CSIR NET MS DEC 2025
(IFAS SOLVED PAPER)

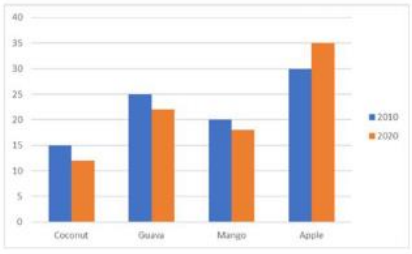
PART A

Q.1 The minimum height of a plane vertical mirror that will allow a 6-feet tall person to see himself fully in it
1. depends on the distance between the person and the mirror
2. is 3 feet
3. is 4.5 feet
4. is 6 feet

Q2. A recent survey suggests that the total fertility rate in a country has fallen below 2.1, the population replacement ratio. This necessarily implies that the
1. infant mortality rate has increased reducing the net fertility ratio.
2. total population will decline.
3. population of young people is going to increase with a faster rate in the long run if the same status continues.
4. proportion of elderly people is going to decrease in the long run if the same status continues.

Q.3 A lady bought some apples, each costing Rs. 25, and some bananas each costing Rs 6, for a total of Rs. 378. In how many ways could she have chosen the numbers of apples and bananas?
1. 1
2. 2
3. 3
4. 4

Q4. The numbers (in millions) of coconut, guava, mango and apple trees in a region in 2010 and 2020 are shown in the following figure.



The maximum relative change in numbers was for
1. coconut trees
2. guava trees
3. mango trees
4. apple trees

Q5. In an exam, questions of three difficulty levels hard, medium, and easy fetch respectively 7, 3, and 2 marks if correct and 0 if incorrect. Three students got 30 marks each but in three different ways, though the total number of questions correctly answered by each student was the same. Then what could be the total number of questions correctly answered by each of these students?
1. 12
2. 10

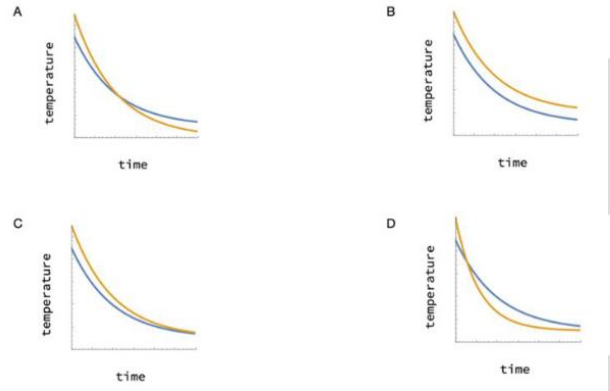
3. 9
4. 6

Q6.The value of $1+\left(\frac{1}{2^1}+\frac{1}{3}\right)+\left(\frac{1}{2^2}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}\right)+\cdots+\left(\frac{1}{2^9}+\cdots+\frac{1}{1023}\right)$ lies between
1. 2 and 10
2. 11 and 20
3. 21 and 30
4. 31 and 40

Q.7 How many 5-digit numbers can be formed from the digits 0, 2, 3, 4, 6, 7 and 9, using each at most once, which are divisible by 5?
1. 120
2. 240
3. 360
4. 720

Q.8 Three periodic events repeat every 24 seconds, 54 seconds, and 56 seconds. If they coincide at 10:20:00, when will they next coincide?
1. 10:35:12
2. 10:45:20
3. 10:45:12
4. 10:35:20

Q9. Two identical metal bars are heated to different temperatures and allowed to cool in the same surroundings. Which one of the following figures correctly shows their cooling curves?



1. A
2. B
3. C
4. D

Q10. Some, but not all, faces of a six-faced cubical fair die are painted red (R) and theremaining green (G); and the die is thrown until red faces come up on top 4 times. Consider the following sequences of colours listed left to right as they appear on the top.
A: GRRRR
B: GRGRRR
Which one of the following is true?
1. A is more probable than B

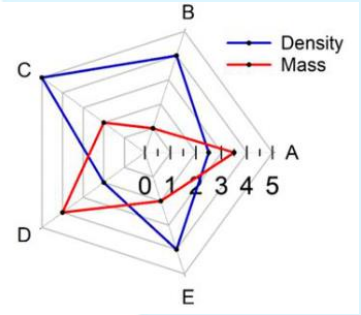
2. B is more probable than A
3. Both have the same probability
4. Whether A or B is more probable depends upon how many faces are painted green

Q11. In a community, some artists are teachers, no teacher is a painter, all painters are artists, and all teachers are professionals. Then it can be definitely asserted that
1. no painter is a professional
2. all artists are professionals
3. no professionals are teachers
4. some artists are professionals

Q12. What is the minimum number of pourings required to transfer exactly 6L of water from a 12L fully filled container to an 8L empty container when a 5L empty container is also available to use?
1. 4
2. 5
3. 6
4. 7

Q13. The geometric mean of 100 observations is 25. If each observation is multiplied by 4, what will be the new geometric mean?
1. 100
2. 50
3. 25
4. $(25 \times 4)^{1/2}$

Q14. The following figure shows densities and masses of five objects (A to E).



The object with the largest volume is
1. A
2. B
3. D
4. E

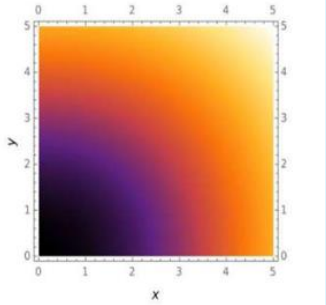
Q15. Five students graduated from a college, not all in the same year, after each has studied for four years. If batchmates Jiten and Anwar were between Ramesh and Prakash but senior to Sam while Ramesh had left the college before Jiten took admission, then it is certain that
1. Anwar was the most senior
2. Ramesh was the most senior
3. Sam was the most junior
4. Prakash was the most junior

Q16. Which among the following cities can be said most appropriately to bear the same relation to Tamil Nadu that Pune bears to Maharashtra; Surat to Gujarat and Asansol to West Bengal?
1. Tirupati
2. Mysore
3. Chennai
4. Coimbatore

Q17. Alloy A is formed by mixing iron (Fe) and nickel (Ni) in the ratio 3:4, while alloy B is formed by mixing Fe and Ni in the ratio 9:5. If equal quantities of alloys A and B are melted together to form a new alloy C, what will be the ratio of Fe to Ni in the alloy C?
1. 4:3
2. 5:3
3. 15:13
4. 13:9

Q18. Suppose $a_1, a_2, \ldots\ldots, a_{300}$ are integers such that $a_{i-1} + a_i + a_{i+1} = 2025$ for all i = 2,3,..., 299. If $a_7 = -5, a_9 = 37$, then the value of a_{106} is
1. 1993
2. 37
3. -5
4. 2030

Q19. The following plot shows temperature as a function of x and y. Along which path is the temperature change minimum?



1. x = constant or y = constant
2. $\frac{y}{x^2}$= constant
3. $y^2 + x^2 =$ constant
4. yx = constant

Q20. In a class, 40% and 20% students passed in Mathematics and Physics, respectively, and 10% students passed in both subjects. What is the probability of a randomly selected student to have passed in Physics if the student already passed in Mathematics?
1. 1/2
2. 1/20
3. 1/4
4. 2/25

PART B

Q.21 Let X_1 and X_2 be a random sample from Uniform $[0, \theta]$ distribution, where $\theta > 0$. For testing the hypothesis

$H_0: \theta = 1 \; against \; H_1: \theta = 2,$

consider a test which rejects H_0 if $X_1 + X_2 > \frac{4}{5}$. Then, the probability of type-I error is

1. $\frac{8}{25}$
2. $\frac{13}{25}$
3. $\frac{17}{25}$
4. $\frac{22}{25}$

Q.22 Let $f: (0, \infty) \to (0, \infty)$ be a uniformly continuous function. Define $g: (0, \infty) \to (0, \infty)$ by

$g(x) = \begin{cases} x^2 & if \; 0 < x \leq 1 \\ x & if \; x > 1. \end{cases}$

Which of the following statements is true?

1. fog is uniformly continuous, but gof is not uniformly continuous.
2. gof is uniformly continuous, but fog is not uniformly continuous.
3. Neither fog nor gof is uniformly continuous.
4. Both fog and gof are uniformly continuous.

Q.23 Let $\{X_n: n \geq 0\}$ be any homogeneous Markov Chain on the state space S = {1,2,3,4} having the transition probability matrix $P = \left(P_{ij}\right)_{i,j \in S}$ given by

$$P = \begin{pmatrix} \frac{1}{4} & 0 & \frac{2}{3} & \frac{1}{12} \\ 0 & 1 & 0 & 0 \\ \frac{1}{12} & 0 & \frac{1}{4} & \frac{2}{3} \\ \frac{2}{3} & 0 & \frac{1}{12} & \frac{1}{4} \end{pmatrix}$$

Which of the following statements about stationary distributions of any such Markov Chain is true?

