?

1. S is dense in $\mathbb{R}$.
2. $S \backslash Q$ is dense in $\mathbb{R}$.
3. $S \backslash Q$ is discrete with subspace topology on S.
4. S is connected.

Q91. Let $D = \{(x, y) \in \mathbb{R}^2 : -1 \leq a \leq 1, -1 \leq y \leq 1\}$, and $f: D \to \mathbb{R}$ be the function defined by $f(x, y) = 1 + \sqrt{y_+}$, where $y_+ = max\{y, 0\}$. Consider the initial value problem (IVP)

$\frac{dy}{dx} = f(x, y), y(0) = 0.$

Then which of the following statements are true?

1. f is a Lipschitz continuous function on D
2. f is NOT a Lipschitz continuous function on D
3. IVP has at least one solution
4. IVP has NO solution

Q92. Consider the ordinary differential equation (ODE)

$\frac{d^2y}{dx^2} + (\cos(x))\frac{dy}{dx} + (\sin(x))y = 0$.

Let $\varphi_1(x), \varphi_2(x)$ be solutions of the ODE, satisfying

$\varphi_1(0) = 1, \frac{d\varphi_1}{dx}(0) = 0$, and $\varphi_2(0) = 0, \frac{d\varphi_2}{dx}(0) = 1$.

Then which of the following statements are true?

1. $\varphi_1(x + 2\pi)$is also a solution of ODE
2. $\varphi_2(x + 4\pi)$ is also a solution of ODE
3. There are NO constants a, b such that $\varphi_2(x + 4\pi) = a\varphi_1(x) + b\varphi_2(x)$
4. There exist $a, b \in \mathbb{R}$ such that $\varphi_1(x + 2\pi) = a\varphi_1(x) + b\varphi_2(x)$.

Q93. For a continuous function q defined on $\mathbb{R}$, consider the ordinary differential equation (ODE)

$\frac{d^2y}{dx^2} + q(x)y = 0, x \in \mathbb{R}.$

Then which of the following statements are FALSE?

1. There exists a q such that cos(x) and e^x cos(x) are solutions of ODE
2. There exists a q such that sin(x) and cos(x) are solutions of ODE
3. There exists a q such that e^x sin(x) and e^x cos(2x) are solutions of ODE
4. There exists a q such that xe^x and $x(x-1)e^x$ are solutions of ODE

Q94. Consider the Cauchy problem (CP)

$x\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 1.$

$u(0, y) = e^y, y \in R.$

Then which of the following statements are true?

1. There is NO neighbourhood of the origin on which (CP) has a solution
2. (CP) has a unique solution defined on some neighbourhood of the origin
3. (CP) has a unique solution defined on some neighbourhood of the point (0, 1) in the xy-plane
4. (CP) has an infinite number of solutions, each of which is defined on some neigh- bourhood of the origin

Q95. Suppose $\mathrm{u} = \mathrm{u}(\mathrm{x}, \mathrm{y})$ is the solution of the boundary value problem

$\frac{\partial^2 \mathrm{u}}{\partial \mathrm{x}^2} + \frac{\partial^2 \mathrm{u}}{\partial \mathrm{y}^2} + \frac{\partial \mathrm{u}}{\partial \mathrm{x}} = 0$ in $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < 1\}$

$\mathrm{u}(\mathrm{x}, \mathrm{y}) = 1 + 2\mathrm{x}^2\mathrm{y}^2$ on $\{(\mathrm{x}, \mathrm{y}) \in \mathbb{R}^2 : \mathrm{x}^2 + \mathrm{y}^2 = 1\}$

Then which of the following statements are true?

1. The minimum value of u is 1
2. The maximum value of u is 3
3. The minimum value of u is 2
4. The maximum value of u is $\frac{3}{2}$

Q96. Consider the initial value problem (IVP) $y' + y = 0, y(0) = 1$. Let (y_n) be the iterates of forward Euler method, applied to the IVP, with step h where 0 < h < 1. Then which of the following statements are true?

