I N D E X

CUET (PG) - PHYSICS

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1. MATHEMATICAL METHODS IN PHYSICS- NUMBER OF QUESTION ASKED IN CUET (PG) PHYSICS (2022-2026)

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1	CALCULUS OF SINGLE AND MULTIPLE VARIABLES, PARTIAL DERIVATIVES, JACOBIAN, IMPERFECT AND PERFECT DIFFERENTIALS	2	-	2	1	-
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5	MULTIPLE INTEGRALS, DIVERGENCE THEOREM, GREEN'S THEOREM, STOKES' THEOREM.	1	3	1	1	1
6	FIRST ORDER EQUATIONS AND LINEAR SECOND ORDER DIFFERENTIAL EQUATIONS WITH CONSTANT COEFFICIENTS.	4	-	2	2	-
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8	ALGEBRA OF COMPLEX NUMBERS.	1	1	1	2	2

CHAPTER 1

MATHEMATICAL METHODS IN PHYSICS

CUET (PG) PHYSICS

CALCULUS OF SINGLE AND MULTIPLE VARIABLES, PARTIAL DERIVATIVES, JACOBIAN, IMPERFECT AND PERFECT DIFFERENTIALS

PREVIOUS YEAR EXAM QUESTIONS

(CUET-PG 2022)

1. Given $A = x^2 + y^2 + z^2$
$x = r\sin\theta\cos\Phi$
$y = r\sin\theta\sin\Phi$
$z = r\cos\theta$
Then, $\frac{\partial A}{\partial r}$ value will be
(a) $2r$ (b) r
(c) r^2 (d) $2r^2$

(CUET-PG 2022)

2. If $x + y + z = a, y + z = ab, z = abc$. The value of $\frac{\partial(x,y,z)}{\partial(a,b,c)}$ is
(a) $-a^2b$ (b) a^2b
(c) ab^2 (d) $-ab^2$

(CUET-PG 2024)

3. The equation $(x^2y - 2xy^2)dx + (3x^2y - x^3)dy = 0$
(A) Is exact
(B) Is inexact
(C) The solution is $\frac{y}{x} - 2\log x + 3\log y = constant$
(D) The solution is $\frac{x}{y} - 2\log x + 3\log y = constant$
Choose the correct answer from the options given below.
(a) (B) and (C) only. (b) (B) and (D) only.
(c) (A) and (D) only. (d) (A) and (C) only.

(CUET-PG 2024)

4. If $J = \frac{\partial(u,v)}{\partial(x,y)}$ and $J' = \frac{\partial(x,y)}{(u,v)}$, then JJ' is equal to
(a) 0 (b) -1
(c) ∞ (d) 1

(CUET-PG 2025)

5. If $x = r\cos_\theta, y = \sin_\theta$ then
Match the LIST-I with LIST-II

	LIST-I		LIST-II
A.	$\frac{\partial r}{\partial x}$	I.	$\frac{1}{r}$
B.	$\frac{\partial r}{\partial y}$	II.	$\frac{y}{r}$
C.	$\frac{\partial(x,y)}{\partial(r,\theta)}$	III.	$\frac{x}{r}$
D.	$\frac{\partial(r,\theta)}{\partial(x,y)}$	IV.	r

Choose the correct answer from the options given below:
(a) A - III, B - II, C - I, D – IV (b) A - III, B - II, C - IV, D - I
(c) A - II, B - III, C - IV, D – I (d) A - II, B - III, C - I, D – IV

ANSWER KEY				
1	2	3	4	5
(a)	(b)	(b)	(d)	(b)

SOLUTIONS

1. Solution:
$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$
$r^2 = x^2 + y^2 + z^2$
$A = r^2$
$\frac{\partial A}{\partial r} = 2r$
Hence option (a) is correct.

2. Solution:
$\because x + y + z = a\ ;\ y + z = ab\ ;\ z = abc$
$\therefore y + z = ab\ ;\ x + ab = a\ ;\ x = a - ab$
$y = ab - z\ ; y = ab - abc$

Jacobian, $J(x,y,z) = \frac{\partial(x,y,z)}{\partial(a,b,c)} = \begin{vmatrix} \frac{\partial x}{\partial a} & \frac{\partial x}{\partial b} & \frac{\partial x}{\partial c} \\ \frac{\partial y}{\partial a} & \frac{\partial y}{\partial b} & \frac{\partial y}{\partial c} \\ \frac{\partial z}{\partial a} & \frac{\partial z}{\partial b} & \frac{\partial z}{\partial c} \end{vmatrix}$

$= \begin{vmatrix} (1-b) & -a & 0 \\ 1-bc & a-ac & -ab \\ bc & ac & ab \end{vmatrix}$
$= (1-b)[(a-ac)(ab) + a^2bc] + a[ab(b-bc) + ab^2c]$
$= (1-b)[a^2b - a^2bc + a^2bc] + a[ab^2 - ab^2c + ab^2c]$
$= a^2b - a^2b^2 + a^2b^2$
$= a^2b$
Hence option (b) is correct.

3. Solution: - We know that $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$
$(x^2y - 2xy^2)dx + (3x^2y - x^3)dy = 0$
$M = (x^2y - 2xy^2)$ and $N = (3x^2y - x^3)$
$\frac{\partial M}{\partial y} = x^2 - 4xy$
$\frac{\partial N}{\partial x} = 6xy - 3x^2$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$.

Therefore, the equation is inexact.