1. There is no stationary distribution
2. Stationary distribution exists and is unique
3. There are exactly two stationary distributions
4. There are infinitely many stationary distributions

Q.24 Let S be a finite subset of $\mathbb{C}$ containing 0. Let $f: \mathbb{C} \backslash S \to \mathbb{C}$ be a holomorphic function which has a simple pole at 0. For $R > 0$, let γ_R denote the path $\gamma_R(t) = Re^{2\pi it}$ for $t \in [0, 1]$. Which of the following statements is necessarily true?

1. The function f(1/z) has a removable singularity at 0.
2. The function f(1/z) has an essential singularity at 0.
3. If $\int_{\gamma_R} f(z)dz = 0$ for some R> 0, then S is not a singleton set.
4. If S is not a singleton set, then $\int_{\gamma_R} f(z)dz = 0$ for some R>0 such that $S \cap \{z \in \mathbb{C}: |z| = R\} = \phi$.

Q.25 For $\alpha \in \mathbb{R}$, let $y_1(x)$ and $y_2(x)$ be solutions of the differential equation

$y'' + (e^{x^2} + cos\, x)\, y = 0$

such that
$y_1(0) = 3, y_1'(0) = -1, y_2(0) = -5, y_2'(0) = a.$
Suppose $W(y_1, y_2)(x)$ denotes the Wronskian of y_1 and y_2. If $W(y_1, y_2)\left(\frac{1}{2}\right) = 4$, then the value of a is
1. 2
2. 3
3. 4
4. -3

Q.26 For a non-negative real number a, let $\sqrt{a}$ denote its non-negative square-root. Consider the function $f: \mathbb{R} \to \mathbb{R}$ given by
$f(x) = x\left(\sqrt{x^2+3} - \sqrt{x^2+2}\right).$
Which of the following statements is true?
1. $\lim_{x\to\infty} f(x) = \infty$
2. $\lim_{x\to\infty} f(x) = 0$
3. $\lim_{x\to\infty} f(x) = 1$
4. $\lim_{x\to\infty} f(x) = \frac{1}{2}$

Q.27 Consider the quadratic form
$f(x, y, z) = [x \quad y \quad z]A\begin{bmatrix} x \\ y \\ z \end{bmatrix}$
where A is an invertible 3×3 symmetric matrix over $\mathbb{Q}$.
Assume that there exists $(\alpha, \beta, \gamma) \in \mathbb{C}^3 \backslash \{(0,0,0)\}$ such that $f(\alpha, \beta, \gamma) = 0$.
Which of the following statements is necessarily true?
1. There exists $(a, b, c) \in \mathbb{R}^3 \backslash \{(0,0,0)\}$ such that f(a, b, c) = 0.
2. If there exists $(a, b, c) \in \mathbb{R}^3 \backslash \{(0,0,0)\}$ such that f(a, b, c) = 0, then $(a, b, c) \in \mathbb{Q}^3$.
3. $\{(a, b, c) \in \mathbb{Z}^3 | f(a, b, c) = 0\}$ is a finite set.
4. If there exists $(a, b, c) \in \mathbb{Q}^3 \backslash \{(0,0,0)\}$ such that f(a, b, c) = 0, then 4 has a positive eigenvalue and a negative eigenvalue.

Q.28 For a non-negative real number a, let $\sqrt{a}$ denote its non-negative square-root. Consider the sequence $\{x_n\}_{x\geq 1}$ defined by
$x_1 = 1 \; and \; x_{n+1} = \sqrt{1 + \frac{x_n^2}{n}} \; for \; n \geq 1.$
Which of the following statements is true?
1. $\limsup_{n\to\infty} x_n = 1$
2. $\limsup_{n\to\infty} x_n = \sqrt{2}$
3. $\limsup_{n\to\infty} x_n = 2$
4. $\limsup_{n\to\infty} x_n = 1 + \sqrt{2}$

Q.29 Let $X|\theta \sim$ Uniform$[0,\theta]$ and θ has an Exponential distribution with mean λ, where $\lambda > 3$ is known. If the realized value of X is 2025, then the posterior mode equals

1. 0
2. 2025
3. $\frac{2025}{\lambda}$
4. 2025λ

Q.30 Let u(x, t) be the solution of the partial differential equation

$u_t = 16u_{xx} + 2, 0 < x < 7, t > 0$

satisfying the conditions

$u_x(0,t) = u(7,t) = 0, t > 0,$

$u(x,0) = 0, 0 < x < 7.$

Then which of the following statements is true?

1. For each $x \in (0,7), u(x,t) \to 0$ as $t \to \infty$.
2. For each $x \in (0,7), u(x,t) \to \frac{x^2(7-x)^2}{16}$ as $t \to \infty$.
3. For each $x \in (0,7), u(x,t) \to \frac{x^2}{16}(7-x)$ as $t \to \infty$.
4. For each $x \in (0,7), u(x,t) \to \frac{1}{16}(49-x^2)$ as $t \to \infty$.

Q.31 Let X be a Binomial (n, p) random variable, where $n \in \{5,6\}$ and $p \in \left\{\frac{1}{4}, \frac{3}{4}\right\}$. If X = 3 is observed, then the maximum likelihood estimate of (n, p) is

1. $\left(5, \frac{1}{4}\right)$
2. $\left(5, \frac{3}{4}\right)$
3. $\left(6, \frac{3}{4}\right)$
4. $\left(6, \frac{1}{4}\right)$

Q.32 Consider the following linear programming problem:

Maximize $3x_1 + 4x_2$

subject to

$3x_1 + 2x_2 \leq 12,$

$3x_1 + 5x_2 \leq 15,$

$2x_1 - x_2 \geq 0,$

$x_2 \leq 2,$

and $x_1, x_2 \geq 0.$

Which of the following values is the optimum value of the objective function in the feasible region?

1. 11
2. 13
3. 14.
4. 15

Q.33 Let $\mathbb{D}$ denote the open unit disc $\{z \in \mathbb{C}: |z| < 1\}$ and

$$X = \left\{ f: D \to C \;\middle|\; f \text{ is holomorphic and satisfies } f(2z) = \frac{f(z)}{1-\left(f(z)\right)^2} \text{ for all } |z| < \frac{1}{2} \right\}.$$

Which of the following statements is true?

1. X is uncountable.
2. X is infinite and countable.
3. Every element of X is an open map.
4. X has exactly one element.

Q.34 Let $X_1, X_2, \ldots$ be a sequence of independent and identically distributed random variables having Binomial $\left(3, \frac{1}{4}\right)$ distribution. For j = 1, 2,..., define

$$Y_j = \begin{cases} 1, & \text{if } X_j \le \sqrt{5} \\ 0, & \text{otherwise.} \end{cases}$$

If $\frac{1}{n}\sum_{j=1}^{n} Y_j^2$ converges almost surely to a constant e as $n \to \infty$, then the value of c equals

1. $\frac{1}{64}$
2. $\frac{37}{64}$
3. $\frac{63}{64}$
4. $\frac{27}{32}$

Q.35 Let $\gamma_R(t) = 2 + i + Re^{2\pi it}$ for $t \in [0, 1]$ and R = 1,2. Which of the following statements is true?

1. $\int_{\gamma_1} tan(z)dz = 2\pi i$
2. $\int_{\gamma_1} tan(z)dz = -2\pi i$
3. $\int_{\gamma_2} tan(z)dz = 2\pi i$
4. $\int_{\gamma_2} tan(z)dz = -2\pi i$

Q.36 Let u(x, t) be the solution of the initial value problem

$u_{tt} - u_{xx} = e^{-t}, x \in \mathbb{R}. t > 0,$

$u(x, 0) = cos\, x, u_t(x, 0) = 0, x \in \mathbb{R}.$

Then which of the following statements is true?

1. For each $x_0 \in \mathbb{R}, e^t u(x_0, t) \to 0$ as $t \to \infty$.
2. For each $x_0 \in \mathbb{R}, |u(x_0, t)| \to 0$ as $t \to \infty$.
3. For each $x_0 \in \mathbb{R}$, there exists a $t_0 > 0$ such that $u(x_0, t_0) \ge 4$.
4. For each $x_0 \in \mathbb{R}$, there exists a $t_0 > 0$ such that $u(x_0, t_0) \le -4$.

Q.37 Let $\mathbb{F}_q$ be a finite field with q elements. For $n \ge 2$, let A be a $2n \times 2n$ matrix with entries in $\mathbb{F}_q$ such that rank(A) = n. Let $W = \{v \in \mathbb{F}_q^{2n} | Av = 0\}$. Which of the following is necessarily the number of (n+2)-dimensional subspaces of $\mathbb{F}_q^{2n}$ that contain W?

1. 1
2. $\frac{(q^{2n}-q^n)(q^{2n}-q^{n+1})}{q^n}$
3. $\frac{(q^{2n}-1)(q^{2n}-q)\ldots(q^{2n}-q^{n-1})}{q^n}$

4. $\frac{(q^n-1)(q^n-q)}{(q^2-1)(q^2-q)}$

Q.38 For a finite group G, let S(G) denote the number of subgroups of G. Which of the following statements is necessarily true?

1. Let G and G' be finite groups such that S(G) = S(G'). Then G is isomorphic to G'.
2. If S(G) = 4, then $|G| = p^m$ for some prime number p and positive integer m.
3. If S(G)=5, then G is a cyclic group.
4. For every positive integer n, there exists a finite group G such that S(G) = n.

