SECTION –A

MULTIPLE CHOICE QUESTION (MCQ)

Q. 1- Q.10 carry one mark each.

1. The value of the limit

$$\lim_{n\to\infty}\left(\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\ldots\left(1+\frac{n}{n}\right)\right)^{\frac{1}{n}}$$

Is equal to

(A) e

(B) $\frac{1}{e}$

(C) $\frac{3}{e}$

(D) $\frac{3}{e}$

2. Let $f:\mathbb{R}\to\mathbb{R}$ be a function defined by

$$f(x)=x^7+5x^3+11x+15 \quad x\in\mathbb{R}$$

Then, which of the following statements id TRUE?

(A) f is both one-one and onto

(B) f is neither one-one nor onto

(C) f is both one-one but NOT onto

(D) f is onto but NOT one-one

3. The value of the limit

$$\lim_{x\to 0}\frac{e^{-3x}-e^{x}+4x}{5(1-\cos x)}$$

Is equal to

(A) 1

(B) 0

(C) $\frac{2}{5}$

(D) $\frac{8}{5}$

4. The value of the limit

$$\lim_{n\to\infty}\sum_{k=0}^{n}\binom{2n}{k}\frac{1}{4^n}$$

Is equal to

(A) 1

(B) $\frac{1}{2}$

(C) 0

(D) $\frac{1}{4}$

5. Let $\{X_n\}_{n\geq 1}$ be sequence of independent and identically distributed random variables with probability density function

$$f(x) = \begin{cases} 1, & if\ 0 < x < 1 \\ 0, & otherwise \end{cases}$$

Then the value of the limit

$$\lim_{n\to\infty} P\left(-\frac{1}{n}\sum_{i=1}^{n} 1n\ X_i \leq 1 + \frac{1}{\sqrt{n}} \right)$$

Is equal to

(A) $\frac{1}{2}$

(B) $\Phi(1)$

(C) 0

(D) $\Phi(2)$

6. Let X be a $U(0,1)$ ransom variable and $Y = X^2$. If ρ is the correlation between the random variables X and Y, then 48 ρ^2 is equal to

(A) 48

(B) 45

(C) 35

(D) 30

7. Let M be a be a 3 x 3 real matrix. Let $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} and \begin{pmatrix} 0 \\ -1 \\ a \end{pmatrix}$ be the eigenvectors of M corresponding to three distinct of M, where a is a real number. Then, which of the following is NOT a possible value of a?

(A) 0

(B) 1

(C) -2

(D) 2

8. If the series $\sum_{n=11}^{\infty} a_n$ converges absolutely, then which of the following series diverges?

(A) $\sum_{n=1}^{\infty} |a_{2n}|$

(B) $\sum_{n=1}^{\infty} \frac{a_n + a_n + 1}{2}$

(C) $\sum_{n=1}^{\infty} (a_n)^3$

(D) $\sum_{n=1}^{\infty} \left(\frac{1}{(1n\ n)^2} + a_n \right)$

9. There are three urns, labeled, Urn 1, Urn 2 and Urn 3. Urn 1 contains 2 white balls and 2 black balls, Urn 2 contains 1 white ball and 3 black balls and Urn 3 contains 3 white balls and 1 black ball Consider two coins with probability of obtaining head in their single trials as 0.2 and 0.3. The two coins are tossed independently once, and an urn is selected according to the following scheme: Urn 1 is selected if 2 heads are obtained: Um 3 is selected if 2 tails obtained; otherwise Urn 2 is selected. A ball is then drawn at random from the selected urn. Then

P (Urn 1 is selected the ball drawn is white)

is equal to

(A) $\frac{6}{109}$

(B) $\frac{12}{109}$

(C) $\frac{1}{18}$

(D) $\frac{1}{9}$

10. Let X be a random variable with probability density function

$$f(x) = \frac{1}{2}e^{-|x|}, -\infty < x < \infty$$

Then, which of the following statements is FALSE?

