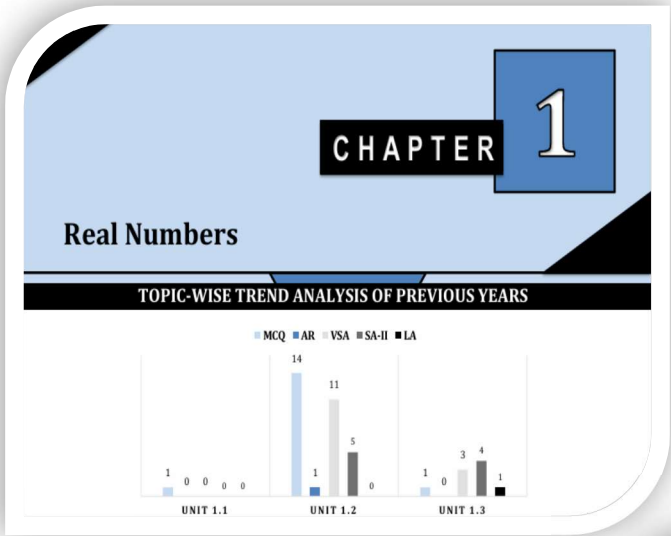
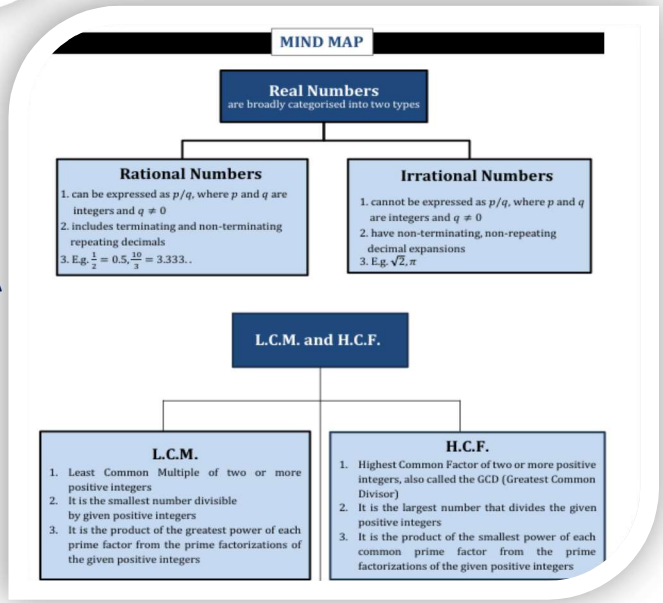
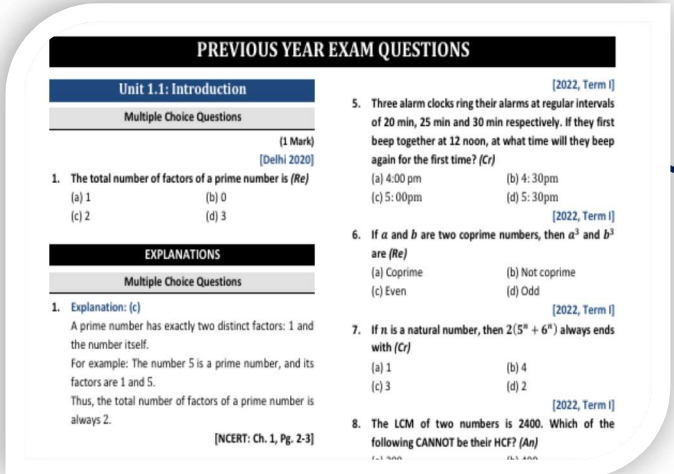
HIGHLIGHTS OF THE CBSE MATHEMATICS BOOK



CBSE Question Trend Analysis by Topic: Understand the Patterns in Frequently Asked Questions.

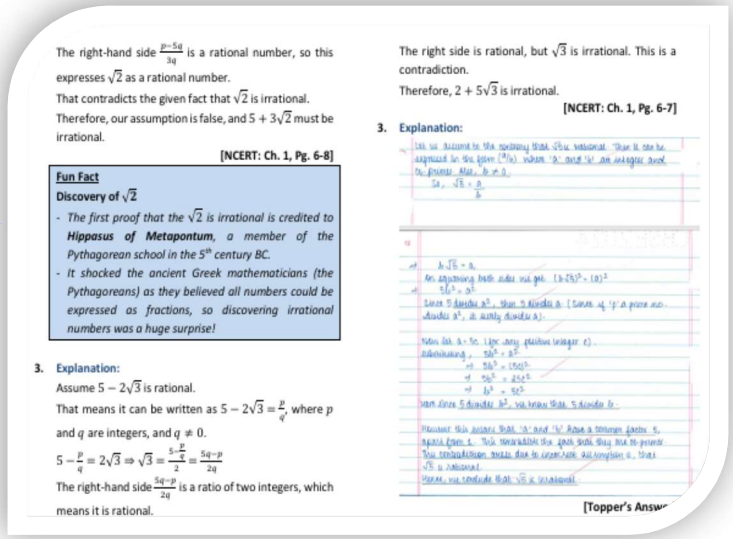


Mind Map: A Visual Tool to Connect and Recall Key Concepts Efficiently.



CBSE Previous Year Questions: Detailed Chapterwise and Topicwise Breakdown with Cognitive Level Tagging for the Last 10 Years.

Toppers' Answer:
Real Exam Responses for Effective Learning and Insight.



The right-hand side $\frac{p-5q}{3q}$ is a rational number, so this expresses $\sqrt{2}$ as a rational number.

That contradicts the given fact that $\sqrt{2}$ is irrational.

Therefore, our assumption is false, and $5+3\sqrt{2}$ must be irrational.

[NCERT: Ch. 1, Pg. 6-8]

Fun Fact

Discovery of $\sqrt{2}$

- *The first proof that the $\sqrt{2}$ is irrational is credited to **Hippasus of Metapontum**, a member of the Pythagorean school in the 5th century BC.*
- *It shocked the ancient Greek mathematicians (the Pythagoreans) as they believed all numbers could be expressed as fractions, so discovering irrational numbers was a huge surprise!*

3. Explanation:

Assume $5-2\sqrt{3}$ is rational.

That means it can be written as $5-2\sqrt{3}=\frac{p}{q}$, where p and q are integers, and $q \neq 0$.

$5-\frac{p}{q}=2\sqrt{3} \Rightarrow \sqrt{3}=\frac{5-\frac{p}{q}}{2}=\frac{5q-p}{2q}$

The right-hand side $\frac{5q-p}{2q}$ is a ratio of two integers, which means it is rational.

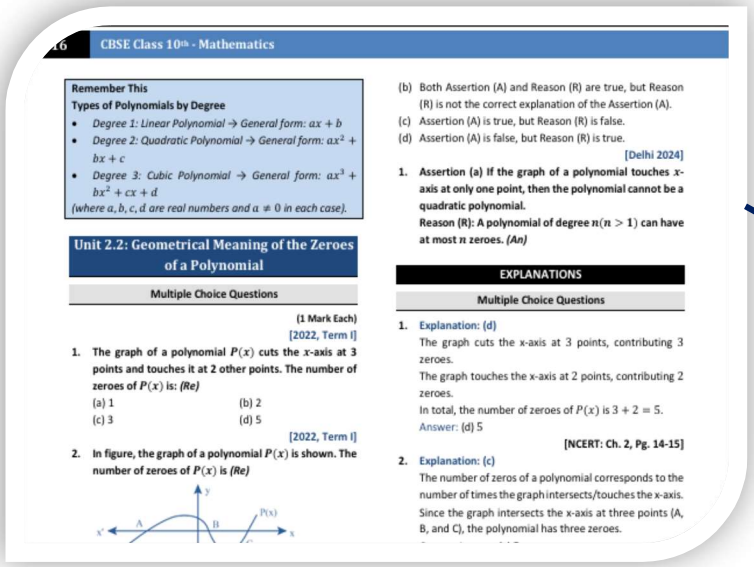
The right side is rational, but $\sqrt{3}$ is irrational. This is a contradiction.

Therefore, $2+5\sqrt{3}$ is irrational.

[NCERT: Ch. 1, Pg. 6-7]

3. Explanation:

[Topper's Answ



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Remember This

Types of Polynomials by Degree

- *Degree 1: Linear Polynomial → General form: $ax+b$*
- *Degree 2: Quadratic Polynomial → General form: ax^2+bx+c*
- *Degree 3: Cubic Polynomial → General form: ax^3+bx^2+cx+d*

(where a, b, c, d are real numbers and $a \neq 0$ in each case).