Q.39 Let $M, N \in M_3(\mathbb{C})$ be such that
$Trace(M^k) = Trace(N^k), 1 \leq k \leq 3.$
Which of the following statements is necessarily true?

1. $M^2 = N^2$
2. $M^3 = N^3$
3. The characteristic polynomials of M and N are the same.
4. The minimal polynomials of M and N are the same.

Q.40 Let S be a subset of the open interval (0, 1) that consists of all the real numbers $\alpha \in (0,1)$ whose infinite decimal expansion $\alpha = 0.\alpha_1\alpha_2\alpha_3 \ldots\ldots$ is such that $\alpha_i \in \{0,2,4\}$ for all $i > 1$. Which of the following statements is true?

1. There is a bijective map from $\mathbb{N}$ to S.
2. There is a surjective map from S onto (0, 1).
3. There is a bijective map from $\mathbb{N}$ to (0, 1)\ S.
4. S is a countable set and (0, 1)\S is uncountable.

Q.41 Let $\{Y_n, n \geq 1\}$ be a sequence of independent and identically distributed random variables, where $Y_1 \sim$ Bernoulli$\left(\frac{1}{2}\right)$. Define $Z = \sum_{n=1}^{\infty} \frac{4Y_n}{5^n}$. Then, which of the following statements is true?

1. $P\left(Z \geq \frac{3}{5}\right) = 0.5, P\left(Z = \frac{4}{25}\right) = 0$
2. $P\left(Z \geq \frac{3}{5}\right) = 0.6, P\left(Z = \frac{4}{25}\right) = 0$
3. $P\left(Z \geq \frac{3}{5}\right) = 0.7, P\left(Z = \frac{4}{25}\right) = 0.16$
4. $P\left(Z \geq \frac{3}{5}\right) = 0.8, P\left(Z = \frac{4}{25}\right) = 0.16$

Q.42 Consider the following statements:

(P) The initial value problem

$$y' = f(y), where\ f(y) = \begin{cases} ycos\frac{1}{y}, & y \neq 0 \\ 0, & y = 0 \end{cases}$$

$y = 0$
has at most one solution.

(Q) If z(x) is the solution of the initial value problem

$$z' = \frac{(1-z^4)^{100}}{1+z^2},$$
$$z(0) = 0,$$

then for each $\alpha \in (0, 2)$, there exists $x_\alpha \in \mathbb{R}$ such that $z(x_\alpha) = 1 - \alpha$.
Then which of the following statements is true?
1. Both (P) and (Q) are true.
2. (P) is true, (Q) is FALSE.
3. (P) is FALSE, (Q) is true.
4. Both (P) and (Q) are FALSE.

Q.43 Let $\alpha, \beta \in (0, \infty)$. Consider the infinite series

$$\sum_{n\geq} \frac{1}{(n(\log_e n)^\alpha (\log_e(\log_e n))^\beta}$$

Which of the following statements is true?
1. The series converges for all $\alpha \in (0, 1)$ and for all $\beta \in (1, \infty)$.
2. The series converges for a = 1 and 3 = 1.
3. The series converges for a=1 and for all $\beta \in (1, \infty)$.
4. The series converges for all $a \in (0, \infty)$ and for all $\beta \in (0, \infty)$.

Q.44 If p(x) is the interpolating polynomial for the data

x	-2	-1	0	1	2
y	-1	3	1	-1	3

then the value of $p\left(\frac{1}{2}\right)$ is
1. $\frac{-3}{8}$
2. $\frac{-5}{8}$
3. $\frac{5}{8}$
4. $\frac{3}{8}$

Q.45 Let A be a 3 x 3 matrix over complex numbers with trace 1 and determinant 1. Suppose, further, that one of the eigenvalues of A is 1. Which of the following statements is necessarily true?
1. The characteristic polynomial of A has repeated roots.
2. Every eigenvalue of A has absolute value 1.
3. A does not have any eigenvalue on the imaginary axis.
4. A^2 is the identity matrix.

Q.46 Let X, Y, and Z be independent Normal random variables with means -1, 0, and 1, respectively, and variances 1, 1, and 3, respectively. Which of the following random variables has a Cauchy distribution with location parameter 0 and scale parameter 1?
1. $\frac{X-1}{|Y|}$
2. $\frac{X+2Y+Z}{X-2Y+Z}$
3. $\frac{Z-1}{Y}$
4. $\frac{X-Y}{X+Y}$

Q.47 Suppose u(x) is the solution of the integral equation

$u(x) = 3 + \int_0^x (x - t)\, u(t)dt.$

Then which of the following statements is true?

1. $u(\pi) = 2e^{\pi}$.
2. $u'(\pi) = e^{\pi}$.
3. $u(\pi) + u'(\pi) = 3e^{\pi}$.
4. $u(\pi) - u'(\pi) = e^{\pi}$.

Q.48 Let $X = \{1, \ldots, 17\}$ and S_{17} be the group of permutations of X. For a subgroup G of S_{17} and $x \in X$, let

$S_G(x) = \{\sigma \in G | \sigma(x) = x\}$.

Which of the following statements is true for every subgroup G of S_{17}?

1. The number of pairs $(\sigma, z) \in G \times X$ such that $\sigma(x) = x$ is strictly greater than $\Sigma_{x \in X}|S_G(x)|$.
2. The number $\frac{1}{|G|}\Sigma_{x \in X}|S_G(x)|$ is always an integer.
3. For all $x \in X, S_G(x)$ is a normal subgroup of G.
4. For all $x, y \in X, S_G(x)$ is isomorphic to $S_G(y)$.

Q.49 Consider the initial-boundary value problem (IBVP)

$u_t + u_x = 0, 0 < x < \infty, t > 0,$

$u(x, 0) = e^x, 0 < x < \infty,$

$u(0, t) = 1 - sint, t > 0.$

Then which of the following statements is true?

1. There exists a unique solution u(x, t) of IBVP such that $u(1, t) = e - sint$, for all t < 1.
2. There does NOT exist a solution u(x, t) of IBVP such that $u(1, t) = e - sint$, for all t> 1.
3. There exists a solution u(x, t) of IBVP such that $u(x, t) = e^{x-t}$, for all t > x.
4. There exists a solution u(x, t) of IBVP such that u(x,t) = e - sin(t - x), for all t<x.

Q.50 Let $X = (X_1, X_2, X_3)^T$ be a 3×1 random vector with $E(X) = (3,2,1)^T$ and $Cov(X) = \sum = \begin{pmatrix} 3 & -2 & 0 \\ -2 & 3 & -2 \\ 0 & -2 & 3 \end{pmatrix}$. Suppose that $Y = (Y_1, Y_2, Y_3)^T = \frac{1}{\sqrt{3}}X$. Then the value of the multiple correlation coefficient between Y_1 and (Y_2, Y_3) equals

1. $\frac{2}{\sqrt{5}}$
2. $\sqrt{\frac{2}{5}}$
3. $\frac{2}{\sqrt{3}}$
4. $\frac{1}{\sqrt{3}}$

Q.51 Let h be a holomorphic function on $\mathbb{C}\backslash\{0\}$ such that

$\lim_{|z| \to \infty} h(z) = 0.$

For every $n \geq 1$, consider

$f_n(z) = \sum_{k=1}^{n} h(z^k)$.

Let $\mathbb{D}$ denote the open unit disc $\{z \in \mathbb{C}: |z| < 1\}$. Which of the following statements is necessarily true?

1. For all with $|z| \geq 1$, the sequence $\{f_n(z)\}_{n\geq 1}$ converges.
2. For all $z \in \mathbb{D}\backslash\{0\}$, the sequence $\{f_n(z)\}_{n\geq 1}$ converges.
3. The sequence $\{f_n\}_{n\geq 1}$ converges pointwise to a holomorphic function on $\mathbb{D}\backslash [0,1)$.
4. The sequence $\{f_n\}_{n\geq 1}$ converges pointwise to a holomorphic function on $\{z \in \mathbb{C}: |z| > 1\}$.

Q.52 Which of the following statements is necessarily true?

1. The set of all finite subsets of $\mathbb{Z}$ is uncountable.
2. Let $f: \mathbb{R} \to \mathbb{R}$ be a continuous and one-one function. If $S \subseteq \mathbb{R}$ is a countably infinite set, then f(S) is a countably infinite set.
3. Let $f: \mathbb{R} \to \mathbb{R}$ be a continuous function. Then there exists an uncountable subset $T \subseteq \mathbb{R}$ such that $f(T) \subseteq \mathbb{R}$ is uncountable.
4. Let $f: \mathbb{R} \to \mathbb{R}$ be a continuous function. If $R \subseteq$ image(f) is a countable set, then $f^{-1}(R)$ is countable.

Q.53 Let B be a 4 x 4 positive-definite real symmetric matrix which is not the identity matrix. Consider the inner product on $\mathbb{R}^4$ given by $(v, w) = v^T Bw$, where v^T denotes the transpose of v. Which of the following statements is FALSE?