1. The sequence (y_n) does NOT converge
2. $y_n \to 0$ as $n \to \infty$
3. $0 \leq y_n \leq 1$ for n =0,1,2,.....
4. $|y(nh) - y_n| \to 0$ as $n \to \infty$

Q97. If $\alpha, \beta \in \mathbb{R}$ are such that the equation $\int_0^3 f(x)dx = \frac{3}{2}[f(a) + f(\alpha + \beta)]$ holds for all polynomials f(x) of degree less then or equal to 2, then which of the following statements are true?

1. $(\alpha, \beta) = \left(\frac{3-\sqrt{3}}{2}, \sqrt{3}\right)$ or $(\alpha, \beta) = \left(\frac{3+\sqrt{3}}{2}, -\sqrt{3}\right)$
2. $(\alpha, \beta) = \left(\frac{3-\sqrt{2}}{2}, \sqrt{2}\right)$ or $(\alpha, \beta) = \left(\frac{3+\sqrt{2}}{2}, -\sqrt{2}\right)$
3. $(\alpha, \beta) = \left(\frac{3-\sqrt{5}}{2}, \sqrt{5}\right)$ or $(\alpha, \beta) = \left(\frac{3+\sqrt{5}}{2}, -\sqrt{5}\right)$
4. $(\alpha, \beta) = \left(\frac{3-\sqrt{7}}{2}, \sqrt{7}\right)$ or $(\alpha, \beta) = \left(\frac{3+\sqrt{7}}{2}, -\sqrt{7}\right)$

Q98. For $\alpha \geq 0$, consider the functional

$$J_{\alpha[y]} = \int_1^2 \frac{(y')^2}{x^\alpha} dx$$

defined for all continuously differentiable functions defined on the interval [1, 2] satisfying the conditions $y(1) = 1, y(2) = 2.$

Then which of the following statements are true?

1. $y(x) = \frac{1}{15}(x^4 + 14)$ is an extremal for J_3
2. $y(x) = \frac{1}{3}(x^2 + 2)$ is an extremal for J_1
3. $y(x) = x$ is an extremal for J_0
4. $y(x) = \frac{1}{2}(x^2 - x + 2)$ is an extremal for J_1

Q99. For $a, b, c \in \mathbb{R}$, consider the variational problem:

Minimize $J[y] = \int_0^2 [(y')^2 + 2byy' + cy^2]\, dx$

subject to $y(0) = 10, y(1) = 100.$

Then which of the following statements are true?

1. If (a, b, c) = (-2, 1, -2), then every admissible extremal is a minimizer
2. If (a, b, c) = (1,0, 2), then every admissible extremal is a minimizer
3. If (a, b, c) = (2,-1, 1), then every admissible extremal is a minimizer
4. If (a, b, c) = (1, -2, 5), then every admissible extremal is a minimizer

Q100. Let u(x) be the solution to the Volterra integral equation

$$u(x) = x^2 + 4\int_0^x (t - x)^2\, u(t) dt$$

Then which of the following statements are true?

1. $u(0) = 0$
2. $u\left(\frac{2\pi}{\sqrt{3}}\right) = \frac{1}{6}\left(e^{\frac{4\pi}{\sqrt{3}}} - e^{-\frac{2\pi}{\sqrt{3}}}\right)$
3. $u\left(\frac{\pi}{2\sqrt{3}}\right) = \frac{1}{6}\left(e^{\frac{\pi}{\sqrt{3}}} - \sqrt{3}e^{-\frac{\pi}{2\sqrt{3}}}\right)$
4. $u\left(\frac{\pi}{2\sqrt{3}}\right) = \frac{1}{6}\left(e^{\frac{\pi}{\sqrt{3}}} + \sqrt{3}e^{-\frac{\pi}{\sqrt{3}}}\right)$

Q101. Let f and K be such that the solution of the initial value problem

$y'' - 3y' + 2y = 4\sin(x), y(0) = 1, y'(0) = -2.$

satisfies the Volterra integral equation

$y(x) = f(x) + \int_0^x K(x,t)\, y(t)\, dt.$

Then which of the following statements are true?