$(x^2y - 2xy^2)dx + (3x^2y - x^3)dy = 0$

$y(x - 2y).dx - x(x - 3y)dy = 0$

Let $y = vx \Rightarrow dy = v.dx + x.dv$..... (1)

Put the value of y and dy in equation (1), then we get,

$vx[x - 2(vx)]dx - x[x - 3(vx)](v.dx + x.dv) = 0$

$v(1 - 2v).dx - (1 - 3v)(v.dx + x.dv) = 0$

$(v - 2v^2 - v + 3v^2)dx = (1 - 3v)x.dv$

$\frac{dv}{dx} = \frac{v^2dv}{(1-3v)x} \Rightarrow \left(\frac{1}{v^2} - \frac{3}{v}\right)dv = \frac{dx}{x}$

$-\frac{1}{v} - 3\ln v = \ln x + c \Rightarrow -\frac{x}{y} - 3\ln\left(\frac{y}{x}\right) = \ln x + c$

$-\frac{x}{y} - 3\ln y + 3\ln x = \ln x + c \Rightarrow -\frac{x}{y} - 3\ln y + 2\ln x = \text{constant}$

$\frac{x}{y} - 2\ln x + 3\ln y = \text{constant}$

Hence, option (b) is correct.

4. Solution:- The product of the Jacobians J and J′ can be represented as follows:

$JJ' = \left|\frac{\partial(u,v)}{\partial(x,y)}\right|\left|\frac{\partial(x,y)}{\partial(u,v)}\right|$

According to the chain rule of partial derivatives, if we compose two transformations, the product of their Jacobians will be 1.

Hence, option (d) is correct.

5. Solution: -

Partial Derivatives with $x = r\cos\theta, y = r\sin\theta$

Since $r^2 = x^2 + y^2$, differentiating implicitly:

$A.\ \frac{\partial r}{\partial x}: 2r\frac{\partial r}{\partial x} = 2x \Rightarrow \frac{\partial r}{\partial x} = \frac{x}{r} \rightarrow$ III

$B.\ \frac{\partial r}{\partial y}: 2r\frac{\partial r}{\partial y} = 2y \Rightarrow \frac{\partial r}{\partial y} = \frac{y}{r} \rightarrow$ II

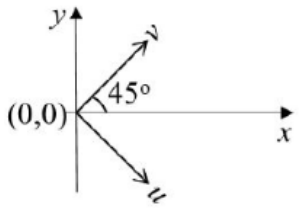
$C.\ \frac{\partial(x,y)}{\partial(r,\theta)}: \begin{vmatrix}\cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta\end{vmatrix} = r\cos^2\theta + r\sin^2\theta = r \rightarrow$ IV

$D.\ \frac{\partial(r,\theta)}{\partial(x,y)} = \frac{1}{r} \rightarrow$ I

A-III, B-II, C-IV, D-I Correct option is (b).

PRACTICE QUESTIONS

1. The Jacobian matrix for transforming from (x, y) to another orthogonal coordinates system (u, v) as shown in the figure is



(a) $\frac{1}{\sqrt{2}}\begin{bmatrix}1 & -1\\ 1 & -1\end{bmatrix}$ (b) $\frac{1}{\sqrt{2}}\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}$

(c) $\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ -1 & 1\end{bmatrix}$ (d) $\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$

ANSWER KEY
1
(b)

SOLUTIONS

1. Solution: (b)

$\begin{bmatrix}u\\ v\end{bmatrix} = \begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}$

$\begin{bmatrix}u\\ c\end{bmatrix} = \begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}\frac{u}{\sqrt{2}} + \frac{v}{\sqrt{2}}\\ -\frac{u}{\sqrt{2}} + \frac{v}{\sqrt{2}}\end{bmatrix}$

$\begin{bmatrix}u\\ c\end{bmatrix} = \begin{bmatrix}\frac{au+av}{\sqrt{2}} + \frac{-bu+bv}{\sqrt{2}}\\ \frac{cu+cv}{\sqrt{2}} + \frac{-du+dv}{\sqrt{2}}\end{bmatrix}$

$u = \frac{(a-b)u}{\sqrt{2}} + \left(\frac{a+b}{\sqrt{2}}\right)v$ & $v = \frac{(c-d)u}{\sqrt{2}} + \frac{(c+d)v}{\sqrt{2}}$

$\frac{a-b}{\sqrt{2}} = 1, \frac{a+b}{\sqrt{2}} = 0, \frac{c-d}{\sqrt{2}} = 0, \frac{c+d}{\sqrt{2}} = 1$

$\frac{2a}{\sqrt{2}} = 1, a = \frac{1}{\sqrt{2}}, \frac{-2b}{\sqrt{2}} = 1, b = -\frac{1}{\sqrt{2}}, c = \frac{1}{\sqrt{2}}, \frac{-2d}{\sqrt{2}} = -1$

Thus, $\begin{bmatrix}a & b\\ c & d\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}$

Hence, option (b) is correct.

TAYLOR EXPANSION

PREVIOUS YEAR EXAM QUESTIONS

(CUET 2023)

1. Match List-I with List-II:

	List-I		List-I
(A)	$\sin z$ for $\lvert z\rvert < \infty$	(I)	$(-1)^{n-1}z^{2n-2}/(2n-2)!$
(B)	$\cos z$ for $\lvert z\rvert < \infty$	(II)	$(-1)^{n-1}z^n/n$
(C)	$\tan^{-1} z$ for $\lvert z\rvert < 1$	(III)	$(-1)^{n-1}z^{2n-1}/(2n-1)!$
(D)	$\ln(1+z)$ for $\lvert z\rvert < 1$	(IV)	$(-1)^{n-1}z^{2n-1}/(2n-1)$

Choose the correct answer from the options given below:

(a) (A)-(II); (B)-(III); (C)-(IV); (D)-(I)

(b) (A)-(I); (B)-(II); (C)-(IV); (D)-(III)