1. If v and w are eigenvectors of B for distinct eigenvalues, then $\langle v, w\rangle = 0$.
2. If v is an eigenvector of B and $w \neq 0$ is such that $\langle w, v\rangle$ 0, then w is an eigenvector of B.
3. For every subspace $W \subseteq \mathbb{R}^4, W + W^{\perp} = \mathbb{R}^4$, where $W^{\perp} = \{v = \mathbb{R}^4 \,|\langle v, w\rangle = 0 \; for \; all \; w \in W\}$.
4. If $W \subseteq \mathbb{R}^4$ is an eigenspace of B, then W has an orthonormal basis.

Q.54 Suppose y(x) is the extremal of the variational problem

$J(y) = \int_0^4 \frac{(y')^2}{y^2} dx \; subject \; to \; y(0) = 1, y(4) = e^8$.

Then which of the following statements is true?

1. $y(\log_e 2) = 2$.
2. $y(\log_e 2) = 9$.
3. $y(\log_e 2) = 4$.
4. $y(\log_e 2) = 5$.

Q.55 Let X and Y be topological spaces. Consider the following statement:

S: For every open subset $U \subseteq X \times Y$ and every $x \in X$ such that $\{x\} \times Y \subseteq U$, there is a neighbourhood W of a in X such that $W \times Y \subseteq U$.

Which of the following statements is true?

1. If $X \times Y$ is Hausdorff, then the statement S is true.
2. If Y is connected, then the statement S is true.
3. If $X \times Y$ is regular, then the statement S is true.
4. If Y is compact, then the statement S is true.

Q.56 For a finite group G, let $H_G = \{g \in G \big| g^{15} = e\}$. Which of the following statements is necessarily true?

1. There exists a finite group G such that $|H_G|$ is even.
2. There exists a finite group G such that $|H_G| = 4n + 1$ for some n ≥ 3.
3. For every finite group G, there exists a non-negative integer n such that $|H_G| = 4n + 1$.
4. For every finite group G, there exists a non-negative integer n such that $|H_G| = 4n + 3$.

Q.57 Consider a finite population of size N = 100. Let T_1 be the sample mean of a study variable based on a sample of size n (1 <n<N) under simple random sampling with replacement scheme. Let T_2 be the sample mean of the same study variable based on a sample of size n under simple random sampling without replacement scheme. If $Var(T_1) = 9Var(T_2)$, then the sample size n equals

1. 33
2. 69
3. 89
4. 93

Q.58 Let X be a single sample from an absolutely continuous distribution with probability density function

$$f(x|\theta) = \begin{cases} \frac{2}{\theta^2}(\theta - x), & if\ 0 < x < \theta \\ 0, & \text{otherwise,} \end{cases}$$

where $\theta > 0$ is unknown. Which of the following intervals is a 95% confidence interval for θ?

1. $\left(X, \frac{X}{1-\sqrt{0.95}}\right)$
2. $\left(X, \frac{X}{0.95}\right)$
3. $\left(X, \frac{X}{\sqrt{0.95}}\right)$
4. $(0.95X, X)$

Q.59 Consider the linear model

$Y_1 = \beta_1 + \beta_2 + \in_1$
$Y_2 = \beta_1 + 2\beta_2 + \in_2$
$Y_3 = \beta_1 + c\beta_2 + \in_3$,

where $\beta_1 + \beta_2 \in \mathbb{R}$ are unknown parameters, and the uncorrelated errors $\in_i, i = 1,2,3$ have zero mean and finite variance $\sigma^2 (> 0)$. The constant c is such that $\hat{\beta}_1$ and $\hat{\beta}_2$ are uncorrelated, where $\hat{\beta}_1$ and $\hat{\beta}_2$ are the best linear unbiased estimators of β_1 and β_2, respectively.

Which of the following statements is the correct option for $\left(Var(\hat{\beta}_1), Var(\hat{\beta}_2)\right)$?

1. $\left(\frac{\sigma^2}{3}, \frac{\sigma^2}{14}\right)$
2. $(3\sigma^2, 14\sigma^2)$
3. $\left(\frac{\sigma^2}{14}, \frac{\sigma^2}{3}\right)$
4. $(14\sigma^2, 3\sigma^2)$

Q.60 Let I denote the 3×3 identity matrix. Given any three distinct matrices $A, B, C \in M_3(\mathbb{R})$, which of the following statements is necessarily true?

1. There exists $D \in M_3(\mathbb{R})$ and a polynomial $f \in \mathbb{R}[X]$ satisfying $f(A) = f(B) = f(C)0$ and f(D) = I.
2. There exists $D \in M_3(\mathbb{R})$ and a polynomial $f \in \mathbb{R}[X]$ satisfying $f(A) = f(B) = 0, f(C) = I$ and f(D) = I.
3. There exists $D \in M_3(\mathbb{R})$ and a polynomial $f \in \mathbb{R}[X]$ satisfying $f(A) = 0, f(B) = f(C) = I$ and $f(D) \neq 0$.
4. There exists $D \in M_3(\mathbb{R})$ and a polynomial $f \in \mathbb{R}[X]$ satisfying $f(A) = f(B) = 0, f(C) = I$ and f(D) = 0.

Q.61 Let $A \in M_4(C)$ be such that $A^2 = l$ and Trace(A) = 0. Which of the following statements are necessarily true?

1. The set $\{B \in M_4(C) | AB + BA = 0\}$ is an 8-dimensional C-vector space.

2. The set $\{B \in M_4(C) | AB - BA = 0\}$ is an 8-dimensional C-vector space.

3. There exists $B \in M_4(C)$ such that $AB + BA = 0, B^2 = I$ and Trace(B) = 0.

4. If $B \in M_4(C)$ is such that AB+BA= 0, then B is diagonalisable.

Q.62 Let $\lambda \in R$ be such that the integral equation $y(x) = \lambda(5xt^3 + 4x^3t + 3xt)y(t)dt$

admits a non-trivial solution $y(x)$ such that $y(1) = \frac{5}{2}$.

Then which of the following statements are true?

1. $y(0) + y'(0) = \frac{3}{2}$

2. $y\left(\frac{1}{2}\right) + y'\left(\frac{1}{2}\right) = \frac{7}{2}$

3. $y(-1) + y'(-1) = -1$

4. $y\left(\frac{1}{3}\right) + y'\left(\frac{1}{3}\right) = \frac{14}{9}$

Q.63 There are six persons. On each day of the year 2024 at least one of them borrowed money from another. A person P is labeled troublesome in a calendar month M of 2024 if P borrowed from the same person at least twice during the month M. Which of the following statements are necessarily true?

1. In every calendar month of 2024, at least one person was troublesome.

2. In any two consecutive calendar months of 2024, at least one person was troublesome in at least one of the two calendar months.

3. At least one person was troublesome in two calendar months of 2024.

4. At least one person was troublesome in January 2024.

Q.64 Suppose $y(x)$ is the extremal of the variational problem

$J(y) = \int_0^1 x^2 (y')^2 \, dx$

subject to

$y(0) = 0, y(1) = 1, \int_0^1 y^2 \, dx = \frac{1}{7}.$

Then which of the following statements are true?

1. $y'\left(\frac{1}{2}\right) = \frac{3}{4}$

2. $y'\left(\frac{1}{\sqrt{3}}\right) = 1$

3. $y'\left(\frac{1}{3}\right) = \frac{1}{3}$

4. $y'\left(\frac{1}{4}\right) = \frac{1}{2}$

Q.65 Suppose that the transition probability matrix of a homogeneous Markov chain with state space {1, 2, 3, 4} is given by

$$P = \begin{pmatrix} \frac{1}{4} & \frac{3}{4} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \frac{1}{8} & 0 & \frac{7}{8} & 0 \\ 0 & 0 & \frac{1}{9} & \frac{8}{9} \end{pmatrix}$$

Which of the following statements are true?

1. State 2 is a positive recurrent state

2. Mean recurrence time of state 1 is $\frac{7}{4}$

3. State 4 is a transient state

4. State 3 is aperiodic and ergodic

Q.66 Let $Y = X\beta + \epsilon$ be a multiple linear regression model with p regressors and an intercept, where $\beta(\beta_0, \beta_1, \ldots, \beta_p)^T$ and the random error $\epsilon \sim N_n(0, \sigma^2 I), \sigma > 0$ and $n > p + 1$. Here, the least squares estimation method provides unique estimator of β, say β. Let the total sum of squares (corrected), sum of squares due to regression/model and sum of squares due to error/residual, based on β, be denoted by $Y^T AY, Y^T BY$ and $Y^T CY$, respectively, so that $Y^T AY = Y^T BY + Y^T CY$. Which of the following statements are always true?

1. $\frac{Y^T AY}{\sigma^2}$ follows a central x^2 distribution with (n-1) degrees of freedom.

2. $\frac{Y^T BY}{\sigma^2}$ follows a central x^2 distribution with p degrees of freedom if $\beta_1 = \beta_2 = \cdots = \beta_P = 0$.

3. $\frac{Y^T BY}{Y^T CY}$ follows a central F distribution with (p, n − p) degrees of freedom

4. $Y^T BY$ and $Y^T CY$ are independently distributed if and only if $\beta_1 = \beta_2 = \cdots = \beta_P = 0$.

Q.67 Let $\{x_n\}$ be a convergent iterative sequence generated by Newton-Raphson method for solving the equation $\sin x - 1 = O$ such that $x_n \to \frac{\pi}{2}$ as$n \to \infty$ no. For $n \in N$, let $e_n = x_n - \frac{\pi}{2}$.