1. $f'(\pi) = 3$
2. $f(\pi) + f'(\pi) = 4 - \pi$
3. $f(\pi) + f'(\pi) = 2 - \pi$
4. $f(0) + f'(0) = -4$

Q102. Suppose f, g are smooth functions of generalized coordinates $q_1, q_2, \ldots, q_n$ the associated conjugate momenta $p_1, p_2, \ldots, p_n$, and time t. Let [f, g] denote the Poisson bracket of f and g. Suppose H is a Hamiltonian of the system.

Then which of the following statements are true?

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1. $\frac{\partial}{\partial t}[f,g] = \left[\frac{\partial f}{\partial t}, g\right] + \left[f, \frac{\partial g}{\partial t}\right]$
2. If f is a constant of motion, and f is independent of t, then [H, f] is a constant of motion
3. $[[H, f.g] + [[g, H.f] + \left[[f,g], H\right] = 0$
4. If f and g are constants of motion, then [f, g] is a constant of motion

Q103. Let X and Y be independent Poisson random variables with means 4 and 2, respectively. Then, which of the following statements are true?

1. The conditional distribution of X given X + Y = 3 is Binomial $\left(3, \frac{1}{3}\right)$
2. $P(X \leq 1 | X + Y = 3) = \frac{7}{27}$
3. $E(X|X + Y = 3) = 2$
4. The value of the characteristic function of $X + Y$ at the point $t = \pi$ is e^{-12}

Q104. Let X be a random variable with the cumulative distribution function

$$F_X(x) = \begin{cases} 0, & if\ x < 0, \\ \frac{z+2}{5}, & if\ 0 \leq x < 2 \\ 1, & if\ x \geq 2 \end{cases}$$

Then, which of the following statements are true?

1. $E(X^2) = \frac{4}{3}$
2. $P\left(\left|X - \frac{4}{5}\right| \geq 1\right) = \frac{19}{25}$
3. The upper bound of $P\left(\left|X - \frac{4}{5}\right| \geq 1\right)$, using Chebyshev's inequality, is $\frac{52}{75}$
4. The value of the moment generating function, $M_x(t)$, at t = 1 is $\frac{e^2-1}{5}$

Q105. Consider a Markov chain $\{X_n : n \geq 1\}$ on state space $\{1,2,3,4,5\}$ with the transition probability matrix

$$P = \begin{pmatrix} 0 & 1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1/3 & 0 & 1/3 & 1/3 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Then, which of the following statements are true?

1. Stationary distribution is $(0,0,0,0,1)$.
2. State 5 is absorbing and recurrent.
3. All states are aperiodic.
4. $\lim_{n\to\infty} P_{55}^{(n)} = 1$

Q106. Let $X_1, X_2, \ldots$ be a sequence of independent and identically distributed random variables with $E(X_1) = 0, E(X_1^2) = 1, E(X_1^3) = 0, E(X_1^4) = 3$. Let $S_n = \sum_{i=1}^n X_i, T_n = \sum_{i=1}^n X_i^2, U_n = \sum_{i=1}^n X_i^4$, Then, which of the following statements are true?

1. $\frac{S_n}{\sqrt{n}}$converges in distribution to a random variable Z, where $Z \sim N(0,1)$
2. $\frac{T_n - n}{\sqrt{3n}}$ converges in distribution to a random variable Z, where $Z \sim N(0,1)$
3. $\frac{\sqrt{n}S_n}{T_n}$ converges in distribution to a random variable Z, where $Z \sim N(0,1)$

4. $\frac{T_n-n}{\sqrt{V_n}}$ converges in distribution to a random variable Z, where $Z\sim N(0,1)$

Q107. Let X and Y be independent and identically distributed N(0, 1) random variables. Let $S-X^2+Y^2$ and $T=e^{-(X^2+Y^2)/2}$, Then, which of the following statements are true?