Unit 2.2: Geometrical Meaning of the Zeroes of a Polynomial

Multiple Choice Questions

(1 Mark Each)

[2022, Term I]

1. **The graph of a polynomial $P(x)$ cuts the x-axis at 3 points and touches it at 2 other points. The number of zeroes of $P(x)$ is: *(Re)***
   (a) 1 (b) 2
   (c) 3 (d) 5

[2022, Term I]

2. **In figure, the graph of a polynomial $P(x)$ is shown. The number of zeroes of $P(x)$ is *(Re)***

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(c) Assertion (A) is true, but Reason (R) is false.
(d) Assertion (A) is false, but Reason (R) is true.

[Delhi 2024]

1. **Assertion (a) If the graph of a polynomial touches x-axis at only one point, then the polynomial cannot be a quadratic polynomial.**
   Reason (R): A polynomial of degree $n(n>1)$ can have at most n zeroes. *(An)*

EXPLANATIONS

Multiple Choice Questions

1. **Explanation: (d)**
   The graph cuts the x-axis at 3 points, contributing 3 zeroes.
   The graph touches the x-axis at 2 points, contributing 2 zeroes.
   In total, the number of zeroes of $P(x)$ is $3+2=5$.
   Answer: (d) 5

[NCERT: Ch. 2, Pg. 14-15]

2. **Explanation: (c)**
   The number of zeros of a polynomial corresponds to the number of times the graph intersects/touches the x-axis.
   Since the graph intersects the x-axis at three points (A, B, and C), the polynomial has three zeroes.

Remember This: Avoid Pitfalls and Enhance Your Performance.

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TREND ANALYSIS (2016-2025)

No. of Questions Asked in Previous Exams

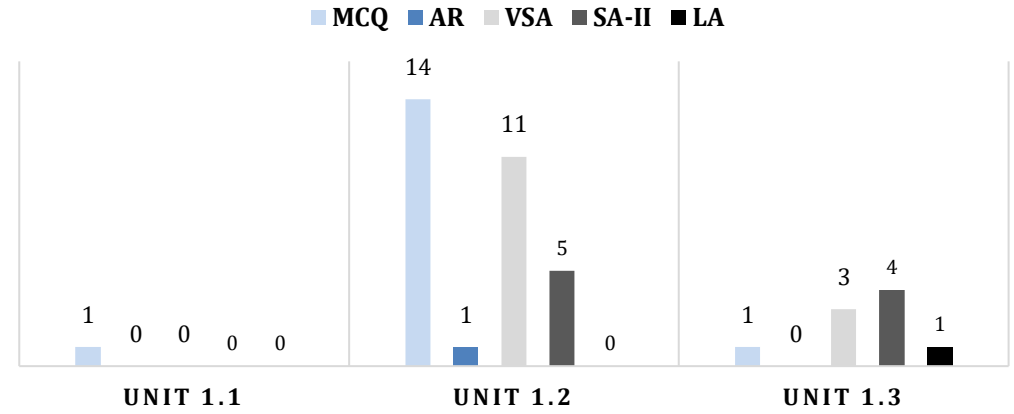
MATHEMATICS		2016	2017	2018	2019	2020	2021	2022	2023	2024	2025
Chapter - 1 Real Numbers											
Unit 1.1	Introduction	-	-	-	-	1	-	-	-	-	-
Unit 1.2	The Fundamental Theorem of Arithmetic	2	-	2	2	2	-	6	6	5	6
Unit 1.3	Revisiting Irrational Numbers	1		1	2	1			1	1	2
Chapter - 2 Polynomials											
Unit 2.1	Introduction	-	-	-	-	2	-	-	1	1	-
Unit 2.2	Geometrical Meaning of the Zeroes of a Polynomial	-	-	-	-	-	-	2	-	1	1
Unit 2.3	Relationship between Zeroes and Coefficients of a Polynomial	1	-	-	2	3	-	3	6	4	3
Chapter - 3 Pair of Linear Equations in Two Variables											
Unit 3.1	Graphical Method of Solution of a Pair of Linear Equations	1	-	-	2	3	-	1	1	3	2
Unit 3.2	Algebraic Methods of Solving a Pair of Linear Equations	2	-	1	4	1	-	2	4	4	3
Chapter - 4 Quadratic Equations											
Unit 4.1	Quadratic Equations	-	2	-	-	1	-	-	-	1	1
Unit 4.2	Solution of a Quadratic Equation by Factorisation	6	1	3	3	2	-	5	3	-	2
Unit 4.3	Nature of Roots	1	5	1	3	1	-	2	4	2	4
Chapter - 5 Arithmetic Progressions											
Unit 5.1	Arithmetic Progressions	2	-	1	-	3	-	1	1	1	-
Unit 5.2	nth Term of an AP	2	5	1	4	2	-	1	2	1	1
Unit 5.3	Sum of First n Terms of an AP	4	3	1	3	5	-	3	4	5	2
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Unit 6.1	Introduction	-	-	-	-	1	-	1	-	-	-
Unit 6.2	Similar Figures	-	-	-	-	1	-	-	-	-	1
Unit 6.3	Similarity of Triangles	3	-	-	1	1	-	1	6	3	3
Unit 6.4	Criteria for Similarity of Triangles	2	-	-	-	1	-	2	2	4	7

Chapter - 7 Coordinate Geometry											
Unit 7.1	Introduction	-	-	-	-	1	-	-	2	-	2
Unit 7.2	Distance Formula	3	3	1	2	1		3	2	5	1
Unit 7.3	Section Formula	2	2	2	4	4	-	3	2	4	5
Chapter - 8 Introduction To Trigonometry											
Unit 8.1	Trigonometric Ratios	1	-	1	-	1	-	1	-	-	1
Unit 8.2	Trigonometric Ratios of Some Specific Angles	3	-	-	2	1	-	3	2	6	3
Unit 8.3	Trigonometric Identities	4	-	1	6	7	-	2	7	4	7
Chapter - 9 Some Applications of Trigonometry											
Unit 9.1	Height and Distance	6	6	1	2	4	-	4	5	2	4
Chapter - 10 Circles											
Unit 10.1	Tangent to a Circle	3	1	-	-	1	-	2	3	4	2
Unit 10.2	Number of Tangents from a Point on a Circle	7	6	1	2	4	-	4	5	4	5
Chapter - 11 Areas Related to Circles											
Unit 11.1	Areas of Sector and Segment of a Circle	2	1	-	1	2	-	4	-	1	3
Chapter - 12 Surface Areas and Volumes											
Unit 12.1	Introduction	-	-	-	-	1	-	1	-	1	-
Unit 12.2	Surface Area of a Combination of Solids	3	2	1	-	1	-	1	-	1	2
Unit 12.3	Volume of a Combination of Solids	3	3	1	1	-	-	-	3	3	2
Chapter - 13 Statistics											
Unit 13.1	Mean of Grouped Data	3	-	1	2	3	-	3	1	2	1
Unit 13.2	Mode of Grouped Data	-	-	-	1	2	-	2	-	1	1
Unit 13.3	Median of Grouped Data	1		1	1	1		2	2	3	3
Chapter - 14 Probability											
Unit 14.1	Probability — A Theoretical Approach	6	6	2	3	8		4	7	7	6

CHAPTER 1

Real Numbers

TOPIC-WISE TREND ANALYSIS OF PREVIOUS YEARS



Quick Revision Sheet

1. **Euclid's Division Algorithm**
   For any two positive integers a and b, there exist unique integers q and r such that: $a = bq + r$, where $0 \leq r < b$.

2. **The Fundamental Theorem of Arithmetic**
   - **Prime Factorization**: Every composite number can be written uniquely as a product of primes in one and only one way, apart from the order in which the primes are written.
   - **Uniqueness**: The prime factorization of a natural number is unique, except for the order of its factors.

   Note:
   1. A **prime number** has no factors other than 1 and itself.
   2. A **composite number** has at least one factor other than 1 and itself.
   3. **1** is neither prime nor composite.
   4. **2** is the smallest and the only even prime number.
   5. Two numbers are **co-prime** if they have no common factors other than 1, i.e., their HCF = 1.

3. **HCF and LCM using Prime Factorization:**
   - **HCF (Highest Common Factor)**: The HCF is the product of the smallest power of each common prime factor from the prime factorizations of the given numbers, also called the **GCD** (Greatest Common Divisor) of the numbers.
   - **LCM (Least Common Multiple)**: The LCM is the product of the greatest power of each prime factor from the prime factorizations of the given numbers.

   Note:
   1. **LCM** (Least Common Multiple) of two numbers is the smallest number divisible by both numbers.
   2. **HCF** (Highest Common Factor) of two numbers is the largest number that divides both numbers exactly.