Let $p > 0$ be such that $\lim_{n\to\infty} \frac{|e_{n+1}|}{|e_n|^p}$ exists and is non-zero.

Then which of the following statements are true?

1. $p = 1$
2. $\lim_{n\to\infty} \frac{|e_{n+1}|}{|e_n|^p} = \frac{1}{2}1/2$
3. $p = 1$
4. $\lim_{n\to\infty} \frac{|e_{n+1}|}{|e_n|^p} = 1$

Q.68 Let $X_1, X_2, X_n, (n \geq 2)$ be a random sample from an Exponential distribution with the probability density function

$$f(x|\mu,\sigma) = \begin{cases} \frac{1}{\sigma}\exp\left(\frac{\mu-x}{\sigma}\right), & if\ x > \mu \\ 0, & \text{otherwise} \end{cases}$$

where parameters μ and σ are unknown and positive. Let $\hat{X}_n, S_n^2$ and $X_{1:n}$ denote the sample mean, the sample variance and the sample smallest order statistic, respectively and let.$\theta = \frac{\sigma}{\mu}$ Then which of the following statements are true?

1. $S_n(X_{1:n})^{-1}$ is a consistent estimator of θ
2. $\left(\hat{X}_n - X_{1:n}\right)(X_{1:n})^{-1}$ is a consistent estimator of θ
3. $\left(\hat{X}_n - X_{1:n}\right)\left(\hat{X}_n - S_n\right)^{-1}$ is a consistent estimator of θ
4. $S_n\left(\hat{X}_n\right)^{-1}$ is a consistent estimator of θ

Q.69 Consider the following 2^5-factorial design with 8 blocks.

Block	Treatment combinations
1	(1), acd, bce, abde
2	e, acde, bc, abd
3	a, cd, abce, bde
4	ab, bcd, ace, de
5	b, abcd, ce, ade
6	ac, d, abe, bcde
7	abc, bd, ae, cde
8	c, ad, be, abcde

Which of the following statements are true?

1. ABC is confounded with blocks

2. BCD is confounded with blocks

3. CDE is confounded with blocks

4. ABDE is confounded with blocks

Q.70 Consider the following real-valued functions F_1 and F_2 defined on R, given by

$$F_1(x) = \begin{cases} 1, & \text{if x} \geq 1, \\ 0, & \text{otherwise,} \end{cases}$$

$$F_2(x) = \begin{cases} \int_0^x \exp(-t)dt, & if\ x \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

Define another function $F: R \to [0,1]$ by $F(x) = \frac{2}{3}F_1(x) + \frac{1}{3}F_2(x)$ for all $x \epsilon R$. Which of the following statements are true?

1. F is non-decreasing on R

2. $\lim_{z \to \infty} F(x) = 1$

3. F is left-continuous on R

4. F is right-continuous on R

Q.71 Consider the following statements:

(P) The initial value problem

$y' + y = e^{-x^2}, y(0) = 0$

has a Taylor series solution about the point x = 0.

(Q) The initial value problem

$y' + y = r(x), y(0) = 0$, where $r(x) = \begin{cases} e^{-\frac{1}{x^2}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

has a Taylor series solution about the point $x = 0$.

Then which of the following statements are true?

1. (P) is true.

2. (P) is FALSE.

3. (Q) is true.

4. (Q) is FALSE.

Q.72 For each positive integer n, let $f_n: [1,1] \to R$ be given by

$$f_n(x) = \begin{cases} -1, & if\ -1 \le x \le -\frac{1}{n}, \\ nx, & if\ -\frac{1}{n} < x < \frac{1}{n}, \\ 1, & if\ \frac{1}{n} \le x \le 1. \end{cases}$$

On which of the following intervals does the sequence $\{f_n\}_{n\ge1}$ converge uniformly?

1. $[0,1]$

2. $(0,1]$

3. $(10^{-2025}, 1)$

4. $(-10^{-2025}, 10^{-2025}]$

Q.73 Let $X_1, X_2, X_n\ (n \ge 2)$ be a random sample from Uniform$\left[-\frac{3\theta}{2}, \frac{\theta}{2}\right]$ distribution, where $\theta > 0$ is an unknown parameter. Let $\hat{X}, X_{(1)}$ and $X_{(n)}$ denote respectively the mean, the smallest order statistic and the largest order statistic of the sample. Then which of the following statements are true?

1. The method of moments estimator of θ is $-2\hat{X}$

2. The method of moments estimator of θ is $2\hat{X}$

3. The maximum likelihood estimator of θ is $min\ \left\{-\frac{2X_{(1)}}{3}, 2X_{(n)}\right\}$

4. The maximum likelihood estimator of is θ max$\left\{-\frac{2X_{(1)}}{3}, 2X_{(n)}\right\}$

Q.74 Suppose $y_1(x)$ and $y_2(x)$ are two linearly independent solutions of the differential equation

$x^2y^n + \frac{(1+sin\,x)x}{2}y' - \frac{3}{2}(cos\,x)y = 0, x > 0$

satisfying $y_2(0) = 0$.

Then which of the following statements are true?

1. $\lim_{x\to0^+} \frac{y_2(x)}{x^2y_1(x)} exists.$

2. $\lim_{x\to0^+} \frac{y_2(x)}{xy_1(x)} exists.$

3. $\lim_{x\to0^+} \frac{xy_1(x)}{y_2(x)} doex\ NOT\ exist.$

4. $\lim_{x\to0^+} \frac{x^2y_1(x)}{y_2(x)} exists.$

Q.75 Let V be a finite-dimensional R-vector space and $T: V \to V$ a linear operator such that T^2 is diagonalizable over R. Which of the following statements are necessarily true?

1. If T is not diagonalizable over R, then T^2 has an eigenvalue ≤ 0.

2. If T^2 has only negative eigenvalues, then dim V is an even integer.

3. If T^2 has only non-negative eigenvalues, then T is diagonalizable over R.

4. For each non-zero $v \epsilon V \{v, Tv, T^2 v\}$ is linearly dependent.

Q.76 Consider the following subset of real numbers

$A = \left\{(1 + (-1)^n)\, n\frac{1}{n} \; n \; is \; a \; positive \; integer\right\}.$

Which of the following statements are true?

1. A is bounded below but not bounded above.

2. A is bounded above but not bounded below.

3. $\inf A = -1$

4. $\sup A = 1$

Q.77 Let G be a group of order 8. Which of the following statements are necessarily true?

1. If there are no elements of order 4 in G, then G is abelian.

2. If there are exactly two elements of order 4 in G, then G is abelian.

3. If there are exactly six elements of order 4 in G, then G is abelian.

4. There is an element of order 4 in G.

Q.78 Let V be a real vector space. Suppose that $\{u, v, w, x, y\} \subseteq V$ CV is a spanning set of V and that $\{u, v, x, y\}$ is linearly independent. Which of the following statements are necessarily true?

1. The dimension of V is 4 or 5.

2. If $2u - 3v + 5w = 0$, then $\{v, w, x, y\}$ is a basis of V.

3. If $u - 6v + 7w = 0$, then the span of $\{v, w, x\}$ is a 2-dimensional vector space.

4. If $u + w = v + x$, then $\{u, v, w, y\}$ is a basis of V.

Q.79 Let $X_1, X_2, , , , , . X_n (n > 5)$ be independent random variables such that

$X_t = a + a^2 t + \epsilon_t$, for $t = 1, \ldots, n$, where, $\epsilon_1, \epsilon_2, \ldots, \epsilon_n$ are independent and identically distributed $N(0, \sigma^2)$ random variables. Here $a \in R$ and $\sigma > 0$ are unknown parameters. Which of the following statements are true?

1. $(X_1, X_2, \ldots., X_n)$ is a sufficient statistic for (a, σ)

2. $(\sum_{t=1}^{n} X_t, \sum_{t=1}^{n} tX_t, \sum_{t=1}^{n} t^2 X_t^2)$ is a jointly minimal sufficient statistic for (a, σ)

3. $X_4 - X_3 - X_2 + X_1$ is an ancillary statistic

4. $\frac{X_4+X_1-X_2-X_3}{X_5+X_2-X_3-X_4}$ is an ancillary statistic

Q.80 Suppose two fair dice are thrown independently at random. Let X and Y be the numbers on the upper face of the first die and that of the second die, respectively. Then which of the following statements are true?

1. $P(X-Y=0|X+Y=2)=P(X-Y=0|X+Y=12)$

2. $E\left(\frac{X-Y}{X+Y}\right)=0$

3. $Cov(X+Y,X-Y)=0$

4. $(X+Y)$ and $(X-Y)$ are independent

Q.81 Fort $\gamma \in [0,2\pi]$, let $\gamma_1(t)=e^{it}$ and $\gamma_2(t)=1+i+e^{it}$. Which of the following statements are true?

1. $\int_{\gamma_1}\frac{dz}{z\sin z}=0$

2. $\int_{\gamma_1}\frac{dz}{z\sin z}=2\pi i$

3. $\int_{\gamma_2}\frac{dz}{z\sin z}=2\pi i$

4. $\int_{\gamma_1}\frac{dz}{z\sin z}=0$

Q82. Consider a dataset of n observations given by $A=\{x_1,x_2,\ldots,x_n\}$. Create a new dataset, given by $B=\{x_1,x_2,\ldots,x_n,-x_1,-x_2,\ldots,-x_n\}$. Which of the following statements are always true?