1. The probability density function of S is $f_s(x)=\begin{cases}\frac{1}{2}e^{-x/2}, & if\ s>0\\ 0, & otherwise\end{cases}$
2. The probability density function of T is $f_x(x)=\begin{cases}1, & if\ 0<t<1\\ 0, & otherwise\end{cases}$
3. Var(S) = 2.
4. $E(T)=\frac{2}{3}$

Q108. Let $(X_1,Y_1),(X_2,Y_2),\ldots.,(X_{10},Y_{10})$ be a random sample from a bivariate normal distribution $BVN(\mu_1,\mu_2,\sigma_1^2,\sigma_2^2,\rho)$ with $\mu_1=5$, $\mu_2=6$, of $\sigma_1^2=4$ of $\sigma_1^2=9$ and $p=\frac{1}{2}$. Then, which of the following statements are true?

1. The distribution of $\frac{1}{\sqrt{7}}\sum_{i=1}^{10}(X_i-Y_i+1)$ is N(0,10)
2. The distribution of $\frac{1}{15}\sum_{i=1}^{10}(X_i-Y_i-11)^2$ is χ^2- distribution with degrees of freedom 10
3. The distribution of $\frac{2\sqrt{2}(X_1-5)}{\sqrt{\sum_{i=3}^{10}(X_i-5)^2}}$ is t-distribution with degrees of freedom 9
4. The distribution of $\frac{2\sum_{i=1}^{3}(Y_i-6)^2}{\sqrt{\sum_{i=3}^{9}(Y_i-6)^2}}$ is F-distribution with degrees of freedom 3 and 6.

Q109. Let $X_1,X_2,\ldots,X_n$ be independent and identically distributed random variables with the probability density function

$f(x|\theta)=\begin{cases}e^{-(x-\theta)} & \text{if } x\geq\theta,\\ 0 & \text{otherwise,}\end{cases}$

where $\theta\in\mathbb{R}$ is the unknown parameter. Further, let $Y=min(X_1,X_2,X_n)$. Which of the following are confidence intervals for θ with the confidence coefficient $(1-\alpha)$, where $\alpha\in(0,1)$?

1. $\left(Y+\frac{1}{n}In\left(1-\frac{\alpha}{2}\right),Y-\frac{1}{n}In\left(1-\frac{\alpha}{2}\right)\right)$
2. $\left(Y+\frac{1}{2n}In(\alpha),Y\right)$
3. $\left(Y+\frac{2}{n}In\left(\frac{\alpha}{2}\right),Y+\frac{2}{n}In\left(1-\frac{\alpha}{2}\right)\right)$
4. $\left(Y+\frac{1}{n}In\left(\frac{\alpha}{2}\right),Y+\frac{1}{n}In\left(1-\frac{\alpha}{2}\right)\right)$

Q110. Let $X_1,X_2,\ldots,X_n\ (n\geq3)$ be a random sample from the uniform distribution on the interval $(\theta_1-\theta_2,\theta_1+\theta_2)$, where $\theta_1\in\mathbb{R}$ and $\theta_2>0$ are unknown parameters. Let $X_{(j)}$ be the j^{th} order statistic, $j=1,2,\ldots,n$, and let $\hat{X}=\frac{1}{n}\sum_{i=1}^{n}X_i$. Here, $\left(X_{(1)},X_{(2)}\right)$ is a complete and sufficient statistic for (θ_1,θ_2). Then, which of the following statements are true?