(A) $E(X|X|) = 0$

(B) $E(X|X|^2) = 0$

(C) $E\left(|X| \sin\left(\frac{X}{|X|}\right)\right) = 0$

(D) $E\left(|X| \sin\left(\frac{X}{|X|}\right)\right) = 0$

Q. 11 – Q.30 carry two marks each

11. Let $f: \mathbb{R}^2 \to \mathbb{R}$ be a function defined by

$$f(x,y) = \begin{cases} \frac{y^3}{x^2+y^2} & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

Let $f_x(x,y)$ and $f_y(x,y)$ denote the first order partial derivatives of $f(x,y)$ with respect to x and y respectively, at the point (x,y). Then which of the following statements is FALSE?

(A) $f_x(x,y)$ exist and is bounded at every at every $(x,y) \in \mathbb{R}^2$

(B) $f_x(x,y)$ exist and is bounded at every at every $(x,y) \in \mathbb{R}^2$

(C) $f_x(0,0)$ exist and $f_x(x,y)$ is continuous at (0, 0)

(D) f is NOT differentiable at (0,0)

12. Let $\{X_n\}_{n\geq 1}$ be a sequence of independent and identically distributed $N(0,1)$ random variables. Then,

$$\lim_{n\to\infty} P\left(\frac{\sum_{i=1}^{n} X_i^4 - 3n}{\sqrt{32n}} \leq \sqrt{6}\right)$$

Is equal to

(A) $\frac{1}{2}$

(B) $\Phi(\sqrt{2})$

(C) 0

(D) $\Phi(1)$

13. Consider a sequence of independent Bernoulli trials with probability of success in each trial as $\frac{1}{3}$. The probability that three successes occur before four failure is equal to

(A) $\frac{179}{243}$

(B) $\frac{179}{841}$

(C) $\frac{233}{729}$

(D) $\frac{179}{1215}$

14. Let X and Y be independent $N(0,1)$ random variables and $Z = \frac{|X|}{|Y|}$. Then, which of the following expectations is finite?

(A) $E\left(\frac{1}{\sqrt{Z}}\right)$

(B) $E(Z\sqrt{Z})$

(C) $E(Z)$

(D) $E\left(\frac{1}{Z\sqrt{Z}}\right)$

15. Consider three coins having probabilities of obtaining head in a single trial as $\frac{1}{4}, \frac{1}{2}$ *and* $\frac{3}{4}$ respectively. A player selects one of these three coins at random (each coin is equally likely to be selected). If the player tosses the selected coin five independently, then the probability of obtaining two tails in five tosses is equal to

(A) $\frac{85}{384}$

(B) $\frac{255}{384}$

(C) $\frac{125}{384}$

(D) $\frac{64}{384}$

16. Let X be a random variable having the probability density function

$$f(x) = \begin{cases} e^{-x} & x > 0 \\ 0, & x \leq 0 \end{cases}$$

Define $Y = [X]$, where [X], where denotes the largest integer not exceeding X.Then, $E(Y^2)$is equal to

(A) $\frac{e(e+1)}{e-1}$

(B) $\frac{e+1}{(e-1)^2}$

(C) $\frac{e(e+1)^2}{e-1}$

(D) $\frac{(e+1)^2}{(e-1)^2}$

17. Let X be a continuous random variable having the moment generating function

$$M(t) = \frac{e^t - 1}{t}, \quad t \neq 0$$

Let $\alpha = P(48\,X^2 - 40\,X + 3 > 0)$ and $\beta = P(1nX)^2 + 2\,1n\,X - 3 > 0)$

Then, the value of $\alpha - 2\,In\,\beta$ is equal to

(A) $\frac{10}{3}$

(B) $\frac{19}{3}$

(C) $\frac{13}{3}$

(D) $\frac{17}{3}$

18. Let $X_1, X_2, \ldots, X_n$ $(n \geq 3)$ be a random sample from Poisson (θ), where $\theta \in (\theta, \infty)$ is unknown and Let $T = \sum_{i=1}^{n} X_i$

Then, the uniformly minimum variance unbiased estimator of $e^{-2\theta}\theta^a$

(A) is $\frac{T}{n}\left(\frac{T}{n} - 1\right)\left(\frac{T}{n} - 2\right)\left(1 - \frac{2}{n}\right)^{T-3}$

(B) is $\frac{T(T-1)(T-2)(n-2)^{T-3}}{n^T}$

(C) does NOT exist

(D) is $e^{-\frac{2T}{n}}\left(\frac{T}{n}\right)^3$

19. Let $X_1, X_2, \ldots, X_n$ $(n \geq 2)$ be a random sample from $U(\theta - 5, \theta + 5)$, where $\theta \in (0, \infty)$ is unknown. Let $T = \max\{X_1, X_2, \ldots, X_n\}$ and $U = \min\{X_1, X_2, \ldots, X_n\}$. Then which of the following statement is TRUE?