(c) (A)-(III); (B)-(I); (C)-(IV); (D)-(II)

(d) (A)-(III); (B)-(I); (C)-(II); (D)-(IV)

(CUET-PG 2024)

2. Match List I with List II

List I		List II	
A.	$\sin x$	I.	$\cos h\, ix$
B.	$\cos x$	II.	$-i\tan h\, ix$
C.	$\tan x$	III.	$\sec h\, ix$
D.	$\sec x$	IV.	$-i\sin h\, ix$

(a) (A) - (I), (B) - (II), (C) - (III), (D) - (IV)
(b) (A) - (IV), (B) - (I). (C) - (II), (D) - (III)
(c) (A) - (I). (B) - (II), (C) - (IV), (D) - (III)
(d) (A) - (III), (B) - (IV), (C) - (1), (D) - (II)

ANSWER KEY	
1	2
(c)	(b)

SOLUTIONS

1. Solution: By Maclaurin Series,
$\sin z = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} Z^{2n-1}}{(2n-1)!}$, $\cos z = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} Z^{2n-2}}{(2n-2)!}$ And $\ln(1+z) = (-1)^{n-1} z^n / n$
Hence, option (c) is correct.

2. Solution:- We know that, $\sin x = \frac{e^{ix} - e^{-ix}}{2i}$, $\cos x = \frac{e^{ix} + e^{-ix}}{2}$, $\tan x = \frac{e^{ix} - e^{-ix}}{i(e^{ix} + e^{-ix})}$
So let, $[I].\ \text{Cosh(ix)} = \frac{e^{ix} + e^{-ix}}{2} = \cos x$
$[II].\ -i \tanh ix = (-i)\frac{e^{ix} - e^{-ix}}{e^{ix} + e^{-ix}} = \frac{i}{i^2} \times \frac{e^{ix} - e^{-ix}}{e^{ix} + e^{-ix}} = \frac{1}{i} \times \frac{e^{ix} - e^{-ix}}{e^{ix} + e^{-ix}} \Rightarrow -i \tanh ix = \tan x$
And $[III].\ \text{Sech (ix)} = \frac{2}{e^{ix} + e^{-ix}} = \text{Sec}x$
$[IV].\ -i \sinh(ix) = -i\left(\frac{e^{ix} - e^{-ix}}{2}\right) = \frac{i}{i^2} \times \frac{e^{ix} - e^{-ix}}{2i} = \frac{i}{i} \times \frac{e^{ix} - e^{-ix}}{2i} \Rightarrow -i \sin h\ ix = \sin x$
Hence, option (b) is correct.

PRACTICE QUESTIONS

1. The function $f(x) = e^{\sin x}$ is expanded as a Taylor series in x, around $x = 0$, in the form $f(x) = \sum_{n=0}^{\infty} a_n x^n$. The value of $a_0 + a_1 + a_2$ is
(a) 0 (b) 3/2
(c) 5/2 (d) 5

2. The function $e^{\cos x}$ is Taylor expanded about $x = 0$. The coefficient of x^2 is
(a) $-\frac{1}{2}$ (b) $-\frac{e}{2}$
(c) $\frac{e}{2}$ (d) Zero

3. The coefficient of x^3 in the Taylor expansion of sin [sin x] around x = 0 is _____.
[Specify your answer up to two digits after the decimal point]
(a) -0.33 (b) 0.8
(c) 1 (d) 4

ANSWER KEY		
1	2	3
(c)	(b)	(a)

SOLUTIONS

1. Solution: Simply by the Taylor series expansion, we can get the value of a_0 , a_1 and a_2 as 1, 1 and ½ respectively. Hence, $a_0 + a_1 + a_2 = 5/2$
Hence, option (c) is correct.

2. Solution: Standard Taylor Series expansions about $X = 0$, are following:
$e^x = 1 + X + \frac{1}{2!}X^2 + \frac{1}{3!}X^3 + \ldots\ldots$
$\cos X = 1 - \frac{1}{2!}X^2 + \frac{1}{4!}X^4 + \ldots\ldots$
The Taylor Series expansion of $e^{\cos x}$ about $x = 0$ will be
$e^{\cos x} = e^{\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots\right)} = e \cdot e^{-\frac{x^2}{2!}} \cdot e^{\frac{x^4}{4!}} \ldots\ldots$
$= e\left(1 - \frac{x^2}{2} + \frac{x^4}{4.2!} \ldots\right) + \left(1 + \frac{x^4}{4!} \ldots\right) = e - \frac{e}{2}x^2 + \cdots$
Therefore, the co-efficient of x^2 is $(-e/2)$.
Hence, option (b) is correct.

3. Solution: Let the given function be denoted as $f(x) = \sin(sinx)$
$f'(x) = \cos(sinx) \,.\, cosx$
$f''(x) = -\sin(sinx).cosx.cosx - sinx.\cos(sinx)$
$f''(x) = -cos^2x.\sin(sinx) - sinx.\cos(sinx)$
$f'''(x) = -[-(2cosxsinx)\sin(sinx) + cos^3xcos(sinx) + cosxcos(sinx) + sinx(-\sin(sinx)\,cosx)]$
$= \left[sin2x.\sin(sinx) - cos^3x.\cos(sinx) - cosx.\cos(sinx) + \frac{1}{2}sin2x.\sin(sinx)\right]$
Now at x=0, $f''(0) = -1 - 1$
$f''(0) = -2$
Therefore, $f(x) = f(x_o) + \frac{(x-x_1)f'(x_o)}{\lfloor 1} + \frac{(x-x_2)^2 f''(x_o)}{\lfloor 2} + \frac{(x-x_o)^3}{\lfloor 3} f'(x_1) + \cdots$.
In the above expression, coefficient of x^3 is $= \frac{1}{\lfloor 3}(-2)$
$= \frac{-2}{3 \times 2 \times 1}$
$= \left(-\frac{1}{3}\right)$
$= -0.33$
Hence, option (a) is correct.