4. **Relationship between HCF and LCM:**
   - HCF $\times$ LCM $=$ Product of two positive integers

     Formulas:
   - LCM$(p, q, r) = \frac{p \times q \times r}{\text{HCF}(p,q,r)}$
   - HCF$(p, q, r) = \frac{p \times q \times r}{\text{LCM}(p,q,r)}$

   Note: The product of three numbers is not equal to the product of their HCF and LCM.

5. **Revisiting Irrational Numbers:**
   - **Rational Numbers**: A number in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is a rational number.
   - **Irrational Numbers**: A number that cannot be expressed as a fraction, i.e., it cannot be written as $\frac{p}{q}$, is an irrational number.

     Key Fact: If p is a prime and p divides a^2, then p also divides a.
   - **Method of Proving Irrationality**: To prove that a number is irrational, we use the method of contradiction.

6. **Some Results Related to Rational and Irrational Numbers:**
   - The **sum** of a rational number and an irrational number is always irrational.
   - The **product** of a non-zero rational number and an irrational number is always irrational.
   - The **sum** of two irrational numbers may or may not be irrational.
   - The **product** of two irrational numbers may or may not be irrational.

MIND MAP

Real Numbers
are broadly categorised into two types

Rational Numbers
1. can be expressed as p/q, where p and q are integers and $q \neq 0$
2. includes terminating and non-terminating repeating decimals
3. E.g. $\frac{1}{2} = 0.5, \frac{10}{3} = 3.333..$

Irrational Numbers
1. cannot be expressed as p/q, where p and q are integers and $q \neq 0$
2. have non-terminating, non-repeating decimal expansions
3. E.g. $\sqrt{2}, \pi$

L.C.M. and H.C.F.

L.C.M.
1. Least Common Multiple of two or more positive integers
2. It is the smallest number divisible by given positive integers
3. It is the product of the greatest power of each prime factor from the prime factorizations of the given positive integers

H.C.F.
1. Highest Common Factor of two or more positive integers, also called the GCD (Greatest Common Divisor)
2. It is the largest number that divides the given positive integers
3. It is the product of the smallest power of each common prime factor from the prime factorizations of the given positive integers

Relationship between L.C.M. and H.C.F.
For any two positive integers a, b,
$\text{L.C.M.}(a, b) \times \text{H.C.F.}(a, b) = a \times b$

PREVIOUS YEAR EXAM QUESTIONS

Unit 1.1: Introduction

Multiple Choice Questions

(1 Mark)

[Delhi 2020]

1. The total number of factors of a prime number is *(Re)*

(a) 1 (b) 0

(c) 2 (d) 3

EXPLANATIONS

Multiple Choice Questions

1. Explanation: (c)

A prime number has exactly two distinct factors: 1 and the number itself.

For example: The number 5 is a prime number, and its factors are 1 and 5.

Thus, the total number of factors of a prime number is always 2.

[NCERT: Ch. 1, Pg. 2-3]

Unit 1.2: The Fundamental Theorem of Arithmetic

Multiple Choice Questions

(1 Mark Each)

[Delhi 2020]

1. The HCF and the LCM of $12, 21, 15$ respectively are *(Ap)*

(a) 3,140 (b) 12,420

(c) 3,420 (d) 420,3

[Outside Delhi 2020]

2. The sum of exponents of prime factors in the prime factorisation of 196 is: *(Re)*

(a) 3 (b) 4

(c) 5 (d) 2

[2022, Term I]

3. The exponent of 5 in the prime factorisation of 3750 is: *(Ap)*

(a) 3 (b) 4

(c) 5 (d) 6

[2022, Term I]

4. What is the greatest possible speed at which a girl can walk 95 m and 171 m in an exact number of minutes? *(Un)*

(a) 17 m/min (b) 19 m/min

(c) 23 m/min (d) 13 m/min

[2022, Term I]

5. Three alarm clocks ring their alarms at regular intervals of 20 min, 25 min and 30 min respectively. If they first beep together at 12 noon, at what time will they beep again for the first time? *(Cr)*

(a) 4:00 pm (b) 4: 30pm

(c) 5: 00pm (d) 5: 30pm

[2022, Term I]

6. If a and b are two coprime numbers, then a^3 and b^3 are *(Re)*

(a) Coprime (b) Not coprime

(c) Even (d) Odd

[2022, Term I]

7. If n is a natural number, then $2(5^n + 6^n)$ always ends with *(Cr)*

(a) 1 (b) 4

(c) 3 (d) 2

[2022, Term I]

8. The LCM of two numbers is 2400. Which of the following CANNOT be their HCF? *(An)*

(a) 300 (b) 400

(c) 500 (d) 600

[Delhi 2023]

9. The ratio of H.C.F. to L.C.M. of the least composite number and the least prime number is: *(Re)*

(a) 1: 2 (b) 2: 1

(c) 1: 1 (d) 1: 3

[Outside Delhi 2023]

10. The LCM of smallest 2 -digit number and smallest composite number is: *(Re)*

(a) 12 (b) 4

(c) 20 (d) 40

[Delhi 2024]

11. If two positive integers p and q can be expressed as $p = 18\,a^2b^4$ and $q = 20\,a^3b^2$, where a and b are prime numbers, then LCM(p, q) is; *(Un)*

(a) $2\,a^2b^2$ (b) $180\,a^2b^2$

(c) $12\,a^2b^2$ (d) $180\,a^3b^4$

[Outside Delhi 2024]

12. If the HCF $(2520, 6600) = 40$ and LCM $(2520, 6600) = 252 \times k$, then the value of k is *(Un)*

(a) 1650 (b) 1600

(c) 165 (d) 1625

[Delhi 2025]

13. If $x = ab^3$ and $y = a^3b$, where a and b are prime numbers, then [HCF (x, y) − LCM (x, y)] is equal to *(Ev)*

(a) $1 - a^3b^3$ (b) $ab(1 - ab)$

(c) $ab - a^4b^4$ (d) $ab(1 - ab)(1 + ab)$

[Outside Delhi 2025]

14. If x is the LCM of $4, 6, 8$ and y is the LCM of $3, 5, 7$ and p is the LCM of x and y, then which of the following is true? *(An)*

(a) $p = 35x$ (b) $p = 4y$
(c) $p = 8x$ (d) $p = 16y$

Assertion and Reason

(1 Mark)

[Delhi 2025]

1. **Two statements are given, one labelled as Assertion (A) and the other is labelled as Reason (R).** *(Re)*

Assertion (A): 4^n ends with digit 0 for some natural number n.

Reason (R): For a number 'x' having 2 and 5 as its prime factors, x^n always ends with digit 0 for every natural number n.

(a) Both (A) and (R) are true, and (R) is the correct explanation of (A).
(b) Both (A) and (R) are true, but (R) is not the correct explanation of (A).
(c) (A) is true, but (R) is false.
(d) (A) is false, but (R) is true.

Very Short Answer Type Questions

[2016, Term I – 2 Marks]

1. **Explain why $(17 \times 5 \times 11 \times 3 \times 2 + 2 \times 11)$ is a composite number?** *(Un)*

[2018 – 1 Mark]

2. **What is the HCF of smallest prime number and the smallest composite number?** *(An)*

[Outside Delhi 2019 – 1 Mark]

3. **If HCF $(336, 54) = 6$, find LCM $(336, 54)$.** *(Re)*

[Outside Delhi 2019 – 2 Marks]

4. **Write the smallest number which is divisible by both 306 and 657.** *(An)*

[Delhi 2023 – 2 Marks]

5. **Two numbers are in the ratio $2:3$ and their L.C.M. is 180. What is the H.C.F. of these numbers?** *(Un)*

[Outside Delhi 2023 – 2 Marks]

6. **(a) Show that 6^n cannot end with digit 0 for any natural number 'n'.** *(Un)*

[Outside Delhi 2023 – 2 Marks]

7. **Find the LCM and HCF of 72 and 120.** *(Ap)*

[Delhi 2024 – 2 Marks]

8. **Show that the number $5 \times 11 \times 17 + 3 \times 11$ is a composite number.** *(Re)*

[Outside Delhi 2024 – 2 Marks]

9. **Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.** *(Un)*

[Outside Delhi 2025 – 2 Marks]

10. **(a) Find the smallest number which is divisible by both 644 and 462.** *(Ap)*

[Outside Delhi 2025 – 2 Marks]

11. **Two numbers are in the ratio 4: 5 and their HCF is 11. Find the LCM of these numbers.** *(Ap)*

Short Answer Type Questions

(3 Marks Each)

[2016, Term I]