(A) $\frac{T+U}{2}$ is the unique MLE of θ

(B) $\frac{2}{T+U}$ is an MLE of $\frac{1}{\theta}$

(C) MLE of $\frac{1}{\theta}$ does NOT exist

(D) $U + 8$ is an MLE of θ

20. Let X and Y be random variables having chi-square distributions with 6 and 3 degrees of freedom, respectively. Then, which of the following statements is TRUE?

(A) $P(X > 0.7) > P(Y > 0.7)$

(B) $P(X > 0.7) < P(Y > 0.7)$

(C) $P(X > 3) < P(Y > 3)$

(D) $P(X < 6) > P(Y < 6)$

21. Let X and Y be random vector with joint moment generating function

$$M(t_1, t_2) = \frac{1}{(1-(t_1+t_2)(1-t_2)}, \quad -\infty < t_1 < \infty, -\infty < t_2 < \min\{1, 1-t_1\}$$

Let $Z = X + Y$. Then, Var(Z) is equal to

(A) 3

(B) 4

(C) 5

(D) 6

22. Let X be a continuous random variable with distribution function

$$F(x) = \begin{cases} 0, & if\ x < 0 \\ ax^2 & if\ 0 \le x < 2, \\ 1 & if\ x \ge 2 \end{cases}$$

For some real constant a. Then $E(X)$ is equal to

(A) $\frac{4}{3}$

(B) $\frac{1}{4}$

(C) 1

(D) 0

23. Let $X_1, X_2, \ldots, X_n$ be a random sample from an exponential distribution with probability density function

$$f(x;\theta) = \begin{cases} \theta e^{-\theta x} & x > 0 \\ 0, & otherwise \end{cases}$$

Where $\theta \in (0, \infty)$ is unknown. Let $a \in (0, 1)$ be fixed and let β be the power of the most powerful test of size a for testing $H_0: \theta = 1$ against $H_1: \theta = 2$

Consider the critical region

$$R = \left\{(x_1, x_2, \ldots, x_n) \in \mathbb{R}^n : \sum_{i=1}^{n} x_i > \frac{1}{2}x_{2n}^2(1-a)\right\}$$

Where for any $\gamma \in (0, 1)$, $x_{2n}^2(\gamma)$ is a fixed point such that $P\left(x_{2n}^2 > x_{2n}^2(\gamma)\right) = \gamma$. Then , the critical region R corresponds to the

(A) most powerful test of size a for testing $H_0: \theta = 1$ against $H_1: \theta = 2$

(B) most powerful test of size $1 - a$ for testing $H_0^*: \theta = 2$ against $H_1^*: \theta = 2$

(C) most powerful test of size β for testing $H_0^*: \theta = 2$ against $H_1^*: \theta = 2$

(D) most powerful test of size $1 - \beta$ for testing $H_0^*: \theta = 2$ against $H_1^*: \theta = 1$

24. Let

$S = \sum_{k=1}^{\infty}(-1)^{k-1}\frac{1}{k}\left(\frac{1}{4}\right)^k$ and $T = \sum_{k=1}^{\infty}\frac{1}{k}\left(\frac{1}{5}\right)^k$

Then, which of the following statements is TRUE?

(A) $S - T = 0$

(B) $5S - 4T = 0$

(C) $4S - 5T = 0$

(D) $16S - 25T = 0$

25. Let E_1, E_2, E_3 and E_4 be four events such that

$$P(E_i|E_4) = \frac{2}{3}, i = 1,2,3; P(E_i \cap E_j^c|E_4) = \frac{1}{6}, i,j = 1,2,3, i \neq j \text{ and } P(E_1 \cap E_2 \cap E_3^c|E_4) = \frac{1}{6}$$

Then $P(E_1 \cup E_2 \cup E_3|E_4)$ is equal to

(A) $\frac{1}{2}$

(B) $\frac{2}{3}$

(C) $\frac{5}{6}$

(D) $\frac{7}{12}$

26. Let $a_1 = 5$ and define recursively

$$a_{n+1} = 3^{\frac{1}{4}}(a_n)^{\frac{3}{4}}, \qquad n \geq 1$$

Then, which of the following statements is TRUE?