FOURIER SERIES

PREVIOUS YEAR EXAM QUESTIONS

(CUET-PG 2025)

1. For an even function, the Fourier coefficients are:

A. $a_0 \neq 0$

B. $a_n \neq 0$

C. $a_n = 0$

D. $b_n = 0$

Choose the correct answer from the options given below:

(a) A, C and D only (b) A, B and D only

(c) C and D only (d) B and D only

ANSWER KEY
1
(b)

SOLUTIONS

1. Solution: - (b)

For an even function, Fourier series has only cosine terms:

$a_0 \neq 0$ (generally, if mean is non-zero)

$a_n \neq 0$ (cosine coefficients can be non-zero)

$b_n = 0$ (sine coefficients are zero)

So, correct option is (b) A, B and D

PRACTICE QUESTIONS

1. For the Fourier series of the following function of period 2π

$$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ 1 & 0 < x < \pi \end{cases}$$

the ratio [to the nearest integer] of the Fourier coefficients of the first and the third harmonic is:

(a) 1 (b) 2

(c) 3 (d) 6

2. In the Fourier series of the periodic function [shown in the figure]

$f(x) = |sinx| = \sum_{n=0}^{\infty}[\alpha_n cosnx + \beta_n sinnx]$

Which of the following coefficients are non-zero?

(a) α_n For odd n (b) α_n for even n

(c) β_n For odd n (d) β_n for even n

3. $f[x]$ is a periodic function of x with a period of 2π. In the interval $-\pi < x < \pi$, $f[x]$ is given by

$$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ sinx, & 0 < x < \pi \end{cases}$$

In the expansion of $f[x]$ as a Fourier series of sine and cosine functions, the coefficient of $\cos[2x]$ is

(a) $\frac{2}{3\pi}$ (b) $\frac{1}{\pi}$

(c) 0 (d) $-\frac{2}{3\pi}$

4. A periodic function can be expressed in a Fourier series of the form, $f(x) = \sum_{n=0}^{\infty}(a_n \cos(nx) + b_n \sin(nx))$. The function $f_1(x) = cos^2 x$ and $f_2(x) = sin^2 x$ are expanded in their respective Fourier series. If the coefficients for the first series are $a_n^{[1]}$ and $b_n^{[1]}$, and the coefficients for the second series are $a_n^{[2]}$ and $b_n^{[2]}$ respectively, then which of the following is correct?

(a) $a_2^{[1]} = \frac{1}{2}\ and\ b_2^{[2]} = \frac{-1}{2}$

(b) $b_2^{[1]} = \frac{1}{2}\ and\ a_2^{[2]} = \frac{-1}{2}$

(c) $a_2^{[1]} = \frac{1}{2}\ and\ a_2^{[2]} = \frac{-1}{2}$

(d) $b_2^{[1]} = \frac{1}{2}\ and\ b_2^{[2]} = \frac{-1}{2}$

ANSWER KEY			
1	**2**	**3**	**4**
(c)	(b)	(d)	(c)

SOLUTIONS

1. Solution:

Calculating $a_o = \frac{1}{2\pi}\int_0^{\pi}(1)\,dx = \frac{1}{2\pi}.\pi$

$a_o = \frac{1}{2}$

$a_n = \frac{1}{\pi}\int_0^{\pi}(1)\cos nx\,dx$

$a_n = \frac{1}{n\pi}[\sin nx]_0^x$

$a_n = 0$

$b_n = \frac{1}{\pi}\int_0^{\pi}(1)\sin nx\,dx$

$b_n = -\frac{1}{n\pi}[cosnx]_0^x$

$b_n = \left(-\frac{1}{n\pi}\right)(-1-1)$

$b_n = \frac{2}{n\pi}$

Hence $\frac{b_1}{b_2} = \frac{2}{\pi} \times \frac{3\pi}{2}$

$\frac{b_1}{b_2} = 3$

Hence, option (c) is correct.

2. Solution: The given function $f(x) = |sinx|$ is an even function.

Therefore, all the B'_n s will be 0.

Now $\alpha_n = \frac{2}{\pi}\int_0^{\pi}|sinx|\,cosnx\,dx$

$\alpha_n = \frac{2}{\pi}\int_0^{\pi} sinx.cosnxdx$

$\alpha_n = \frac{2}{2\pi}\int_0^{\pi}[\sin(n+1)x - \sin(n-1)x]\,dx$

$\alpha_n = \frac{1}{\pi}\left[\frac{-\cos(n+1)x}{(n+1)} + \frac{\cos(n-1)x}{(n-1)}\right]_0^x$

For odd n,

$\alpha_n = \frac{1}{\pi}\left[-\frac{1}{(n+1)} + \frac{1}{(n-1)} + \frac{1}{(n+1)} - \frac{1}{(n-1)}\right]$

$\Rightarrow \alpha_n = 0$, for oddn.

NOW, For even n

$\alpha_n = \frac{1}{\pi}\left[\frac{1}{(n+1)} + \frac{1}{(n-1)} + \frac{1}{(n+1)} - \frac{1}{(n-1)}\right]$

$\Rightarrow \alpha_n = \frac{2}{\pi}\left[\frac{1}{(n+1)} - \frac{1}{(n-1)}\right]$

$\alpha_n = \frac{2}{\pi}\left[\frac{n-1-n-1}{(n^2-1)}\right]$

$\alpha_n = \frac{4}{\pi(n^2-1)}$

Therefore for even n, α_n is nonzero.