1. **The length, breadth and height of a room are 8 m 50 cm, 6 m 25 cm and 4 m 75 cm respectively. Find the length of the longest rod that can measure the dimensions of the room exactly.** *(Ap)*

(2018)

2. **Find HCF and LCM of 404 and 96 and verify that HCF × LCM = Product of the two given numbers.** *(Re)*

[Outside Delhi 2023]

3. **Find the HCF and LCM of 26, 65 and 117, using prime factorisation.** *(Ap)*

[Delhi 2024]

4. **In a teachers' workshop, the number of teachers teaching French, Hindi and English are 48, 80 and 144 respectively. Find the minimum number of rooms required if in each room the same number of teachers are seated and all of them are of the same subject.** *(Un)*

[Delhi 2025]

5. **Let x and y be two distinct prime numbers and $p = x^2y^3, q = xy^4, r = x^5y^2$. Find the HCF and LCM of p, q and r. Further check if HCF $(p, q, r) \times$ LCM$(p, q, r) = p \times q \times r$ or not.** *(Un)*

EXPLANATIONS

Multiple Choice Questions

1. **Explanation: (c)**

Prime Factorization of the given numbers are

$12 = 2^2 \times 3$

$21 = 3 \times 7$

$15 = 3 \times 5$

The HCF is found by multiplying the lowest powers of common factors:

$\therefore$ HCF $= 3$

The LCM is found by multiplying the highest powers of all factors:

$\therefore$ LCM $= 2^2 \times 3 \times 7 \times 5 = 420$

So, HCF $= 3$ and LCM $= 420$.

Correct Answer: (c) $3, 420$

[NCERT: Ch. 1, Pg. 2-3]

2. **Explanation: (b)**

$$196 = 14^2 = (2 \cdot 7)^2 = 2^2 \cdot 7^2$$

$\therefore$ Sum of exponents $= 2 + 2 = 4$

[NCERT: Ch. 1, Pg. 2-3]

Remember This

Prime Factorization

- Every integer greater than 1 can be written as a product of prime numbers.
- This factorization is unique (apart from the order of the primes).

3. **Explanation: (b)**

The prime factorization of 3750 is: $3750 = 5^4 \times 2 \times 3$.

Thus, the exponent of 5 in the prime factorization is 4.

[NCERT: Ch. 1, Pg. 2-4]

4. **Explanation: (b)**

The greatest possible speed is determined by the highest common factor (HCF) of 95 and 171, as the distance should be a multiple of the speed.

$95 = 5 \times 19$

$171 = 3^2 \times 19$

$\therefore HCF\ of\ (95,171) = 19$

Thus, the greatest possible speed is 19 meters per minute.

Therefore, the answer is (b) $19\ m/min$.

[NCERT: Ch. 1, Pg. 2-4]

5. **Explanation: (c)**

Given:

Three alarm clocks ring at regular intervals of 20 minutes, 25 minutes, and 30 minutes respectively.

They all beep together at $12:00$ noon.

To find when all three alarms will beep together again, we need to find the Least Common Multiple (LCM) of the three intervals: 20 minutes, 25 minutes, and 30 minutes.

Prime Factorization:

$20 = 2^2 \times 5$

$25 = 5^2$

$30 = 2 \times 3 \times 5$

The LCM is found by taking the highest power of each prime factor:

$\text{LCM}(20,25,30) = 2^2 \times 3 \times 5^2 = 4 \times 3 \times 25 =$ 300 minutes

Convert 300 minutes to hours:

$$300 \text{ minutes} = \frac{300}{60} = 5 \text{ hours}$$

Since they beep together at $12:00$ noon, adding 5 hours gives:

$12:00$ noon $+ 5$ hours $= 5:00$ PM

Answer: (c) $5:00$ PM.

[NCERT: Ch. 1, Pg. 2-3]

6. **Explanation: (a)**

If a and b are coprime, it means their greatest common divisor (GCD) is 1:

Now, consider a^3 and b^3.

Since a and b are coprime, their cubes will also be coprime. (because a^3 and b^3 have same prime factors as that of a and b, just with different multiplicities)

So, a^3 and b^3 will also have a GCD of 1, meaning they are coprime.

Answer: (a) Coprime.

[NCERT: Ch. 1, Pg. 2-4]

Expert Tip

Prime vs. Coprime

- *A prime number is a number greater than 1 that has exactly two factors: 1 and itself (e.g., 2, 3, 5, 7).*
- *Coprime numbers are two numbers that have no common factor other than 1 (e.g., 8 and 15 are coprime, though neither is prime).*
- *Important: Coprime numbers need not be prime. For example, 14 and 15 are coprime, but both are composite.*

7. **Explanation: (d)**

We are given the expression $2(5^n + 6^n)$.

Powers of 5: The units digit of 5^n is always 5, regardless of n.

Example: $5^1 = 5, 5^2 = 25, 5^3 = 125, \ldots$. The last digit is always 5.

Powers of 6: The units digit of 6^n is always 6, regardless of n.

Example: $6^1 = 6, 6^2 = 36, 6^3 = 216, \ldots$ — The last digit is always 6.

Now, consider the sum $5^n + 6^n$:

The units digit of 5^n is 5, and the units digit of 6^n is 6, so the units digit of $5^n + 6^n$ is:

$5 + 6 = 11$ (units digit is 1).

Finally, we multiply by 2: $2 \times 1 = 2$.

Answer: (d) 2

[NCERT: Ch. 1, Pg. 3-4]

8. **Explanation: (c)**

Note that, HCF of two numbers divides their LCM.

So, for HCF to be valid, it must divide 2400. Let's check:

$\frac{2400}{300} = 8$ (valid)

$\frac{2400}{400} = 6$ (valid)

$\frac{2400}{500} = 4.8$ (invalid)

$\frac{2400}{600} = 4$ (valid)

Thus, the HCF cannot be 500.

Answer: (c) 500

[NCERT: Ch. 1, Pg. 2-3]

Remember This

For any two positive integers a and b:
$HCF(a,b)$ divides $LCM(a,b)$, i.e. $LCM(a,b)$ is a multiple of $HCF(a,b)$.

9. Explanation: (a)

The least composite number is 4. A composite number has more than two factors. The factors of 4 are $1, 2$, and 4.

The least prime number is 2. A prime number has only two factors: 1 and itself. The only factors of 2 are 1 and 2.

Least composite number $= 4$

Least prime number $= 2$

The H.C.F. is the greatest number that divides both numbers completely.

H.C.F. $(4,2) = 2$

The L.C.M is the smallest number that is a multiple of both numbers.

L.C.M $(4,2) = 4$

Ratio of H.C.F to L.C.M $= 2:4 = 1:2$

Answer: $(a)\ 1:2$

[NCERT: Ch. 1, Pg. 2-3]

Remember This

Prime vs. Composite

A prime number has exactly two distinct factors: 1 and itself (e.g., 2, 3, 5, 7).

A composite number has more than two factors (e.g., 4, 6, 8, 9, 12).

Note: 1 is neither prime nor composite.

10. Explanation: (c)

Smallest 2-digit number = 10

Smallest composite number = 4

Prime factorization of these numbers are:

$10 = 2 \times 5$

$4 = 2^2$

So, LCM = $2^2 \times 5 = 20$.

Answer: **(c) 20**

[NCERT: Ch. 1, Pg. 2-3]

11. Explanation: (d)

$p = 18a^2b^4 = 2 \times 3^2 \times a^2 \times b^4$

$q = 20a^3b^2 = 2^2 \times 5 \times a^3 \times b^2$

The LCM is found by multiplying the highest powers of all factors:

$\therefore \text{LCM}(p,q) = 2^2 \times 3^2 \times 5 \times a^3 \times b^4 = 180a^3b^4$

Answer: $180a^3b^4$

[NCERT: Ch. 1, Pg. 2-3]

12. Explanation: (a)

Given:

HCF $= 40$, LCM $= 252k$

HCF $\times$ LCM $=$ Product of the two numbers

$2520 \times 6600 = 40 \times (252k)$

$252 \times 660 \times 100 = 40 \times 252k$

$660 \times 100 = 40k$

$66000 = 40k$

$k = \dfrac{66000}{40} = 1650$

Correct answer: (a) 1650

[NCERT: Ch. 1, Pg. 4]

Concept Applied

Relation between Product, LCM, and HCF

For any two positive integers a and b:

$a \times b = LCM(a,b) \times HCF(a,b)$

i.e., the product of two numbers is equal to the product of their LCM and HCF.