(A) $\{a_n\}$ is monotone increasing and $\lim_{n\to\infty} a_n = 3$

(B) $\{a_n\}$ is monotone decreasing and $\lim_{n\to\infty} a_n = 3$

(C) $\{a_n\}$ is non- monotone and $\lim_{n\to\infty} a_n = 3$

(D) $\{a_n\}$ is decreasing and $\lim_{n\to\infty} a_n = 0$

27. Consider the problem of testing $H_0: X \sim f_0$ against $H_1: X \sim f_1$ based on a sample of size 1, where $f_0(x) = \begin{cases} 1, & 0 \leq x \leq 1 \\ 0, & otherwise \end{cases}$ and $f_1(x) = \begin{cases} 2-2x & 0 \leq x \leq 1 \\ 0, & otherwise \end{cases}$

Then, the probability of type II error of the most powerful test of size $a = 0.1$ is equal to

(A) 0.81

(B) 0.91

(C) 0.1

(D) 1

28. For $a \in \mathbb{R}$ consider the system of linear equation

$$\begin{aligned} ax + ay &= a + 2 \\ x + ay + (a-1)z &= a - 4 \\ ax + ay + (a-1)z &= -8 \end{aligned}$$

In the unknown x, y and z. Then, which of the following statements is TRUE?

(A) The given system has a unique solution for $a = 1$

(B) The given system has infinitely many solution for $a = 2$

(C) The given system has a unique solution for $a = -2$

(D) The given system has infinitely many solution for $a = -2$

29. Let $\{a_n\}_{n\geq 1}$ be a sequence of real numbers such that $a_n \geq 1$. Then, which of the following conditions imply the divergence of $(a_n)_{n\geq 1}$?

(A) $\{a_n\}_{n\geq 1}$ is non-increasing

(B) $\sum_{n=1}^{\infty} b_n$ converges, where $b_1 = a_1$ and $b_n = a_{n+1} - a_n$, for all $n > 1$

(C) $\lim_{n\to\infty} \frac{a_{2n+1}}{a_{2n}} = \frac{1}{2}$

(D) $\{\sqrt{a_n}\}_{n\geq 1}$ converges

30. Let E_1, E_2 and E_3 be three events such that $P(E_1) = \frac{4}{5}, P(E_2) = \frac{1}{2}$ and $P(E_2) = \frac{9}{10}$

Then, which of the following statements is FALSE?

(A) $P(E_1 \cup E_2 \cup E_3) \geq \frac{9}{10}$

(B) $P(E_2 \cap E_3) \leq \frac{1}{2}$

(C) $P(E_1 \cap E_2 \cap E_3) \leq \frac{1}{6}$

(D) $P(E_1 \cup E_2) \geq \frac{4}{5}$

SECTION-B
MULTIPLE SELECT QUECTION (MSQ)

Q.31 – Q.40 carry two marks each

31. Consider the linear system $A\underline{x} = \underline{b}$, where A is an m x n matrix, $\underline{x}$ is an n x 1 vector of unknowns anb $\underline{b}$ is an m x 1 vector. Further, suppose there exists an m x 1 vector $\underline{c}$ such that is /are necessarily TRUE?

(A) If $m \leq n$ and $\underline{d}$ is the first column of A, then the linear system $A\underline{x} = \underline{d}$ has a unique solution

(B) if $m \leq n$, then rank $(A) < n$

(C) Rank $(A) < m$

(D) If $m > n$, then the linear system $A\underline{x} = \underline{0}$ has a solution other than $\underline{x} = \underline{0}$

32. Let A be 3 x 3 real matrix such that $A \neq I_3$ and the sum of the entries in each row of A is 1 Then, which of the following statements is /are necessarily TRUE?