Hence, option (b) is correct.

3. Solution:

a_2 is the coefficients of $cos2x$.

Therefore, $a_2 = \frac{1}{\pi}\left[\int_{-\pi}^{0} 0.cos2xdx + \int_{o}^{\pi} sinx.cos3xdx\right]$

$\Rightarrow a_2 = \frac{1}{2\pi}\int_0^{\pi}(sin3x - sinx)\,dx$

$a_2 = \frac{1}{2\pi}\left[-\frac{cos3x}{3} + cosx\right]_0^{\pi}$

$a_2 = \frac{1}{2\pi}\left[\left(-\frac{cos3x}{3} + cos\pi\right) - \left(-\frac{1}{3} + 1\right)\right]$

$a_2 = \frac{1}{2\pi}\left[\frac{1}{3} - 1 + \frac{1}{3} - 1\right]$

$a_2 = -\frac{2}{3\pi}$

Hence, option (d) is correct.

4. Solution:

Given that $f_1(x) = \frac{1}{2} + \frac{1}{2}cos2x$ *and* $f_2(x) = \frac{1}{2} - \frac{1}{2}cos2x$

Hence, $a_2^{(1)} = \frac{1}{2}$ and $a_2^{(2)} = -\frac{1}{2}$

Since the given functions are odd so all the b_n s of each of the series are zero. There is no sine terms in any of the two given functions.

Hence, option (c) is correct.

VECTOR ALGEBRA, VECTOR CALCULUS

PREVIOUS YEAR EXAM QUESTIONS

(CUET PG 2022)

1. Find $\vec{a} \times (\vec{b} \times \vec{c})$ Where $\vec{a} = \hat{\imath} + \hat{\jmath} - \hat{k}$, $\vec{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$, $\vec{c} = \hat{\imath} - \hat{\jmath} - \hat{k}$

(a) $\hat{\imath} - \hat{\jmath}$ (b) $2\hat{\imath} - 2\hat{\jmath}$

(c) $2\hat{\jmath} - 2\hat{\imath}$ (d) $\hat{\jmath} - \hat{\imath}$

(CUET PG 2022)

2. Which of the statements given below are correct?

(A) If the dot product of two vectors is zero, then vectors are perpendicular to each other.

(B) Work done is a dot product of force and displacement.

(C) Vector product is commutative.

(D) Vector product is associative with respect to a scalar.

(E) The scalar product of two non-zero vectors is always positive.

Choose the correct answer from the options given below:

(a) A and C (b) B and C

(c) A, B and D (d) C and E

(CUET 2023)

3. Given below are two statements, one is labelled as Assertion (A) and the other is labelled as Reason (R):

Assertion (A): A given vector F is irrational i.e., $\vec{V} \times \vec{F} = 0$.

Reason (R): The vector $\vec{F}$ is conservative.

In the light of the above statements, choose the most appropriate answer from the options given below.

(a) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(b) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

(c) (A) is correct but (R) is not correct

(d) (A) is not correct but (R) is correct

(CUET-PG 2024)

4. The vector $r^n\vec{r}$ is solenoidal, if

(a) $n = 3$ (b) n = 1

(c) $n = -1$ (d) $4\,n = 0$

(CUET-PG 2024)

5. If $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ is a position vector.

Match List I with List II

List I		List II	
A.	div $\vec{r}$	I.	$-\frac{\vec{r}}{\lvert\vec{r}\rvert^3}$
B.	curl $\vec{r}$	II.	3
C.	grad $\lvert\vec{r}\rvert$	III.	0
D.	Grad $\left\lvert\frac{1}{\vec{r}}\right\rvert$	IV.	$\frac{\vec{r}}{\lvert\vec{r}\rvert}$

Choose the correct answer from the options given below:

(a) (A) - (III). (B) - (II). (C) - (I), (D) - (IV)

(b) (A) - (III). (B) - (II). (C) - (IV). (D) - (I)

(c) (A) - (II). (B) - (III). (C) - (IV), (D) - (I)

(d) (A) - (II). (B) - (III), (C) - (I). (D) - (IV)

(CUET PG 2025)

6. If $\vec{A} = \vec{\nabla}\phi$ and $\phi = xy + yz + zx$, then the true statements are:

A. $\vec{\nabla} \cdot \vec{A} = 0$ B. $\vec{\nabla} \cdot \vec{A} \neq 0$

C. $\vec{\nabla} \times \vec{A} = 0$ D. $\vec{\nabla} \times \vec{A} \neq 0$

Choose the correct answer from the option given below:

(a) A and C only (b) A and D only

(c) B and D only (d) B and C only

(CUET PG 2025)

7. The projection of vector $\vec{A} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$ on vector $\vec{B} = 4\hat{\imath} - 4\hat{\jmath} + 7\hat{k}$ is:

(a) $\frac{17}{9}$ (b) $\frac{17}{7}$

(c) $\frac{19}{7}$ (d) $\frac{19}{9}$

(CUET PG 2025)

8. If $|\vec{A}+\vec{B}|=|\vec{A}-\vec{B}|$ then the angle between vectors $\vec{A}$ and $\vec{B}$ is:

(a) 0 (b) $\pi/4$

(c) $\pi/2$ (d) $3\pi/4$

(CUET PG 2026)

9. Match List - I with List - II.

List - I	List - II
A. $\vec{\nabla}\cdot\vec{A}=0$	I. $\vec{A}$ is conservative
B. $\oint \vec{A}\cdot\overrightarrow{dr}=0$	II. $\vec{A}$ is irrotational
C. $\vec{A}=\vec{\nabla}_{\phi}$	III. $\vec{A}$ is solenoidal
D. $\vec{\nabla}\times\vec{A}=0$	IV. $\vec{A}$ points in the direction of maximum increase of the function ϕ

Choose the correct answer from the options given below:

(a) A-III, B-II, C-I, D-IV (b) A-II, B-III, C-IV, D-I

(c) A-IV, B-III, C-I, D-II (d) A-III, B-I, C-IV, D-II

(CUET PG 2026)

10. The temperature of the points in space is given by $T(x,y,z)=x^3+y^2+z$. A mosquito located at $(0,1,2)$ desires to fly in such a direction that it will get warm as soon as possible. In what direction should it move?