1. Mean of the observations in dataset B is 0

2. Median of the observations in dataset B is 0

3. Variance of the observations in dataset $B \geq$ Variance of the observations in dataset A

(Here variance is calculated with divisor as the number of observations in the corresponding dataset)

4. Range of the observations in dataset $B \geq$ Range of the observations in dataset A

Q83. Let V be a real vector space and $L(V)$ denote the space of linear operators on V. Let $T \in L(V)$ be a non-zero operator such that $T^2=T$. Consider the subspace W of $L(V)$ spanned by $\{I,T^n: n$ is a positive integer$\}$.

Which of the following statements are necessarily true?

1. The set $\{S \in W: S^2=S\}$ contains exactly 2 elements.

2. $\dim_R(W)=2$

3. If $U \in L(V)$ is such that $U^2 = U$ and $(T+U)^2 = T+U$, then $TU = 0$.

4. If $U \in L(V)$ is such that $U^2 = U$ and $(T+U)^2 = T-U$, then $(TU)^2 = TU$.

Q84. Let $F: R^3 \to R$ be a continuously differentiable function such that

$F(0,0,0) = -1, \frac{\partial F}{\partial x}(0,0,0) = 2, \frac{\partial F}{\partial y} = (0,0,0)$ and $\frac{\partial F}{\partial z} = (0,0,0) = 3$.

For $\in > 0$, define $\Omega = (-\epsilon, \epsilon) \times (-\epsilon, \epsilon) \subset R^2$. Which of the following statements are necessarily true?

1. There exist $\epsilon > 0, \delta > 0$ and a continuously differentiable function $g: \Omega_\epsilon \to (-\delta, \delta)$ such that $g(0,0) = 0$ and $F\big(x, y, g(x,y)\big) = -1$ for all $(x,y) \in \Omega_\epsilon$

2. There exist $\epsilon > 0, \delta > 0$ and a continuously differentiable function $h: \Omega_\epsilon \to (-\delta, \delta)$ such that $h(0,0) = 0$ and $F(x, h(x,z), z) = -1$ for all $(x,y) \in \Omega_\epsilon$

3. There existe $\epsilon > 0, \delta > 0$ and continuously differentiable functions $k_1, k_2: (-\epsilon, \epsilon) \to (-\delta, \delta)$ such that $k_1(0) = k_2(0) = 0$ and $F\big(x, k_1(x), k_2(x)\big) = -1$ for all $x \in (-\epsilon, \epsilon)$.

4. There exist $\epsilon > 0, \delta > 0$ and continuously differentiable functions $j_1, j_2: (-\epsilon, \epsilon) \to (-\delta, \delta)$ such that $j_1(0) = j_2(0) = 0$ and $F(j_1(z), j_2(z), z) = -1$ for all $z \in (-\epsilon, \epsilon)$

q85. Let l^2 denote the vector space of all square summable sequences $\{a_n\}_{n \geq 1}$ of real numbers with the inner product given by $\langle \{a_n\}, \{b_n\} \rangle = \sum_{n=1}^{\infty} a_n b_n$.

Let $H = \left\{ \{a_n\} \in l^2 : |a_n| \leq \frac{1}{n} \; for \; all \; positive \; integers \; n \right\}$

Which of the following statements are true?

1. H contains an orthonormal basis of l^2.

2. H is a linear subspace of l^2.

3. H is a bounded subset of l^2.

4. H is a convex subset of l^2.

Q86. Let $X_1, X_2, \ldots, X_{20}$ be a random sample from $N_{12}(\mu \sum)$, where $\mu \in R^{12}$ and $\sum$ is positive definite matrix. Suppose $\hat{X} = \frac{1}{20} \sum_{i=1}^{20} X_i$ and $S == \frac{1}{20} \sum_{i=1}^{20} \big(X_i - \hat{X}\big)\big(X_i - \hat{X}\big)^T$ .Then which of the following statements are true?

1. $19\big(\hat{X}^T S \hat{X}\big)\left(\hat{X}^T E \hat{X}\right)^{-1} \sim X_{19}^2$

2. $E(S^{-1}) = \frac{19}{6} \sum^{-1}$

3. $S \sim W_{12}(19, 19 \sum)$

4. $trace\ (19 \sum^{-1} S) \sim X_{228}^2$

Q87. Let $X_1, X_2, \ldots, X_{15}$ be a random sample from an Exponential distribution with the probability density function

$$f(x|\sigma) = 0, \begin{cases} \frac{1}{\sigma}\exp\left(-\frac{x}{\sigma}\right), & if\ x > 0, \\ & elsewhere, \end{cases}$$

where the unknown parameter σ is positive. Let $\hat{X} = \frac{1}{15}\sum_{i=1}^{15} X_i$. Suppose that ϕ denotes the likelihood ratio test for testing $H_O: \sigma \leq 1$ against $H_1: \sigma > 1$ at level $\sigma = 0.1$. It is given that $X^2_{15,0.1} = 22.307, X^2_{15,0.9} = 8.547, X^2_{30,0.1} = 40.256, X^2_{30,0.9} = 20.599$, where $P(W > X^2_{n,a}) = a$ and $W \sim X^2_n$. Then which of the following statements are true?

1. If the observed value of $\hat{X}$ is 0.6, then ϕ does not reject H_O

2. If the observed value of $\hat{X}$ is 1.6, then ϕ rejects H_O

3. If the observed value of $\hat{X}$ is 1.3, then ϕ rejects H_O

4. If the observed value of $\hat{X}$ is 1.2, then ϕ does not reject H_O

Q88. Suppose $X|\theta \sim$Binomial $(7, \theta), 0 < \theta < 1$, and the prior distribution of θ is Beta(a, β) where $a > 0$ and $\beta > 0$ are known. Then which of the following statements MAY NOT be true?

1. Posterior mean of θ given X = 2 is less than the prior mean

2. Posterior mean of θ given X = 3 is greater than the prior mean

3. Posterior mean of θ given X = 4 is not equal to the prior mean

4. Posterior mean of θ given X = 5 is equal to the prior mean

Q89. Let X and Y be two independent random variables such that the moment generating functions of X and Y are $M_x(t) = e^{3(e^t-1)}, t \in R$,

and $M_y(t) = \left(\frac{1}{2}e^{3t} + \frac{1}{2}e^{3t}\right) t \in R$,

respectively. Then which of the following statements are true?

1. $P(XY = 0) = \frac{1+e^{-3}}{2}$

2. $E(X + Y) = 3$

3. $Var(X + Y) = 21$

4. $Cov(X + Y, X - Y) = 0$

Q90. For $n \geq 3$, consider the space $M_n(C)$ of $n \times n$ complex matrices endowed with the inner product $\langle A, B\rangle = Trace(A * B)$. For $0 \leq k \leq n$, let

$W_k = \{A \in M_n(C) : \langle A, B\rangle = 0 \; for\; all\; B \in M_n(C)\; with\; rank\; k\}.$

Which of the following statements are necessarily true?

1. $W_0 = \{0\}$
2. $W_1 = \{0\}$
3. $W_2 = \{0\}$
4. $W_3 = \{0\}$

Q.91 Suppose y(x) is the extremal of the variational problem $J(y) = \int_0^1 ((y')^2 sin\, x + (2\, cos\, x)y)dx$ subject to $y(0) = 0, y(1) = 1.$
Then which of the following statements are true?

1. $y\left(\frac{1}{2}\right) = 1$
2. $y'(0) = 1$
3. $y\left(\frac{1}{4}\right) = 2$
4. $y'\left(\frac{1}{2}\right) = 1$

Q.92 Let $p \geq 3$ be a prime number and $f(x) \in \mathbb{Q}[x]$ an irreducible polynomial of degree p. Suppose that $a_1, , , a_p \in \mathbb{C}$ are the roots off and that
$a_1 \notin \mathbb{R}, a_2 \notin \mathbb{R}$ and $a_3 \notin \mathbb{R}$ for all $3 \leq i \leq p$.
Let $K = \mathbb{Q}(a_1, , a_p)$ be the subfield of $\mathbb{C}$ generated by the roots of f. Consider the Galois group G of K over $\mathbb{Q}$ as a subgroup of s_p, the group of permutations $\{a_1, , a_p\}$ Which of the following statements are true?

1. The transposition (1 2) belongs to G.
2. |G| is divisible by p.
3. A p-cycle belongs to G.
4. G = s_p

Q.93 Let a, b be distinct positive real numbers. Consider the function $f : R^2 \to R$ given by

$$f(x,y) = \begin{cases} \dfrac{(ax+by)^2}{ax^2+by^2}, & \text{if } (x,y) \neq (0,0), \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$$

Which of the following statements are necessarily true?

1. $\lim_{(xy)\to(0,0)} f(x,y)$ does not exist.
2. The partial derivatives of ƒ at (0,0) do not exist.
3. $\lim_{x\to 0}$ f(x, 0)=. $\lim_{y\to 0}$ f(0, y)
4. f is differentiable at (0,0).