(A) $A - I_3$ is an invertible matrix

(B) The set $\{\underline{x} \in \mathbb{R}^3 : (A - I_3)\underline{x} = \underline{0}\}$ has at least two elements ($\underline{x}$ is a column vector)

(C) The characteristics polynomials, $p(\lambda)$, of $A + 2A^2 + A^3$ has $(\lambda - 4)$ as a factor

(D) A cannot be an orthogonal matrix

33. Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\theta, 1)$, where $\theta, 1)$ where $\theta \in (-\infty, \infty)$ is unknown. Consider the problem of testing $H_0 : \theta \leq 0$ against $H_1 : \theta > 0$. Let $\beta(\theta)$ denote the power function of the likelihood ratio test of the size $a(0 < a < 1)$ for testing H_0 against H_1. Then, which of the following statement is /are TRUE?

(A) $\beta(\theta) > \beta(0)$ for all $\theta > 0$

(B) $\beta(\theta) < \beta(0)$ for all $\theta > 0$

(C) The critical region of the likelihood test of size a is

$$\left\{(x_1, x_2, \ldots, x_n) \in \mathbb{R}^n : \sqrt{n}\frac{\sum_{i=1}^n x_i}{n} > \tau_{a/2}\right\}$$

Where $\tau_{a/2}$ is a fixed point such that $P\left(Z > \tau_{\frac{a}{2}}\right) = a, Z \sim N(0,1)$.

(D) The critical region of the likelihood test of size a is

$\left\{(x_1, x_2, \ldots, x_n \in \mathbb{R}^n : \sqrt{n}\frac{\sum_{i=1}^n x_i}{n} < \tau_a\right\}$

Where τ_a is a fixed point such that $P(Z > \tau_a) = a, Z \sim N(0,1)$

34. Consider the function

$$f(x, y) = 3x^2 + 4xy + y^2, \quad (x, y) \in \mathbb{R}^2$$

If $S = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$, then which of the following statement is /are TRUE?

(A) The maximum value of f on S is $3 + \sqrt{5}$

(B) The maximum value of f on S is $3 - \sqrt{5}$

(C) The maximum value of f on S is $2 + \sqrt{5}$

(D) The minimum value of f on S is $2 - \sqrt{5}$

35. Let $f : \mathbb{R} \to \mathbb{R}$ be a twice differentiable function. Then, which of the following statements is /are necessarily TRUE?

(A) f' is continuous

(B) If $f'(0) = f'(1)$. Then $f''(x) = 0$ has a solution in (0,1)

(C) f' is bounded on [8, 10]

(D) f'' is bounded on (0, 1)

36. **Let $X_1, X_2, \ldots, X_n$ $(n \geq 2)$ be independent and identically distributed random variable with probability density function**

$$f(x) = \begin{cases} \frac{1}{x^2} & x \geq 1 \\ 0, & otherwise \end{cases}$$

Then, which of the following random variables has/have finite expectation?

(A) X_1

(B) $\frac{1}{X_2}$

(C) $\sqrt{X_1}$

(D) $\min\{X_1, \ldots, X_n\}$.

37. **A sample of size n is drawn randomly (without replacement) from an urn containing $5n^2$ balls, of which $2n^2$ are red balls and $3n^2$ are black balls. Let X_n denote the number of red bulls in the selected sample. If $\ell = \lim_{n\to\infty} \frac{E(X_n)}{n}$ and $= \lim_{n\to\infty} \frac{Var(X_n)}{n}$, then which of the following statements is /are TRUE?**

(A) $\ell + m = \frac{16}{25}$

(B) $\ell - m = \frac{3}{25}$

(C) $\ell m = \frac{14}{125}$

(D) $\frac{\ell}{m} = \frac{5}{3}$

38. **Let $X_1, X_2, \ldots, X_n$ $(n \geq 2)$ be a random sample from a distribution with probability density function**

$$f(x;\theta) = \begin{cases} \frac{1}{2\theta} & -\theta \leq x \leq \theta \\ 0, & |x| > \theta \end{cases}$$

Where $\theta \in (0, \infty)$ is unknown. If $R = \min\{X_1, X_2, \ldots, X_n\}$ and $S = \max\{X_1, X_2, \ldots, X_n\}$ then which of the following is /are TRUE?