(a) $\frac{1}{\sqrt{2}}(2\hat{j}+\hat{k})$ (b) $\frac{1}{\sqrt{5}}(\hat{j}+2\hat{k})$

(c) $\frac{1}{\sqrt{5}}(2\hat{j}+\hat{k})$ (d) $\frac{1}{\sqrt{14}}(3\hat{i}+2\hat{j}+\hat{k})$

(CUET PG 2026)

11. $\oint_C (2xyz\,dx+x^2z\,dy+2y\,dy+x^2y\,dz)$, where C is the curve $x^2+y^2+z^2=9$

(a) 3 (b) 6

(c) 9 (d) 0

(CUET PG 2026)

12. Find the volume of parallelopiped if $\vec{a}=-3\hat{i}+7\hat{j}+5\hat{k}$, $\vec{b}=-3\hat{i}+7\hat{j}-3\hat{k}$ and $\vec{c}=7\hat{i}-5\hat{j}-3\hat{k}$ are the three co-terminous edge of the parallelopiped.

(a) 68 units (b) 272 units

(c) 148 units (d) 488 units

(CUET PG 2026)

13. The ratio of volume of tetrahedron having $\vec{A}+\vec{B}$, $\vec{B}+\vec{C}$, $\vec{C}+\vec{A}$ as concurrent edge to that of volume of tetrahedron having $\vec{A},\vec{B},\vec{C}$ as concurrent edges is:

(a) $\frac{4}{3}$ (b) $\frac{2}{1}$

(c) $\frac{8}{1}$ (d) $\frac{4}{1}$

ANSWER KEY							
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
(c)	(c)	(a)	(a)	(c)	(a)	(d)	(c)
9	**10**	**11**	**12**	**13**			
(d)	(c)	(d)	(b)	(b)			

SOLUTIONS

1. Solution: Determent of $(\vec{b}\times\vec{c})$ is

$$|\vec{b}\times\vec{c}|=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&-1&-1\\1&-1&-1\end{vmatrix}$$

$=\hat{i}(1+1)-\hat{j}(-1-1)+\hat{k}(-1+1)$

$=2\hat{i}+2\hat{j}$

Now determinant of $\vec{a}\times(\vec{b}\times\vec{c})$ is

$$|\vec{a}\times(\vec{b}\times\vec{c})|=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&1&-1\\2&2&0\end{vmatrix}$$

$=\hat{i}(0+2)-\hat{j}(0+2)+\hat{k}(2-2)$

$=2\hat{i}-2\hat{j}$

Hence option (c) is correct.

2. Solution:

$\vec{A}.\vec{B}=0=ABcos\theta=ABcos90^0=0$, Then vectors are perpendicular to each other.

$\int dw=F.dx$

$\vec{A}\times\vec{B}=-\vec{B}\times\vec{A}$, Vectors are not commutative.

$$\vec{A}(\vec{B}\times\vec{C})=\vec{B}(\vec{C}\times\vec{A})=\vec{C}(\vec{A}\times\vec{B})$$

Acute angle is positive and obtuse angle is negative

So, A, B and D are correct Statement.

Hence, option (c) is correct.

3. Solution: If a vector is conservative then its curl will be zero i.e., it is irrotational. Hence option

Hence, option (a) is correct.

4. Solution:- Given that $f=\boldsymbol{r^n\vec{r}}$

$\nabla.f=0$

$\nabla(r^n\vec{r})\Rightarrow\overline{r}=x\hat{i}+y\hat{j}+z\hat{k}$

$|r|=(x^2+y^2+z^2)^{1/2}$

$r^n=(x^2+y^2+z^2)^{n/2}$

$r^n\vec{r}=(x^2+y^2+z^25)^{n/2}\left(x\hat{i}+y\hat{j}+z\hat{k}\right)$

$\overline{\nabla}(r^n\vec{r})=\left(\hat{i}\frac{\partial}{\partial x}+\hat{j}\frac{\partial}{\partial x}+\hat{k}\frac{\partial}{\partial x}\right)(x^2+y^2+z^2)^{n/2}\left(x\hat{i}+y\hat{j}+z\hat{k}\right)$

$\overline{\nabla}(r^n\vec{r})=\frac{\partial}{\partial x}\left[(x^2+y^2+z^2)^{n/2}\,x\right]+\frac{\partial}{\partial y}\left[(x^2+y^2+z^2)^{n/2}\,y\right]+\frac{\partial}{\partial z}\left[(x^2+y^2+z^2)^{n/2}\,z\right]$

$=(x^2+y^2+z^2)^{n/2}[nx^2(x^2+y^2+z^2)^{-1}+1]+(x^2+y^2+z^2)^{n/2}[ny^2(x^2+y^2+z^2)^{-1}+1]+(x^2+y^2+z^2)^{n/2}[nz^2(x^2+y^2+z^2)^{-1}+1]$

$=(x^2+y^2+z^2)^{n/2}[n(x^2+y^2+z^2)^{-1}(x^2+y^2+z^2)+3]$

$=(x^2+y^2+z^2)^{\frac{n}{2}}(n+3)$

$Solenoid=\Delta.f=0$

$\nabla.(r^n\vec{r})=0\Rightarrow(n+3)\,(x^2+y^2+z^2)^{\frac{n}{2}}=0$

$\therefore(x^2+y^2+z^2)^{\frac{n}{2}}\neq0$

$n+3=0\Rightarrow n=-3$

Hence, option (a) is correct.