13. Explanation: (d)

Given: $x = ab^3, y = a^3b$

The HCF is found by multiplying the lowest powers of common factors:

$\therefore \text{HCF}(x,y) = ab$

The LCM is found by multiplying the highest powers of all factors:

$\therefore \text{LCM}(x,y) = a^3b^3$

$$\text{HCF}(x,y) - \text{LCM}(x,y) = ab - a^3b^3 = ab(1 - a^2b^2)$$
$$= ab(1-ab)(1+ab)$$

Correct option: (d) $ab(1-ab)(1+ab)$

[NCERT: Ch. 1, Pg. 4]

14. Explanation: (a)

$4 = 2^2$

$6 = 2 \times 3$

$8 = 2^3$

$\therefore \text{LCM} = 2^3 \times 3 = 24$

So, $x = 24$

LCM (3,5,7) $= 3 \times 5 \times 7 = 105$

So, $y = 105$

Now,

$24 = 2^3 \times 3$

$105 = 3 \times 5 \times 7$

$\therefore \text{LCM} = 2^3 \times 3 \times 5 \times 7 = 24 \times 35 = x \times 35 = 35x$

Final Answer: (a) $p = 35x$

[NCERT: Ch. 1, Pg. 2-3]

Assertion and Reason

1. Explanation: (d)

Assertion (A): 4^n ends with digit 0 for some natural number n.

This is false. The only prime in the factorization of 4^n is 2. And no power of 2 ends in 0.

Reason (R): For a number 'x' having 2 and 5 as its prime factors, x^n always ends with digit 0 for every natural number n.

This is true. Such a number x will have factor of 10 ($= 2 \times 5$) and will always end in 0 when raised to any power, so true.

Answer: (d) Assertion (A) is false but Reason (R) is true.

[NCERT: Ch. 1, Pg. 3-4]

Very Short Answer Type Questions

1. Explanation:

$17 \times 5 \times 11 \times 3 \times 2 + 2 \times 11$

$= 2 \times 11 \times (17 \times 5 \times 3 + 1)$

$= 2 \times 11 \times (255 + 1) = 2 \times 11 \times 256$

This number is the product of 3 integers: 2, 11, and 256.

Since the number is divisible by 2, 11, and 256, it has more than two factors, meaning it is not prime.

Since the number has divisors other than 1 and itself (specifically $2, 11, and\ 256$), it is a composite number.

[NCERT: Ch. 1, Pg. 2-3]

2. Explanation:

The smallest prime number $= 2$

The smallest composite number $= 4 = 2^2$

Now, HCF$(2,4) = 2$

Final Answer: 2

[NCERT: Ch. 1, Pg. 4]

Fun Fact

- *There is no largest prime number — primes go on infinitely.*
- *Similarly, there is no largest composite number — composites also continue without end.*
- *Mathematicians keep discovering record-breaking largest known primes with the help of powerful computers. The current largest known prime has millions of digits!*

3. Explanation:

To find the LCM of two numbers, we use the relation:

$\text{LCM}(a, b) = \frac{a \times b}{\text{HCF}(a,b)}$

$\text{LCM}(336,54) = \frac{336 \times 54}{6} = 336 \times 9 = 3024$

The LCM of 336 and 54 is 3024.

[NCERT: Ch. 1, Pg. 4]

4. Explanation:

The smallest number divisible by two numbers is their LCM.

Prime factorization:

$306 = 2 \times 3^2 \times 17$

$657 = 3^2 \times 73$

For LCM, we take highest powers of all prime factors:

$\text{LCM}(306, 657) = 2^1 \times 3^2 \times 17^1 \times 73^1 = 22{,}338$

The smallest number divisible by both is $22,338$.

[NCERT: Ch. 1, Pg. 4]

Remember This

LCM and HCF

- *LCM (Least Common Multiple): The smallest number that is divisible by both given numbers.*
- *HCF (Highest Common Factor): The largest number that divides both given numbers exactly.*
- *Also called GCD (Greatest Common Divisor).*

5. Explanation:

Let the two numbers be $2x$ and $3x$, where x is a common factor. According to the property of LCM and HCF:

$$\text{L.C.M.} \times \text{H.C.F.} = \text{Product of the numbers}$$

So, the product of the numbers is:

$$(2x) \times (3x) = 6x^2$$

We are given that the L.C.M. of the numbers is 180, so:

$$\text{L.C.M.} = 180$$

Now, using the formula:

$$180 \times \text{H.C.F.} = 6x^2$$

The H.C.F. of $2x$ and $3x$ is x, because x is the greatest common divisor of $2x$ and $3x$.

$$\therefore 180 \times x = 6x^2$$

$$180 = 6x \Rightarrow x = \frac{180}{6} = 30$$

Therefore, the H.C.F. of the numbers is $x = 30$.

[NCERT: Ch. 1, Pg. 2-3]

6. Explanation:

If the number 6^n, for any n, were to end with the digit zero, then it would be divisible by 5. That is, the prime factorization of 6^n would contain the prime 5. This is not possible because $6^n = (2 \times 3)^n = 2^n 3^n$; so, the only primes in the factorization of 6^n are 2 and 3.

Thus, 6^n cannot end with digit 0 for any natural number n.

[NCERT: Ch. 1, Pg. 4]

7. Explanation:

Prime factorization:

$72 = 2^3 \times 3^2$

$120 = 2^3 \times 3^1 \times 5^1$

$\text{HCF} = 2^3 \times 3^1 = 8 \times 3 = 24$

$\text{LCM} = 2^3 \times 3^2 \times 5^1 = 8 \times 9 \times 5 = 360$

Answer: HCF = 24, LCM = 360

[NCERT: Ch. 1, Pg. 2-3]

8. Explanation:

$5 \times 11 \times 17 + 3 \times 11$

$= 11(5 \times 17 + 3) = 11(85 + 3) = 11 \times 88$

So, given number has factors other than 1 and itself, like 11 and 88.

So, it is a composite number.

[NCERT: Ch. 1, Pg. 2-3]

9. Explanation:

(i) $7 \times 11 \times 13 + 13 = 13(7 \times 11 + 1) = 13(77 + 1) = 13 \times 78$

Since it has factors other than 1 and itself; $13, 78$, it is composite.

(ii) $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 = 5(7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1)$

Hence, it has factor other than 1 and itself; 5, so, it is composite.

[NCERT: Ch. 1, Pg. 2-3]

10. Explanation:

We want the smallest number divisible by both 644 and 462, i.e., their LCM.

Prime factorization:

$644 = 2^2 \times 7 \times 23$

$462 = 2 \times 3 \times 7 \times 11$

For LCM, take the highest powers of all primes

$\text{LCM} = 2^2 \times 3^1 \times 7^1 \times 11^1 \times 23^1 = 21252$

Final Answer: 21252

[NCERT: Ch. 1, Pg. 4]

11. Explanation:

Given, ratio of numbers = $4:5$ and HCF = 11

If HCF is 11 and ratio is $4:5$, the numbers are:

$11 \times 4 = 44$ and $11 \times 5 = 55$

For two numbers:

HCF × LCM = Product of the numbers

$11 \times \text{LCM} = 44 \times 55$

$\text{LCM} = \dfrac{44 \times 55}{11} = 4 \times 55 = 220$

Final Answer: 220

[NCERT: Ch. 1, Pg. 2-3]

Short Answer Type Questions

1. Explanation:

Given, dimensions of room,

Length, $l = 8$ m 50 cm $= 850$ cm $= 2 \times 5^2 \times 17$ cm

Breadth, $b = 6$ m 25 cm $= 625$ cm $= 5^4$ cm

Height, $h = 4$ m 75 cm $= 475$ cm $= 5^2 \times 19$ cm

To find the length of the longest rod that can measure the dimension of the room exactly, we have to find the HCF.

HCF of $l, b, h = 5^2 = 25$

Thus, the Length of the longest rod $= 25\ cm$.

[NCERT: Ch. 1, Pg. 2-3]

2. Explanation:

$404 = 2^2 \times 101$

$96 = 2^5 \times 3$

So, HCF $= 2^2 = 4$ and

LCM $= 2^5 \times 3 \times 101 = 9696$

Now, LCM × HCF $= 4 \times 9696 = 38784$ and

Product of 96 and 404 $= 38784$

Hence, LCM × HCF = Product of two numbers is verified.

[NCERT: Ch. 1, Pg. 4]

3. Explanation:

Prime factorization:

$26 = 2 \times 13$

$65 = 5 \times 13$

$117 = 3^2 \times 13$

So, HCF $= 13$ and

LCM $= 2 \times 3^2 \times 5 \times 13 = 1170$

Final Answer: HCF $= 13$, LCM $= 1170$

[NCERT: Ch. 1, Pg. 2-3]

4. Explanation:

The minimum number of teachers in each room should be the greatest common divisor (GCD) of $48, 80$ and 144 because this will ensure that the number of teachers in each room is the same for all subjects.