(A) (R, S) is jointly sufficient for θ

(B) S is an MLE of θ

(C) max $\{|X_1|, |X_2|, .., |X_n|\}$ is a complete and sufficient statistic for θ

(D) Distribution of $\frac{R}{S}$ does NOT depend on θ

39. **Let $X_1, X_2, \ldots, X_n$ $(n \geq 2)$ be a random sample from a distribution with probability density function**

$$f(x;\theta) = \begin{cases} \frac{3\theta^2}{\theta} e^{-x^3/\theta} & x > 0 \\ 0, & otherwise \end{cases}$$

Where $\theta \in (0, \infty)$ is unknown

IF $T = \sum_{i=1}^{n} X_i^3$, which of the following statements is /are TRUE?

(A) $\frac{n-1}{T}$ is the unique uniformly minimum variance unbiased estimator of $\frac{1}{\theta}$

(B) $\frac{n}{T}$ is the unique uniformly minimum variance unbiased estimator of $\frac{1}{\theta}$

(C) $(n-1)\sum_{i=1}^{n} \frac{1}{X_i^3}$ is the unique uniformly minimum variance unbiased estimator of $\frac{1}{\theta}$

(D) $\frac{n}{T}$ is the MLE of $\frac{1}{\theta}$

40. Let $X_1, X_2, \ldots, X_n$ $(n \geq 2)$ be a random sample from a distribution with probability density function

$$f(x;\theta) = \begin{cases} \theta x^{\theta-1} & 0 \leq x \leq 1 \\ 0, & otherwise \end{cases}$$

Where $\theta \in (0, \infty)$ is known. Then, which of the following statements is /are TRUE?

(A) Cramer-Rao lower bound, based on $X_1, X_2, \ldots, X_n$ for the estimated θ^3 is $9\frac{\theta^6}{n}$

(B) Cramer-Rao lower bound, based on $X_1, X_2, \ldots, X_n$ for the estimand θ^3 is $\frac{\theta^2}{n}$

(C) There does not exist any unbiased estimator of $\frac{1}{\theta}$ which attains the Cramer- Rao lower bound

(D) There exist an unbiased estimator of $\frac{1}{\theta}$ which attains the Cramer Rao lower bound

SECTION – C
NUMERICAL ANSWER TYPE (NAT)

Q.41 – Q.50 carry one mark each

41. Let a, β and γ be the eigenvalues of $= \begin{bmatrix} 0 & 1 & 0 \\ 1 & 3 & 3 \\ -1 & 2 & 2 \end{bmatrix}$. If $\gamma = 1$ and $a > \beta$ then the value of $2a + 3\beta$ is

_____

42. Let $M = \begin{pmatrix} 5 & -6 \\ 3 & -4 \end{pmatrix}$ be a 2 x 2 matrix. If $a = \det(M^4 - 6I_2)$, then the value a^2 is_____

43. Let $S = \{(x, y) \in \mathbb{R}^2 : 2 \leq x \leq y \leq 4\}$. Then the value of the integral

$$\iiint xyz\, dx\, dy\, dz$$

Then 384 a is equal to _____

45. Let f_0 and f_1 be the probability mass function given by

x	1	2	3	4	5	6
$f_0(x)$	0.1	0.1	0.1	0.1	0.1	0.5
$f_1(x)$	0.1	0.1	0.2	0.2	0.2	0.2

Consider the problem of testing the null hypothesis $H_0: X \sim f_0$ against $H_1: X \sim f_1$ based on a single sample X. IF a and β respectively, denote the size and power of the test with critical region $\{x \in \mathbb{R}: x > 3\}$, then $10(a + \beta)$ is equal to _____

46. Let $5, 10, 4, 15, 6$ be an observed random sample of size 5 from a distribution with probability density function

$$f(x;\theta) = \begin{cases} e^{-(x-\theta)} & x \geq \theta \\ 0, & otherwise \end{cases}$$

$\theta \in (-\infty, 3]$ is unknown. Then the maximum likelihood estimate of θ based on the observed sample is equal to _____