Q.94 Let $X_1, X_2, \ldots, X_7$ be a random sample drawn from a continuous distribution with unknown unique median M. The null hypothesis H_0: $M = 2$ is tested against the alternative $H_1 : M > 2$ at level of significance 0.05 using the right-tailed test based on the Sign test statistic K, which is the number of observations in the sample greater than 2. If the observed sample is -3, -6, 1, 9, 4, 10, 12, which of the following statements are true?

1. Ho is rejected
2. The p-value of the test is greater than 0.01
3. Under $H_0, E(K) = 3.5$
4. If $X_{(i)}$ denotes the i-th smallest observation of the sample, then $[X_{(2)}, X_{(6)}]$ is a confidence interval for M with confidence coefficient at least 0.95

Q.95 Let $u(x, t)$ be the solution of the initial-boundary value problem

$$u_{tt} + u_{xx} = 0, 0 < x < \pi \; t > 0,$$
$$u(0, t) = 0, u(\pi, t) = 0, t > 0,$$
$$u(x, 0) = \sin x, u_t(x, 0) = 0, 0 < x < \pi.$$

For t > 0, define

$$E(t) = \int_0^{\pi} (u_t^2 + u_x^2) dx.$$

Then which of the following statements are true?

1. $E(t) \geq \pi$ for all t > 0.
2. $E(t) \leq \pi$ for all t > 0.
3. $E(t) \geq \frac{\pi}{2}$ for all t > 0.
4. $E(t) \leq \frac{\pi}{2}$ for all t > 0.

Q.96 A mechanical system is described using generalized position q and generalized momentum p. Let Q and P denote new generalized position and generalized momentum variables respectively, generated by the generating function $F(g, P) = q^2 e^p$ and Q, P are canonical coordinates. Let G(p, Q) be a function such that G(2, e) = 0, and it generates the same canonical coordinates Q, P. Then which of the following statements are true?

1. $G(p, Q) = -Q\left[1 + log_e\left(\frac{p^2}{4Q}\right)\right]$
2. $G(p, Q) = -pQ\left[1 + log_e\left(\frac{Q^2}{4p}\right)\right]$
3. $p = 2qe^p, Q = q^2 e^p$
4. $p = -2qe^p, Q = q^2 e^p$

Q.97 Consider a series system comprising four components, having independent and identically distributed lifetimes with hazard rate $\lambda(t) = \frac{1}{1+t}$, $t < 0$, and survival function S(t), t > 0. If Y denotes the lifetime of the series system, then which of the following statements are true?

1. Cumulative hazard function of each component is $H(t) = 2\ln(1 + t), t > 0$
2. $S(t) = \frac{1}{(1+t)^2}, t > 0$
3. $P(Y < \frac{1}{2}) = \left\{S\left(\frac{1}{2}\right)\right\}^4$
4. $P(Y < \frac{1}{2}) = \frac{65}{81}$

Q.98 Let $\alpha = \sum_{t=1}^{2025} \frac{a_\iota}{10^t}$ with $a_\iota \epsilon \{0, 1, \ldots 9\}$ for all $1 \leq l \leq 2025$ and $0_{2025} \neq 0$. Let $(\beta_n)_{n\geq 1}$ be a strictly increasing sequence of positive real numbers in (0, 1) that converges to α. For each $n \geq 1$, write

$\beta_n = \sum_{l-1}^{\infty} \frac{b_n l}{10^l}$ with $b_n l \epsilon \{0, 1, \ldots 9\}$

for the infinite decimal expansion of β_n. Which of the following statements are necessarily true?

1. There exists a positive integer N such that for all $n \geq N$, $b_n l = a_\iota$, for all $1 \leq l \leq 2023$.

2. There exists a positive integer N such that for all $n \geq N$, $b_{n.2025} = a_{2025} - 1$.

3. B is rational for infinitely many n.

4. There exists a positive integer N such that for all $=n \geq N$, $b_{n.2024} = a_{2024} - 1$.

Q.99 Let (X, d) be a metric space. For a non-empty subset A of X, and x = X, define $d(x, A) = \inf_{a \epsilon A} d(x, a)$.

Which of the following statements are necessarily true?

1. For all x, y Є X and every non-empty subset A of X, we have $d(x, A) - d(y, A) \leq d(x, y)$.

2. For every non-empty subset A of X, the function → d(x, A) is uniformly continuous on X.

3. A non-empty subset A of X is closed if and only if d(x, A) > 0 for all z Є X \ A.

4. If X is compact and $C_1, \ldots, C_2$ are non-empty closed sets of X such that $\bigcap_{1\leq i\leq k} C_i = 0$. then the minimum value of the function $x \to \sum_{1\leq i\leq k} d(x, C;)$ on X is 0.

Q.100 Consider the simple linear regression model $Y_i = \beta x_i + \epsilon_i, , i = 1, 2, \ldots, n$, where $\beta > 0, \sum_{i=1}^{n} x_i^2 > 0$. Let $\hat{\beta}_1 = \sum_{i=1}^{n} \hat{a}_i Y_i$, where $a_i^{*\prime}$s minimize $E(\sum_{i=1}^{n} a_i Y_i - \beta)^2$ with respect to scalars $a_i, a_2, \ldots . a_n$ Let $\hat{\beta}_2$ e the ordinary least squares estimator of 3. Which of the following statements are true?

1. $Var(\hat{\beta}_1) > Var(\hat{\beta}_2), E(\hat{\beta}_1 - \beta)^2 > E(\hat{\beta}_2 - \beta)^2$

2. $Var(\hat{\beta}_1) < Var(\hat{\beta}_2), E(\hat{\beta}_1 - \beta)^2 < E(\hat{\beta}_2 - \beta)^2$

3. $Var(\hat{\beta}_1) > Var(\hat{\beta}_2), E(\hat{\beta}_1 - \beta)^2 < E(\hat{\beta}_2 - \beta)^2$

4. $Var(\hat{\beta}_1) < Var(\hat{\beta}_2), E(\hat{\beta}_1) > E(\hat{\beta}_2)$

Q.101 Let y(x) be the solution of the integral equation (IE)

$y(x) = e^x + e + 1 + \frac{1}{3}\int_0^1 y(t)\,dt$, and $R\left(x, t, \frac{1}{3}\right)$ Then which of the following statements are true?

1. $R\left(0,\ 1,\ \frac{1}{3}\right) = \frac{3}{2}$

2. $R\left(\frac{1}{2}\ ,\ \frac{1}{2}\ ,\ \frac{1}{3}\right) = \frac{2}{3}$

3. $y(1) = 3e + 1$

4. $y(1) = e + 3$

Q.102 Let f : [0,1] → [0,1] be a monotonically increasing function, that is, a ≤ b implies that f(a) ≤ f(b). For any α ∈ (0,1), let $L_\alpha^+ = \lim_{x\to\alpha^+} f(x)$ and $L_\alpha^- = \lim_{x\to\alpha-} f(x)$

denote the right hand and left hand limits respectively, provided they exist. For α ∈ (0,1), if L_α^+ and L_α^- exist, then define

$$U_\alpha = \begin{cases} (L_\alpha^-, L_\alpha^+), & if\ L_\alpha^- < L_\alpha^+, \\ 0, & if\ L_\alpha^- \le L_\alpha^+. \end{cases}$$

Which of the following statements are true?

1. L_α^+ and L_α^- exist for every α ∈ (0,1).

2. If f is surjective, then f is continuous.

3. If f is Riemann integrable, then f is continuous.

4. If the left and right hand limits exist at α, β ∈ (0,1), α ≠ β, then $U_\alpha \cap U_\beta = \emptyset$.

Q.103 Which of the following statements are true?

1. $x^2 + xy^2 - x - y + 1$ is irreducible in $\mathbb{Q}[x, y]$.

2. $x^3 + 100x^2 - 25x + 10$ is irreducible in $\mathbb{Z}[x]$.

3. $2x^2 + 3xy - y^2 - 3x + 2y + 1$ is irreducible in $\mathbb{Q}[x, y]$.

4. $x^4 + 4x^2 + 3$ is irreducible in $\mathbb{Z}[x]$.

Q.104 Consider a commutative ring R with unity with at least five elements such that for any two elements a, b ∈ R, there exists c∈R such that a = bc or b=ac. Which of the following statements are necessarily true?

1. R has a unique maximal ideal.

2. Every finitely generated ideal of R is principal.

3. R is an integral domain.

4. R is a Euclidean domain.

Q.105 Let $\mathcal{T}_E$ denote the Euclidean topology on $\mathbb{R}$. Which of the following statements are necessarily true?

1. $(\mathbb{R}, \mathcal{T}_E)$ is a normal space.

2. Let $\mathcal{T}$ be a topology on $\mathbb{R}$. If the identity function of $\mathbb{R}$ is a continuous map from $(\mathbb{R}, \mathcal{T}_E)$ to $(\mathbb{R}, \mathcal{T})$, then $(\mathbb{R}, \mathcal{T})$ is a regular space.

3. Let $\mathcal{T}$ be a topology on $\mathbb{R}$. If the identity function of $\mathbb{R}$ is a continuous map from $(\mathbb{R}, \mathcal{T})$ to $(\mathbb{R}, \mathcal{T}_E)$ then $(\mathbb{R}, \mathcal{T})$ is a Hausdorff space.