Let's find the GCD of $48, 80$, and 144:

Prime factorization of 48: $48 = 2^4 \times 3$

Prime factorization of 80: $80 = 2^4 \times 5$

Prime factorization of 144: $144 = 2^4 \times 3^2$

The GCD is the product of the lowest powers of the common prime factors:

So, GCD$(48,80,144) = 2^4 = 16$.

Now, divide the number of teachers by the GCD (16) to get the number of rooms required for each subject:

For French: $\frac{48}{16} = 3$ rooms, For Hindi: $\frac{80}{16} = 5$ rooms, For English: $\frac{144}{16} = 9$ rooms.

The total number of rooms required is: 3 (French) $+$ 5 (Hindi) $+ 9$ (English) $= 17$ rooms

The minimum number of rooms required is 17.

[NCERT: Ch. 1, Pg. 2-3]

5. Explanation:

Given: $p = x^2y^3, q = xy^4, r = x^5y^2$

HCF$(p, q, r) = x^1y^2 = xy^2$

LCM$(p, q, r) = x^5y^4$

$p \times q \times r = x^8y^9$

HCF × LCM $= x^6y^6$

Since $x^6y^6 \neq x^8y^9$, the equation is not true.

Conclusion: HCF$(p, q, r) \times$ LCM$(p, q, r) \neq p \times q \times r$

[NCERT: Ch. 1, Pg. 4]

Expert Tip

LCM, HCF and Product

- *The relation $a \times b = LCM(a,b) \times HCF(a,b)$ holds only for two numbers.*
- *For three or more numbers, this relation does not hold.*

Counter example:

Take $a = 4, b = 6, c = 8$

- *Product $= 4 \times 6 \times 8 = 192$*
- *$LCM(4,6,8) = 24$*
- *$HCF(4,6,8) = 2$*
- *$LCM \times HCF = 24 \times 2 = 48 \neq 192$*

So, the relation fails for three numbers.

Unit 1.3: Revisiting Irrational Numbers

Multiple Choice Questions

(1 Mark)

[Delhi 2025]

1. $(1+\sqrt{3})^2 - (1-\sqrt{3})^2$ is: *(Re)*

(a) a positive rational number.

(b) a negative integer.

(c) a positive irrational number.

(d) a negative irrational number.

Very Short Answer Type Questions

(2 Marks Each)

[2016, Term I]

1. Show that $3\sqrt{7}$ is an irrational number. *(An)*

[2018]

2. Given that $\sqrt{2}$ is irrational, prove that $(5+3\sqrt{2})$ is an irrational number. *(Ap)*

[Delhi 2024]

3. Prove that $5-2\sqrt{3}$ is an irrational number. It is given that $\sqrt{3}$ is an irrational number. *(Cr)*

Short Answer Type Questions

(3 Marks Each)

[Delhi 2019/Outside Delhi 2023/Delhi 2025]

1. Prove that $\sqrt{2}$ is an irrational number. *(Re)*

[Outside Delhi 2019]

2. Prove that $2+5\sqrt{3}$ is an irrational number, given that $\sqrt{3}$ is an irrational number. *(An)*

[Delhi 2023/ Outside Delhi 2024]

3. Prove that $\sqrt{5}$ is an irrational number. *(Re)*

[Outside Delhi 2025]

4. Prove that $\left(5\sqrt{3}+\frac{2}{3}\right)$ is an irrational number given that $\sqrt{3}$ is an irrational number. *(Ap)*

Long Answer Type Questions

(4 Marks)

[Outside Delhi 2020]

1. Prove that $\sqrt{5}$ is an irrational number. *(Un)*

EXPLANATIONS

Multiple Choice Questions

1. Explanation: (c)

$(1+\sqrt{3})^2 - (1-\sqrt{3})^2$

$= (1+2\sqrt{3}+3) - (1-2\sqrt{3}+3)$

$= 4\sqrt{3}$

which is a positive irrational number.

Answer: (c) a positive irrational number.

[NCERT: Ch. 1, Pg. 6]

Very Short Answer Type Questions

1. Explanation:

Let us assume that $3\sqrt{7}$ is rational number.

$\Rightarrow 3\sqrt{7} = \frac{a}{b}$ where a, b are integers.

Since, 3, a and b are integers, $\frac{a}{3b}$ can be written in the form of $\frac{p}{q}$, where p, q are integers.

So, $\frac{a}{3b}$ is rational number.

But $\sqrt{7} = \frac{a}{3b}$, implies, $\sqrt{7}$ is also rational number.

But this is a contradiction, as $\sqrt{7}$ is irrational.

That means, our assumption was wrong.

Thus, $3\sqrt{7}$ is irrational number.

[NCERT: Ch. 1, Pg. 6]

Expert Tip

Proving Irrational Numbers

- *To prove a particular number is irrational, always start by assuming it is rational (i.e., can be written as $\frac{p}{q}$, where p, q are integers and $q \neq 0$).*
- *Then, show that this assumption leads to a contradiction.*
- *This method is called Proof by Contradiction (also known as Reductio ad Absurdum).*

2. Explanation:

Assume, for contradiction, that $5+3\sqrt{2}$ is rational.

Then there exist integers p, q with $q \neq 0$ and $\gcd(p,q) = 1$ such that

$5 + 3\sqrt{2} = \frac{p}{q}$

$3\sqrt{2} = \frac{p}{q} - 5 = \frac{p-5q}{q}$

$\sqrt{2} = \frac{p-5q}{3q}$

The right-hand side $\frac{p-5q}{3q}$ is a rational number, so this expresses $\sqrt{2}$ as a rational number.

That contradicts the given fact that $\sqrt{2}$ is irrational.

Therefore, our assumption is false, and $5+3\sqrt{2}$ must be irrational.

[NCERT: Ch. 1, Pg. 6-8]

Fun Fact

Discovery of $\sqrt{2}$

- *The first proof that the $\sqrt{2}$ is irrational is credited to **Hippasus of Metapontum**, a member of the Pythagorean school in the 5th century BC.*
- *It shocked the ancient Greek mathematicians (the Pythagoreans) as they believed all numbers could be expressed as fractions, so discovering irrational numbers was a huge surprise!*

3. Explanation:

Assume $5-2\sqrt{3}$ is rational.

That means it can be written as $5-2\sqrt{3}=\frac{p}{q}$, where p and q are integers, and $q\neq 0$.

$5-\frac{p}{q}=2\sqrt{3}\Rightarrow\sqrt{3}=\frac{5-\frac{p}{q}}{2}=\frac{5q-p}{2q}$

The right-hand side $\frac{5q-p}{2q}$ is a ratio of two integers, which means it is rational.

But this contradicts the assumption that $\sqrt{3}$ is irrational.

Therefore, the assumption that $5-2\sqrt{3}$ is rational must be false.

Hence, $5-2\sqrt{3}$ is irrational.

[NCERT: Ch. 1, Pg. 6]

Short Answer Type Questions

1. Explanation:

Assume $\sqrt{2}$ is rational, so $\sqrt{2}=\frac{p}{q}$, where p and q are coprime integers.

Squaring both sides: $2=\frac{p^2}{q^2}\implies p^2=2q^2$.

This means p^2 is even, so p is even.

Let $p=2k$. Substituting into the equation: $2q^2=4k^2\implies q^2=2k^2$.

This shows q^2 is also even, so q is even.

Both p and q being even contradicts the assumption that they are coprime.

Therefore, $\sqrt{2}$ is irrational.

[NCERT: Ch. 1, Pg. 6-7]

2. Explanation:

Assume $2+5\sqrt{3}=\frac{p}{q}$, where p and q are integers.