47. Let

$$a = \lim_{n\to\infty} \sum_{m=n^2}^{2n^2} \frac{1}{\sqrt{5n^4 + n^3 + m}}$$

Then $10\sqrt{5}a$ is equal to _____

48. Let X be a random variable having the probability density function

$$f(x) = \frac{1}{8\sqrt{2\pi}}\left(2e^{-\frac{x^2}{2}} + 3e^{-\frac{x^2}{8}}\right) - \infty < x < \infty$$

Then $4E(X^4)$ is equal to _____

49. Let X be a random variable with moment generating function

$$M_X(t) = \frac{1}{12} + \frac{1}{6}e^t + \frac{1}{3}e^{2t} + \frac{1}{4}e^{-t} + \frac{1}{6}e^{-2t}, t \in \mathbb{R}$$

Then, 8 $E(X)$ is equal to _____

50. Let β denote the length of the curve $y = 1n\,(\sec x)$ from $x = 0$ to $x = \frac{\pi}{4}$

Then the value of $3\sqrt{2}(e^{\beta} - 1)$ is equal to_____

Q.1 – Q.60 carry two marks each

51. Let $S \subseteq \mathbb{R}^2$ be the region bounded by the parallelogram with vertices at the points (1, 0), (3, 2), (3, 5) and (1, 3). Then the value of the integral $\iint_S (x + 2y)dx\,dy$ is equal to _____

52. Let $A = \left\{(x, y) \in \mathbb{R}^2 : x^2 - \frac{1}{2\sqrt{x}} < y < x^2 + \frac{1}{2\sqrt{\pi}}\right\}$ and let the joint probability density function of (X, Y) be

$$f(x, y) = \begin{cases} e^{-(x-1)^2} & (x, y) \in A \\ 0, & otherwise \end{cases}$$

Then, the covariance between the random variables X and Y is equal to _____

53. Let X_1 and X_2 be independent $N(0, 1)$ random variables. Define

$$sgn\,(u) = \begin{cases} -1, & if\ u < 0 \\ 0, & if\ u = 0 \\ 1, & if\ u > 0 \end{cases}$$

Let $Y_1 = X_1 sgn(X_2)$ and $Y_2 = X_2 sgn\,(X_1)$. If the correlation coefficient between $Y_1\ and\ Y_2$ is a, then πa is equal to _____

54. Let

$$a_n = \sum_{k=2}^{n} \binom{n}{k} \frac{2^k (n-2)^{n-k}}{n^n}, n = 2, 3 \ldots\ldots$$

Then $e^2 \lim_{n\to\infty}(1 - a_n)$ is equal to _____

55. Let E_1, E_2 and E_3 be four independent events such that $P(E_1) = \frac{1}{2}, P(E_2) = \frac{1}{3}, P(E_3) = \frac{1}{4}$ and $P(E_4) = \frac{1}{5}$. Let p be the probability that at most two events among E_1, E_2, E_3 and E_4 occur. Then 240p is equal to _____

56. Let the random vector (X, Y) have the joint probability mass function

$$f(x, y) = \begin{cases} \binom{10}{x}\binom{5}{4}\binom{1}{4}^{x-y+5}\binom{3}{4}^{y-x+10} & x = 0.1, .., 10; y = 0, 1, \ldots., 5 \\ 0, & otherwise \end{cases}$$

Let $Z = Y - X + 10$. If $a = E(Z)$ and $= Var(Z)$, then $8a + 48\beta$ is equal to _____

57. Let $S = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq \pi, \min\{sinx, cosx\} \leq y \leq \max\{\sin x, \cos x\}\}$.
If a is the area of S, then the value of $2\sqrt{2}a$ is equal to _____

58. The number of real roots of the polynomial

$$f(x) = x^{11} - 13x + 5$$

Is_____

59. Let $a = \lim_{n\to\infty}(1 + nsin\frac{3}{n^2})$. Then, 1n a is equal to _____

60. Let $\phi: (-1, 1) \to \mathbb{R}$ be defined by

$$\phi(x) = \int_{x^7}^{x^4} \frac{1}{1+t^3}\,dt$$

If $a = \lim_{x\to 0} \frac{\phi(x)}{e^{2x^4}-1}$ then 42 a is equal to _____

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