5. Solution:-

A. Given that the position vector $\Rightarrow \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\nabla.\left(x\hat{\imath} + y\hat{\jmath} + z\hat{k}\right) = \frac{\partial}{\partial x}(x) + \frac{\partial}{\partial y}(y) + \frac{\partial}{\partial z}(z) \Rightarrow 1 + 1 + 1 = 3$

Divergence of any position vector $= 3$

B. Curl $\vec{r} = 0 \Rightarrow$The curl ∇ × r → of a position vector (where represents the vector differential operator) can be calculated as follows: curl ∇ × r →0. This zeroes out because a position vector field doesn't rotate around its points. The field flows in a direct or linear manner out of the origin, not circulating around it in a swirl.

C. grad $|\vec{r}| \Rightarrow |\vec{r}| = \frac{1}{\sqrt{x^2+y^2+z^2}} \Rightarrow \nabla.|\vec{r}| = \frac{2.x.\hat{x}}{2\sqrt{x^2+y^2+z^2}} + \frac{2.y.\hat{y}}{2\sqrt{x^2+y^2+z^2}} + \frac{2.z.\hat{z}}{2\sqrt{x^2+y^2+z^2}} \Rightarrow \frac{x.\hat{x}+y\hat{y}+z\hat{z}}{\sqrt{x^2+y^2+z^2}} = \frac{\vec{r}}{|\vec{r}|}$

D. Grad $\left|\frac{1}{\vec{r}}\right| = -\frac{\vec{r}}{|\vec{r}|^3}$

Hence, option (c) is correct.

6. Solution: - (a)

$\vec{A} = \vec{\nabla}\phi$, $\phi = xy + yz + zx$

$\vec{A} = (y + z)\hat{\imath} + (x + z)\hat{\jmath} + (y + x)\hat{k}$

A. $\nabla \cdot \vec{A} = 0 + 0 + 0 = 0$ TRUE

C. $\vec{A} = \nabla\phi$ is a gradient field (irrotational),

so $\nabla \times \vec{A} = 0$ TRUE

Hence, option (a) is correct.

7. Solution: - (d)

Projection of $\vec{A}$ on $\vec{B}$

$\vec{A} \cdot \vec{B} = (1)(4) + (-2)(-4) + (1)(7) = 4 + 8 + 7 = 19$

$|\vec{B}| = \sqrt{16 + 16 + 49} = \sqrt{81} = 9$

Projection $= \frac{\vec{A} \cdot \vec{B}}{|\vec{B}|} = \frac{19}{9}$.

Hence, option (d) is correct.

8. Solution: - (c)

$|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$

Squaring both sides:

$A^2 + 2\vec{A} \cdot \vec{B} + B^2 = A^2 - 2\vec{A} \cdot \vec{B} + B^2$

$4\vec{A} \cdot \vec{B} = 0 \Rightarrow \cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}$

Hence, option (c) is correct.

9. Solution: (d)

A. $\nabla^{\rightarrow} \cdot A^{\rightarrow} = 0$ → This means the divergence of A is zero, so A is solenoidal. A → III

B. $\oint A^{\rightarrow} \cdot (dr)^{\rightarrow} = 0$ → The line integral over a closed path is zero, which implies A is conservative. B → I

C. $A^{\rightarrow} = \nabla^{\rightarrow}\phi$ → This means A is the gradient of a scalar function ϕ. The gradient points in the direction of the maximum increase of ϕ .C → IV

D. $\nabla^{\rightarrow} \times A^{\rightarrow} = 0$ → The curl of A is zero, so A is irrotational. D → II

So, the correct matching is: A-III, B-I, C-IV, D-II That corresponds to option (4).

10. Solution: (c)

$T(x, y, z) = x^3 + y^2 + z$

$\frac{\partial T}{\partial x} = 3x^2, \frac{\partial T}{\partial y} = 2y, \frac{\partial T}{\partial z} = 1$

So, $\nabla T = 3x^2\,\hat{\imath} + 2y\,\hat{\jmath} + 1\,\hat{k}$

$\nabla T(0,1,2) = 3(0)^2\,\hat{\imath} + 2(1)\,\hat{\jmath} + 1\,\hat{k} = 0\,\hat{\imath} + 2\,\hat{\jmath} + 1\,\hat{k}$

So, the direction vector is: $2\hat{\jmath} + \hat{k}$

Magnitude: $|2\hat{\jmath} + \hat{k}| = \sqrt{2^2 + 1^2} = \sqrt{5}$

Unit vector: $\frac{1}{\sqrt{5}}(2\hat{\jmath} + \hat{k})$

Hence, option (3) is correct.

11. Solution:- (d)

$\oint_C (2xyz\,dx + x^2z\,dy + 2y\,dy + x^2y\,dz)$

$= \oint_C (2xyz\,dx + x^2z\,dy + x^2y\,dz) + \oint_C 2y\,dy$

$2y\,dy = d(y^2)$ (exact differential) and $\oint_C d(y^2) = 0$ (closed curve)

$\vec{F} = (2xyz, x^2z, x^2y)$

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial_x & \partial_y & \partial_z \\ 2xyz & x^2z & x^2y \end{vmatrix}$$

$\hat{x}(x^2 - x^2) - \hat{y}(2xy - 2xy) + \hat{z}(2xz - 2xz) = \vec{0}$

Since $\nabla \times \vec{F} = \vec{0}$, the field is conservative.