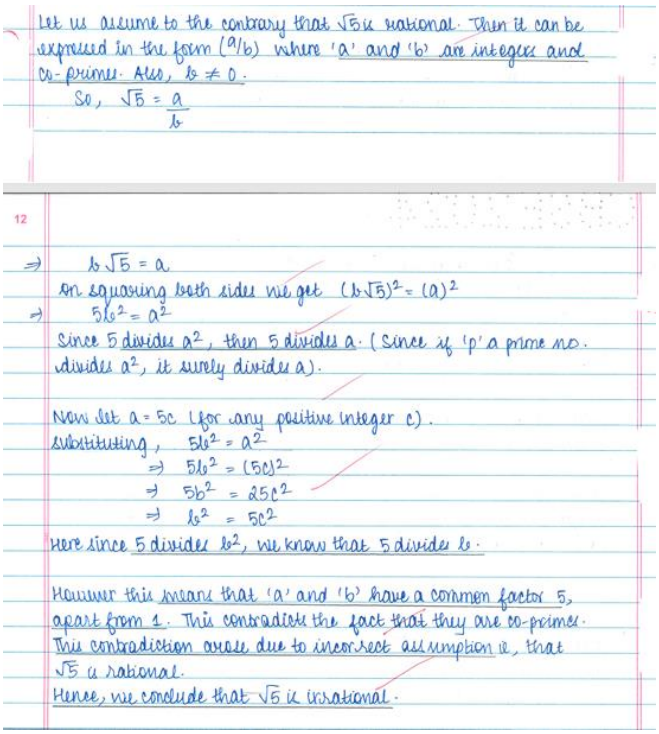
$5\sqrt{3}=\frac{p-2q}{q}\Rightarrow\sqrt{3}=\frac{p-2q}{5q}$

The right side is rational, but $\sqrt{3}$ is irrational. This is a contradiction.

Therefore, $2+5\sqrt{3}$ is irrational.

[NCERT: Ch. 1, Pg. 6-7]

3. Explanation:


Let us assume to the contrary that $\sqrt{5}$ is rational. Then it can be expressed in the form (a/b) where 'a' and 'b' are integers and co-prime. Also, $b\neq 0$.

So, $\sqrt{5}=\frac{a}{b}$

12

$\Rightarrow b\sqrt{5}=a$

On squaring both sides we get $(b\sqrt{5})^2=(a)^2$

$\Rightarrow 5b^2=a^2$

Since 5 divides a^2, then 5 divides a. (Since if 'p' a prime no. divides a^2, it surely divides a).

Now let $a=5c$ (for any positive integer c).

Substituting, $5b^2=a^2$

$\Rightarrow 5b^2=(5c)^2$

$\Rightarrow 5b^2=25c^2$

$\Rightarrow b^2=5c^2$

Here since 5 divides b^2, we know that 5 divides b.

However this means that 'a' and 'b' have a common factor 5, apart from 1. This contradicts the fact that they are co-primes. This contradiction arose due to incorrect assumption is, that $\sqrt{5}$ is rational.

Hence, we conclude that $\sqrt{5}$ is irrational.

[Topper's Answer]

4. Explanation:

Suppose $5\sqrt{3}+\frac{2}{3}$ is rational.

Then, it can be expressed as $\frac{p}{q}$, where p and q are integers with no common factors (i.e., $\gcd(p,q)=1$), and $q\neq 0$.

$5\sqrt{3}+\frac{2}{3}=\frac{p}{q}$

$5\sqrt{3}=\frac{p}{q}-\frac{2}{3}=\frac{3p-2q}{3q}$

$\therefore\sqrt{3}=\frac{3p-2q}{15q}$

Since p and q are integers, $3p-2q$ and $15q$ are also integers.

Thus, $\sqrt{3}$ is expressed as a ratio of two integers, $\frac{3p-2q}{15q}$, which would make $\sqrt{3}$ rational.

However, it is given that $\sqrt{3}$ is irrational.

Our assumption that $5\sqrt{3}+\frac{2}{3}$ is rational leads to the false conclusion that $\sqrt{3}$ is rational.

Therefore, the assumption must be false.

Hence, $5\sqrt{3}+\frac{2}{3}$ is irrational.

[NCERT: Ch. 1, Pg. 6]

Long Answer Type Questions

1. **Explanation:**

We prove by contradiction.

Assume $\sqrt{5}$ is rational. Then it can be written as a reduced fraction $\sqrt{5} = \frac{p}{q}$

where p, q are integers with no common factor ($\gcd(p, q) = 1$) and $q \neq 0$.

So, $\sqrt{5} = \frac{p}{q}$

Square both sides:

$$5 = \frac{p^2}{q^2}$$

$$p^2 = 5q^2$$

Since, 5 divides p^2, then 5 divides p. (if a prime divides a square, it divides the base).

Write $p = 5k$ for some integer k.

Substitute back:

$(5k)^2 = 5q^2 \implies 25k^2 = 5q^2 \implies q^2 = 5k^2$

Thus, 5 divides q^2, and thus 5 divides q.

Now both p and q are divisible by 5, which contradicts the assumption that the fraction $\frac{p}{q}$ was in lowest terms ($\gcd(p, q) = 1$).

This contradiction shows our assumption was false. Therefore $\sqrt{5}$ is irrational.

[NCERT: Ch. 1, Pg. 6]

Out of the Syllabus Questions

(2016, Term I: 4 Marks)

1. **Find HCF of numbers 134791,6341 and 6339 by Euclid division algorithm. *(Re)***

(Delhi 2019: 1 Mark)

2. **Find a rational number between $\sqrt{2}$ and $\sqrt{3}$. *(Re)***

(Delhi 2019: 2 Marks)

3. **Find the HCF of 1260 and 7344 using Euclid's algorithm. *(Re)***

(Delhi 2019: 2 Marks)

4. **Show that every positive odd integer is of the form $(4q + 1)$ or $(4q + 3)$, where q is some integer. *(Cr)***

(Outside Delhi 2019: 3 Marks)

5. **Using Euclid's Algorithm, find the HCF of 2048 and 960. *(Re)***

(Delhi 2020: 4 Marks)

6. **Show that the square of any positive integer cannot be of the form $(5q + 2)$ or $(5q + 3)$ for any integer q. *(Ap)***

(Delhi 2020: 4 Marks)

7. **Prove that one of every three consecutive positive integers is divisible by 3. *(Un)***

(Outside Delhi 2020: 1 Mark)

8. **Euclid's division Lemma states that for two positive integers a and b, there exists unique integer q and r satisfying $a = bq + r$, and *(An)***

(a) $0 < r < b$ (b) $0 < r \leq b$

(c) $0 \leq r < b$ (d) $0 \leq r \leq b$

(2022, Term I: 1 Mark)

9. **The greatest number which when divides 1251,9377 and 15628 leaves remainder 1,2 and 3 respectively is *(Un)***

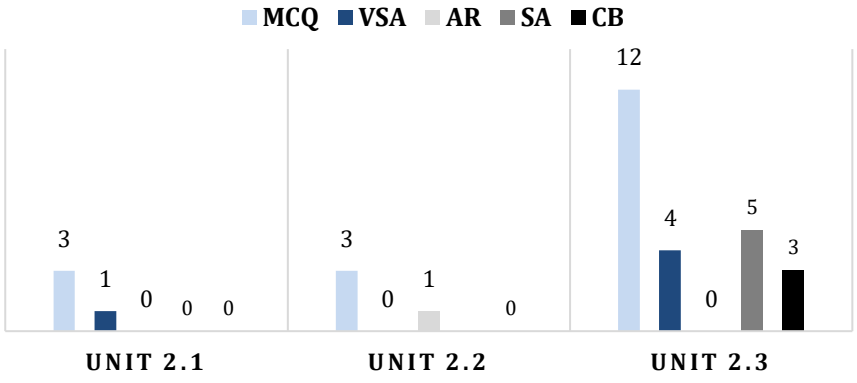
(a) 575 (b) 450

(c) 750 (d) 625

CHAPTER 2

Polynomials

TOPIC-WISE TREND ANALYSIS OF PREVIOUS YEARS



Quick Revision Sheet

1. Definition of a Polynomial

- An expression of the form $p(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$, where coefficients are real numbers and $a_n \neq 0$.
- The **degree** of the polynomial = highest power of x with a non-zero coefficient.

2. Types of Polynomials by Degree

- Degree 1 → Linear Polynomial (e.g., $2x - 3$).
- Degree 2 → Quadratic Polynomial (e.g., $ax^2 + bx + c, a \neq 0$).
- Degree 3 → Cubic Polynomial (e.g., $ax^3 + bx^2 + cx + d, a \neq 0$).

3. Zeroes of a Polynomial

- A real number k is a zero if $p(k) = 0$.
- Graphical meaning: Zeroes are the **x-coordinates of the points where the graph of $y = p(x)$ intersects the x-axis**.

4. Geometrical Meaning of Zeroes

- Linear polynomial: exactly 1 zero (line cuts x-axis once).
- Quadratic polynomial:
  Two distinct zeroes (curve cuts x-axis at two points).
  One double zero (curve touches x-axis).
  No zero (curve does not intersect x-axis).
- Cubic polynomial: at most 3 zeroes (graph can intersect x-axis in 1, 2, or 3 points).
- In general, a polynomial of degree n has at most n zeroes.