4. $\mathbb{R}$ is a regular space in the finite-complement topology.

Q.106 Consider the ordinary differential equation (ODE)

$y'' + r(x)y = 0,$

where

$$r(x) = \begin{cases} x^3 \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Then which of the following statements are true?

1. There exists a non-trivial solution φ of ODE, and $\alpha, \beta \in \mathbb{R}, \alpha < \beta$ such that has infinitely many zeros in $[\alpha < \beta]$.

2. There exists a non-trivial solution φ of ODE, and a sequence $\{x_n\}$ such that $x_n \to \infty$ and $\varphi(x_n) = 0$ for all $n \in N$.

3. If φ_1, φ_2 are two linearly independent solutions of ODE, then their Wronskian is a constant function on $\mathbb{R}$.

4. There exists a non-trivial solution of ODE which vanishes at most at one point in $(0, \infty)$.

Q.107 Let F be the set of all functions f: $[0, \infty) \to [0, \infty)$ that are Riemann integrable on [0,1] for all $t \in [0, \infty)$. Which of the following statements are necessarily true?

1. If $f \in F$ is a continuous function and f(n) = 0 for all positive integers n, then $\lim_{x\to\infty} \int_0^x f(t)dt$ exists in $\mathbb{R}$.

2. If $f \in F$ is uniformly continuous and $\lim_{x\to\infty} \int_0^x f(t)dt$ exists in $\mathbb{R}$ but $\lim_{x\to\infty} \int_0^x f(t) = 0$

3. There exists a function $f \in F$ such that $\lim_{x\to\infty} \int_0^x f(t)dt$ exists in $\mathbb{R}$ but $\lim_{x\to\infty} \int_0^x f(t) \neq 0$

4. If $f \in F$ is Lebesgue measurable and $\lim_{x\to\infty} \int_0^x f(t)dt = 0$ then f=0 almost everywhere on $[0, \infty)$.

Q.108 Let V be a 4-dimensional complex vector space and A a linear operator on V. Which of the following statements are necessarily true?

1. There exist $\lambda \in C$ and a non-zero $v \in V$ such that $Av = \lambda v$.

2. There exist $\lambda, \mu \in C$ and linearly independent vectors $v, w \in V$ such that $Av = \lambda v$ and $Aw = \mu w$.

3. There exist $\lambda, \mu, \delta \in C$ and linearly independent vectors $v, w \in V$ such that $Av = \lambda v$ and $Aw = \mu v + \delta \omega$.

4. There exists a three-dimensional subspace W such that $Aw \in W$ for all $w \in W$.

Q.109 Let $f: (0, \infty) \to (0, \infty)$ be the function defined by $f(x) = \int_0^x \frac{\sqrt{t}}{1+t^2} dt$, where $\sqrt{t}$ denotes the positive square root for $t > 0$. Which of the following statements are true?

1. f is a uniformly continuous function.

2. f is a bounded function.

3. There exists a $x \in (0, \infty)$ such that $f(x) = 0$.

4. The derivative of f is continuous.

Q.110 Let A be a non-zero 3×3 matrix with integer entries. Let $\lambda_i; \in C, 1 \leq i \leq 3$ be all the eigenvalues of A (not necessarily distinct). Which of the following statements are necessarily true?

1. There exists a cubic polynomial $f(X) \in Q[X]$ such that $f(\lambda_i) = 0$ for all $1 \leq i \leq 3$.

2. There exists a quadratic polynomial $f(X) \in Q[X]$ such that $f(\lambda_i) = 0$ for all $1 \leq i \leq 3$.

3. If $f(X) \in Q[X]$ is such that $f(\lambda_i) = 0$ for all $1 \leq i \leq 3$, then $f(A) = 0$.

4. If $f(X) \in Q[X]$ is a cubic polynomial such that $f(\lambda_i) = 0$ for all $1 \leq i \leq 3$, then $f(A) = 0$.

Q.111 Let $\mathbb{N}$ denote the set of all positive integers. For $n \in \mathbb{N}$, let $f_n: [0, 1] \to R$ be given by

$$f(x) = \begin{cases} n(1 - nx), & if\ 0 \leq x \leq \frac{1}{n}, \\ 0, & \text{if x} > \frac{1}{\text{n}}, \end{cases}$$

Which of the following statements are necessarily true?

1. The sequence of functions $\{f_n\}_{n \geq 1}$ is uniformly bounded.

2. The sequence of functions $\{f_n\}_{n \geq 1}$ does not converge pointwise.

3. The sequence of functions $\{f_n\}_{n \geq 1}$ converges uniformly to the constant function 0.

4. The set $\{f_n: n \in \mathbb{N}\}$ is compact in $C[0, 1]$, where $C[0, 1]$ denotes the space of all real valued continuous functions on $[0, 1]$ equipped with the supremum norm.

Q.112 Let S be the set of all 2×2 matrices A such that the iterative sequence generated by the Gauss-Seidel method converges for every initial guess, when employed to solve the system of equations $A\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

Then which of the following statements are true?

1. $\begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix} \in S$

2. $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \in S$

3. $\begin{pmatrix} 1 & 5 \\ 1 & 10 \end{pmatrix} \in S$

4. $\begin{pmatrix} -5 & 2 \\ 1 & -4 \end{pmatrix} \in S$

Q.113 Let f be an entire function. Consider the function g given by

$g(z) = f(z) - \frac{1}{z}$

for $z \in C\backslash\{0\}$. Which of the following statements are necessarily true?

1. The function g has a pole at 0.

2. If $g(\alpha) = 0$, then $\alpha \neq 1$.

3. The function g has only finitely many zeros.

4. $\max_{|z|=1} |g(z)| \geq 1$

114 Let $D = \{z \in C : |z| < 1\}$ and $f : D \to D$ be a holomorphic function which satisfies $f(-1/2) = 0$. Which of the following statements are necessarily true?

1. $|f(-1/5)| \leq 1/5$

2. $|f(-1/5)| \leq 1/3$

3. $|f(-1/2)| \leq 1/2$

4. $|f(-1/2)| \leq 4/3$

Q.115 Let G be a finite non-abelian group. Which of the following statements are necessarily true?

1. If d is a positive integer that divides $|G|$, then G has a subgroup of order d.

2. The map $f : G \times G \to G$ given by $f(a, b) = ab$ is not a group homomorphism.

3. Suppose that for every positive integer d that divides $|G|$, there exists a subgroup of G of order d. Then G has at least three normal subgroups.

4. $|G| \neq 16$

Q.116 Let $\{X_n : n \geq 1\}$ be a sequence of independent and identically distributed random variables, where X_1 has an Exponential distribution with mean 1. Define

$T_n = max\{X_1, X_2, \ldots, X_n\} - In\, n, n \geq 1.$

Suppose $T_n \xrightarrow{d} Y$ as $n \to \infty$. Then which of the following statements are true?

1. Y has a Double Exponential distribution with location parameter In(In 2) and scale parameter 1

2. Median of $Y = \ln(ln\, 2)$

3. $P(Y \leq 0) = e^{-1}$

4. The derivative of the cumulative distribution function of Y at In 3 is $e^{-\frac{1}{3}}$

Q.117 Consider an $M/M/2$ queuing system with the birth rate $\lambda = 4$ per minute, the death rate $\mu = 1$ per minute, and the total capacity of 3 customers (including the ones that are being served). Let p_0 and p_3 denote the long-run probabilities that the system will be empty (i.e. without customers) and will be blocked (i.e. full), respectively. Which of the following statements are true?

1. $P_0 + P_3 = \frac{17}{29}$

2. $P_0 > P_3$

3. $\frac{P_3}{P_0} = 2$

4. $P_3 - P_0 = \frac{15}{29}$

Q118. Let $f : C \to C$ be defined by

$f(z) = sin^2 z + cos^2|z|.$

Which of the following statements are true?

1. f is a real valued function.

2. $f(z) = 1$ for all $z \in C$.

3. f is not an entire function.

4. f has finitely many zeros on the imaginary axis.

Q119. Consider the boundary value problem (BVP)

$U_{xx} + U_{yy} = 0$ in $\Omega = \{(x, y) \in R^2 : x^2 + y^2 < 1\}$,

$u(x, y) = e^{x+y}$ on $\partial\Omega = \{(x, y) \in R^2 : x^2 + y^2 = 1\}$.

Then which of the following statements are true?

1. There exists a unique solution to BVP.

2. The BVP does NOT have a solution.

3. There exists a solution u to BVP such that $u(x, y) = \frac{1+e}{2}$ for some $(\mathrm{x}, \mathrm{y}) \in \Omega \cup \partial\Omega$.

4. There exists a solution u to BVP such that $u(x, y) = \frac{1+e^3}{2}$ for some $(\mathrm{x}, \mathrm{y}) \in \Omega \cup \partial\Omega$.

Q.120 Consider $X = \prod_{c\in(0,\infty)} [0, c]$ together with the product topology, where $[0, c]$ is equipped with the euclidean topology. Which of the following stssatements are necessarily true?

1. X is metrizable.

2. X is compact.

3. X is Hausdorff.

4. X is connected.