1. $\hat{X}$ is an unbiased estimator of θ_1
2. $\left(\hat{X}-X_{(1)}\right)$ is an unbiased estimator of θ_2
3. $\frac{X_{(1)}+X_{(2)}}{2}$ is the uniformly minimum variance unbiased estimator of θ_1

4. $\frac{(n+1)(X_{(n)}-X_{(1)})}{2(n-1)}$ is the uniformly minimum variance unbiased estimator of θ_2

Q111. Let $X_1, X_2, \ldots\ldots, X_n (n \geq 2)$ be a random sample from a continuous distribution with the probability density function $f(x|0) - \frac{1}{20} e^{-\frac{|x-\theta|}{\theta}}, x \in \mathbb{R}$.

where $\theta(> 0)$ is an unknown parameter. Let $U_n - \frac{1}{n}\sum_{i-1}^{n} X_i, V_n - \frac{1}{n}\sum_{i-1}^{n} X_1^2$. and

$S_n^2 = \frac{1}{n-1}\sum_{i-1}^{n}(X_i - U_n)^2$. Then, which of the following statements are true?

1. U_n is an unbiased estimator of θ
2. S_n^2 is an unbiased estimator of θ^2
3. $\frac{1}{3}V_n$ is a consistent estimator of θ^2
4. The statistic (U_n, V_n) is complete

Q112. Let X be a random sample of size one from the probability density function

$$f(x|\theta) = \begin{cases} \theta r^{-0((x-1)} & if\ x > 1, \\ o. & \text{otherwise,} \end{cases}$$

where $\theta(> 0)$ is the unknown parameter. Suppose we want to test the null hypothesis $H_o: \theta = 1$ against the alternative hypothesis $H_1: \theta \neq 1$, based on the observed value x of X. Then, which of the following statements are true?

1. The likelihood function is maximized at $\theta = \frac{1}{x-1}$
2. The maximum value of the likelihood function is $\frac{e^{-1}}{x-1}$
3. The likelihood ratio test for testing H_o against H_1rejects H_o if $(x-1)e^{-x} < k$, for some $k > 0$
4. The likelihood ratio test for testing H_o against H_1 rejects H_o if $x > c$, for some $c > 1$

Q113. Let $(X_1, X_2, \ldots, X_7)$ and $(Y_1, Y_2, \ldots, Y_9)$ be two independent random samples from the continuous distribution functions $F(x-\mu)$ and $F(x-\theta)$, respectively, where F, μ and θ are all unknown. Further, let μ be the unique median of $F(x-\mu)$ and θ be the unique median of $F(x-\theta)$. Let R_i be the rank of Y_i in the combined sample, $i = 1, 2, \ldots, 9$. For testing $H_0: \mu = \theta$ against $H_1: : \mu > \theta$, the test statistic $T = \sum_{i=1}^{9} R_i$ is proposed. Then, which of the following statements are true?

1. The maximum possible value of T is 115
2. Right-tailed test based on T is appropriate for testing H_0 against H_1
3. Under $H_0. E(T) = 76$
4. Under $H_0, P(R_1 = 1, R_2 = 16) = \frac{1}{240}$

Q114. Let $\mathrm{X}_1, \mathrm{X}_2, \ldots\ldots \mathrm{X}_n$ be a random sample from $N(\theta, 1)\theta \in \mathbb{R}$, If A is the Bayes estimator of θ with respect to some prior $\pi(\theta)$ and loss function $\mathrm{L}(\theta, \mathrm{d})$.Then, which of the following statements are true?

1. $\ddot{\theta} = \frac{\sum_{i=1}^{n} x_i}{n+r^2}$, if the prior is $N\left(\theta, \frac{1}{r^2}\right), r^2$ known and $L(\theta, d) = (\theta - d)^2$
2. $\ddot{\theta} = \frac{\sum_{i=1}^{n} x_i}{n+r^2}$if the prior is $N(\theta, r^2), r^2$ known and $L(\theta, d) = |\theta - d|$
3. $\ddot{\theta} = \frac{\sum_{i=1}^{n} x_i}{n+\frac{1}{r^2}}$, if the prior is $N\left(\theta, \frac{1}{r^2}\right), r^2$ known and $L(\theta, d) = |\theta - d|$
4. $\ddot{\theta} = \frac{\sum_{i=1}^{n} x_i}{n}$, if the prior is the Jeffreys prior and $L(\theta, d) = (\theta - d)^2$

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Q115. Consider a paired data (x, y); $i = 1, 2, 3, 4, 5$, where $(x_1, x_2, x_3, x_4, x_5) = (-2, -1, 0, 1, 2)$ and $y_i = x_i^2$ for all i = 1, 2, 3, 4, 5. On this data, a simple linear regression model with an intercept term and a simple linear regression model without an intercept term are fitted using the method of least squares. Which of the following statements are true?