5. Relationship between Zeroes and Coefficients

For quadratic $p(x) = ax^2 + bx + c$:

(i) Sum of zeroes $(\alpha + \beta) = -\frac{b}{a}$

(ii) Product of zeroes $(\alpha\beta) = \frac{c}{a}$

For cubic $p(x) = ax^3 + bx^2 + cx + d$:

(i) Sum of zeroes $(\alpha + \beta + \gamma) = -\frac{b}{a}$

(ii) Sum of products of zeroes taken two at a time $(\alpha\beta + \beta\gamma + \gamma\alpha) = \frac{c}{a}$

(iii) Product of zeroes $(\alpha\beta\gamma) = -\frac{d}{a}$

MIND MAP

Polynomial
is an algebraic expression of the form $p(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$, where $a_0, a_1, a_2, \ldots, a_n$ are real numbers and n is a non-negative integer

Types of Polynomials

Linear Polynomial
A polynomial of degree one

Quadratic Polynomial
A polynomial of degree two

Cubic Polynomial
A polynomial of degree three

Important Definitions

Degree of a Polynomial $p(x)$
is the highest power of x in $p(x)$.

Zero of a Polynomial $p(x)$
is a real number k such that $p(k) = 0$ or it is the x-coordinate of the point where the graph of $y = p(x)$ intersects the x-axis.

Relationship between Zeroes and Coefficients
If α and β are the zeroes of a quadratic polynomial $ax^2 + bx + c$, then
$\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

PREVIOUS YEAR EXAM QUESTIONS

Unit 2.1 Introduction

Multiple Choice Questions

(1 Mark Each)

[Delhi 2020]

1. If one of the zeroes of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value of k is *(Ap)*

(a) 10 (b) -10

(c) -7 (d) -2

[Outside Delhi 2023)

2. If one zero of the polynomial $x^2 + 3x + k$ is 2, then the value of k.*(Ap)*

(a) -10 (b) 10

(c) 5 (d) -5

[Outside Delhi 2024]

3. Value of k for which $x = 2$ is a solution of the equation $5x^2 - 4x + (2 + k) = 0$, is *(Ap)*

(a) 10 (b) -10

(c) 14 (d) -14

Very Short Answer Type Questions

[Outside Delhi 2020]

1. A teacher asked 10 of his students to write a polynomial in one variable on a paper and then to handover the paper. The following were the answers given by the students:

$$2x + 3, 3x^2 + 7x + 2, 4x^3 + 3x^2 + 2, x^3 + \sqrt{3x} + 7, 7x + \sqrt{7}$$

$$5x^3 - 7x + 2, 2x^2 + 3 - \frac{5}{x}, 5x - \frac{1}{2}, ax^3 + bx^2 + cx + d, x + \frac{1}{x}$$

Answer the following questions: *(Cr)*

(i) How many of the above ten, are not polynomials?

(ii) How many of the above ten, are quadratic polynomials?

EXPLANATIONS

Multiple Choice Questions

1. Explanation: (b)

Since 2 is one of the zeroes, we substitute $x = 2$ into the polynomial equation:

$2^2 + 3(2) + k = 0$

$4 + 6 + k = 0 \Rightarrow 10 + k = 0 \Rightarrow k = -10$

Correct Answer: (b) -10

[NCERT: Ch. 2, Pg. 11]

2. Explanation: (a)

If $x = 2$ is a zero of $x^2 + 3x + k$, substitute $x = 2$:

$2^2 + 3(2) + k = 0$

$4 + 6 + k = 0$

$10 + k = 0$

$k = -10$

Answer: **(a) -10**

[NCERT: Ch. 2, Pg. 11]

Expert Tip

In polynomials, a root and a zero are the same: both are values of x that make the polynomial equal to zero.

3. Explanation: (d)

Substitute $x = 2$ into $5x^2 - 4x + (2 + k) = 0$:

$5(2)^2 - 4(2) + (2 + k) = 20 - 8 + 2 + k = 14 + k = 0 \Rightarrow k = -14$

[NCERT: Ch. 2, Pg. 11]

Very Short Answer Type Questions

1. Explanation:

Let's analyze each of the 10 expressions:

Expression	Polynomial	Type
1. $2x + 3$	Yes	Linear
2. $3x^2 + 7x + 2$	Yes	Quadratic
3. $4x^3 + 3x^2 + 2$	Yes	Cubic
4. $x^3 + \sqrt{3}x + 7$	Yes	Cubic
5. $7x + \sqrt{7}$	Yes	Linear
6. $5x^3 - 7x + 2$	Yes	Cubic
7. $2x^2 + 3 - \frac{5}{x}$	No	Not a polynomial (term with x^{-1})
8. $5x - \frac{1}{2}$	Yes	Linear
9. $ax^3 + bx^2 + cx + d$	Yes	Cubic (general form)
10. $x + \frac{1}{x}$	No	Not a polynomial (term with x^{-1})

(i) Not polynomials: Expression 7 and 10 contain $\frac{1}{x}$, which is not allowed in polynomials.

Answer: 2

(ii) Quadratic polynomials: Only Expression 2 is quadratic.

Answer: 1

[NCERT: Ch. 2, Pg. 10-11]

Remember This

Types of Polynomials by Degree

- *Degree 1: Linear Polynomial → General form:* $ax + b$
- *Degree 2: Quadratic Polynomial → General form:* $ax^2 + bx + c$
- *Degree 3: Cubic Polynomial → General form:* $ax^3 + bx^2 + cx + d$

(where a, b, c, d *are real numbers and* $a \neq 0$ *in each case).*

Unit 2.2: Geometrical Meaning of the Zeroes of a Polynomial

Multiple Choice Questions

(1 Mark Each)

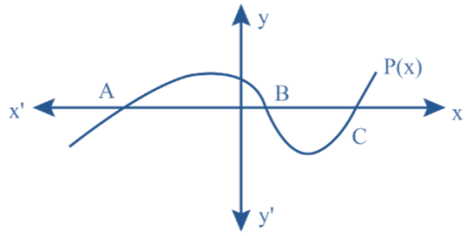
[2022, Term I]

1. The graph of a polynomial $P(x)$ cuts the x-axis at 3 points and touches it at 2 other points. The number of zeroes of $P(x)$ is: *(Re)*

(a) 1 (b) 2
(c) 3 (d) 5

[2022, Term I]

2. In figure, the graph of a polynomial $P(x)$ is shown. The number of zeroes of $P(x)$ is *(Re)*



(a) 1 (b) 2
(c) 3 (d) 4

[Delhi 2025]

3. Which of the following statements is true for a polynomial $p(x)$ of degree 3? *(Re)*

(a) $p(x)$ has at most two distinct zeroes.
(b) $p(x)$ has at least two distinct zeroes.
(c) $p(x)$ has exactly three distinct zeroes.
(d) $p(x)$ has at most three distinct zeroes.

Assertion and Reason

(1 Mark)

Questions below are Assertion and Reason based questions. Two statements are given, one labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(c) Assertion (A) is true, but Reason (R) is false.
(d) Assertion (A) is false, but Reason (R) is true.

[Delhi 2024]

1. Assertion (a) If the graph of a polynomial touches x-axis at only one point, then the polynomial cannot be a quadratic polynomial.
Reason (R): A polynomial of degree $n(n > 1)$ can have at most n zeroes. *(An)*

EXPLANATIONS

Multiple Choice Questions

1. Explanation: (d)
The graph cuts the x-axis at 3 points, contributing 3 zeroes.
The graph touches the x-axis at 2 points, contributing 2 zeroes.
In total, the number of zeroes of $P(x)$ is $3 + 2 = 5$.
Answer: (d) 5

[NCERT: Ch. 2, Pg. 14-15]

2. Explanation: (c)
The number of zeros of a polynomial corresponds to the number of times the graph intersects/touches the x-axis. Since the graph intersects the x-axis at three points (A, B, and C), the polynomial has three zeroes.
Correct Answer: (c) 3

[NCERT: Ch. 2, Pg. 14-15]

3. Explanation: (d)
A cubic polynomial (degree 3) can have at most 3 distinct zeroes. (since polynomial of degree n has at most n zeroes.)
Answer: (d) $p(x)$ has at most three distinct zeroes.

[NCERT: Ch. 2, Pg. 11-17]

Assertion and Reason

1. Explanation: (d)
Assertion (A): If the graph of a polynomial touches the x-axis at only one point, then the polynomial cannot be a quadratic polynomial.
This is incorrect. A quadratic polynomial can touch the x-axis at exactly one point (if the discriminant is zero), in which case it has one repeated real root. E.g. x^2.
Reason (R): A polynomial of degree n $(n > 1)$ can have at most n zeroes.
This statement is correct. A polynomial of degree n can have at most n roots (zeroes), according to the Fundamental Theorem of Algebra.