1. The two fitted lines have the same slope
2. The two fitted models have the same intercept
3. The model with intercept passes through at least one of the observed data points
4. The model without intercept passes through at lesast one of the observed data points

Q116. An analyst fits a multiple linear regression model $Y_1 = \beta_0 + \beta_0 X_{1i} + +\beta_4 X_{4i} + \epsilon_i, i = 1,2,\ldots,n$, using the method of least squares. However, his coordinator insists that the intercept and two regressors $Z_1 - X_1 + X_3, \text{ and } Z_2 - X_2 - X_4$, are enough to represent the model. Suppose the coordinator's claim can be tested in the form of a general linear hypothesis, viz., $H_0: L^T\beta = 0$ against $H_A: H_0$ is not true, where $\beta^T = (\beta_0, \beta_1, \ldots, \beta_n)$. Suppose we have n observations on the response Y and each regressor. Further, assume that the errors with or without restrictions are independent $N(0, \sigma^2)$ variables. Then, which of the following statements are true?

1. A possible choice of L is $\begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 0 & 1 \end{pmatrix}$
2. The test statistic for testing H_0 against H_A follows an F-distribution with $(2, n-4)$ degrees of freedom under H_0
3. Sum of squares residuals under the restrictions $L^T\beta = 0$ follows $\sigma^2 x^2_{n-1}$ distribution
4. Sum of squares residuals without the restrictions $L^T\beta = 0$ follows a non-central $\sigma^2 x^2_{n-1}$ distribution

Q117. Let the random vector $\begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix}$ have the positive definite dispersion matrix $\begin{pmatrix} 1 & \rho & \rho \\ \rho & 1 & \rho \\ \rho & \rho & 1 \end{pmatrix}$. Then, which of the following statements are true?

1. ρ may be 0.47
2. The first principal component can only explain 32% of the total variation for some ρ
3. The second principal component can explain more than 32% of the total variation for any ρ
4. The variance of the first principal component is 1+ 2ρ for any ρ

Q118. If the incidence matrix of a block design is given by $N = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$, then which of the following statements are true?

1. The design is incomplete
2. The design is connected
3. The design is balanced
4. The design is orthogonal

Q119. A system has two components C_1 and C_2 put in parallel. The components C_1 and C_2 have independent lifetimes X_1 and X_2, respectively. The probability distribution of X_j is exponential with mean

$\frac{1}{j}, \mathrm{j} = 1,2$. Suppose that R(t) and h(t) are the reliability and the hazard rate functions of the system, respectively. Then, which of the following statements are true?

1. $\mathrm{R(t)} = \mathrm{e}^{-2\mathrm{t}} + \mathrm{e}^{\mathrm{t}} - \mathrm{e}^{-3\mathrm{t}}, \mathrm{t} > 0$
2. The expected lifetime of the given system is 1
3. $\mathrm{h}(1) = \frac{2\mathrm{e}+\mathrm{e}^2-3}{\mathrm{e}+\mathrm{e}^2-1}$
4. $\mathrm{h}(1) = \frac{4\mathrm{e}+\mathrm{e}^6-5}{\mathrm{e}^3+\mathrm{e}^6-1}$

Q120. Consider the M/M/1 queue in which customers arrive according to a Poisson process with rate 3 and successive service times are independent exponential random variables having mean$\frac{1}{9}$. Let P_n be the long run probability that there are exactly n customers in the system. Then, which of the following statements are true?

1. $P_0 = \frac{1}{3}$
2. $P_1 = \frac{2}{9}$
3. The average number of customers in the system is 1
4. The average amount of time that a customer spends in the system is $\frac{1}{6}$