Conservative field over closed curve: $\oint_C \vec{F} \cdot d\vec{l} = 0$

Hence, option (4) is correct.

12. Solution:- (b)

The volume of the parallelepiped $= |\vec{a} \cdot (\vec{b} \times \vec{c})|$

$\vec{a} = -3\hat{\imath} + 7\hat{\jmath} + 5\hat{k}$, $\vec{b} = -3\hat{\imath} + 7\hat{\jmath} - 3\hat{k}$, $\vec{c} = 7\hat{\imath} - 5\hat{\jmath} - 3\hat{k}$

$$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -3 & 7 & -3 \\ 7 & -5 & -3 \end{vmatrix}$$

$= \hat{\imath}\begin{vmatrix} 7 & -3 \\ -5 & -3 \end{vmatrix} - \hat{\jmath}\begin{vmatrix} -3 & -3 \\ 7 & -3 \end{vmatrix} + \hat{k}\begin{vmatrix} -3 & 7 \\ 7 & -5 \end{vmatrix}$

$= \hat{\imath}(7(-3) - (-3)(-5)) - \hat{\jmath}((-3)(-3) - (-3)(7)) + \hat{k}((-3)(-5) - (7)(7))$

$= \hat{\imath}(-21 - 15) - \hat{\jmath}(9 + 21) + \hat{k}(15 - 49)$

$= -36\hat{\imath} - 30\hat{\jmath} - 34\hat{k}$

$\vec{a} \cdot (\vec{b} \times \vec{c}) = (-3)(-36) + (7)(-30) + (5)(-34)$

$= 108 - 210 - 170$

$= 108 - 380 = -272$

Volume $= |-272| = 272$

Hence, option (2) is correct.

13. Solution: (b)

$V = \frac{1}{6}|\vec{u} \cdot (\vec{v} \times \vec{w})|$

Edges: $\vec{A}, \vec{B}, \vec{C}$ then $V_1 = \frac{1}{6}\left|\vec{A} \cdot (\vec{B} \times \vec{C})\right|$

Edges: $\vec{A}+\vec{B}, \vec{B}+\vec{C}, \vec{C}+\vec{A}$ then $V_2 = \frac{1}{6}\left|(\vec{A}+\vec{B})\cdot((\vec{B}+\vec{C})\times(\vec{C}+\vec{A}))\right|$

First, compute: $(\vec{B}+\vec{C})\times(\vec{C}+\vec{A}) = \vec{B}\times\vec{C}+\vec{B}\times\vec{A}+\vec{C}\times\vec{C}+\vec{C}\times\vec{A}$

$= \vec{B}\times\vec{C}+\vec{B}\times\vec{A}+\vec{C}\times\vec{A}$

$= \vec{B}\times\vec{C}-\vec{A}\times\vec{B}-\vec{A}\times\vec{C}$

$(\vec{A}+\vec{B})\cdot(\vec{B}\times\vec{C}-\vec{A}\times\vec{B}-\vec{A}\times\vec{C}) = \vec{A}\cdot(\vec{B}\times\vec{C})-\vec{A}\cdot(\vec{A}\times\vec{B})-\vec{A}\cdot(\vec{A}\times\vec{C})+\vec{B}\cdot(\vec{B}\times\vec{C})-\vec{B}\cdot(\vec{A}\times\vec{B})-\vec{B}\cdot(\vec{A}\times\vec{C})$

$= \vec{A}\cdot(\vec{B}\times\vec{C})-\vec{B}\cdot(\vec{A}\times\vec{C})$

Now, $\vec{B}\cdot(\vec{A}\times\vec{C}) = \vec{A}\cdot(\vec{C}\times\vec{B}) = -\vec{A}\cdot(\vec{B}\times\vec{C})$.

Thus:$= \vec{A}\cdot(\vec{B}\times\vec{C})-(-\vec{A}\cdot(\vec{B}\times\vec{C})) = 2\,\vec{A}\cdot(\vec{B}\times\vec{C})$

$V_2 = \frac{1}{6}|2\,\vec{A}\cdot(\vec{B}\times\vec{C})| = \frac{1}{6}\cdot 2\left|\vec{A}\cdot(\vec{B}\times\vec{C})\right|$

$V_1 = \frac{1}{6}\left|\vec{A}\cdot(\vec{B}\times\vec{C})\right|$

So, $\frac{V_2}{V_1} = \frac{2}{1}$

Hence, option (2) is correct.

PRACTICE QUESTIONS

1. Which of the following fields has non-zero curl?

(a) $y^2\hat{\imath}+(2xy+z^2)\hat{\jmath}+2yz\hat{k}$

(b) $(y+z)\hat{\imath}+(x+z)\hat{\jmath}+(x+y)\hat{k}$

(c) $x\hat{\imath}+y\hat{\jmath}+z\hat{k}$

(d) $xy\hat{\imath}+2yz\hat{\jmath}+3xz\hat{k}$

2. A unit vector perpendicular to the plane containing $\vec{A} = \hat{\imath}+\hat{\jmath}-2\hat{k}$ and $\vec{B} = 2\hat{\imath}-\hat{\jmath}+\hat{k}$ is

(a) $\frac{1}{\sqrt{26}}(-\hat{\imath}+3\hat{\jmath}-4\hat{k})$ (b) $\frac{1}{\sqrt{19}}(-\hat{\imath}+3\hat{\jmath}-3\hat{k})$

(c) $\frac{1}{\sqrt{35}}(-\hat{\imath}+5\hat{\jmath}-3\hat{k})$ (d) $\frac{1}{\sqrt{35}}(-\hat{\imath}-5\hat{\jmath}-3\hat{k})$

3. Identify the CORRECT statements for the following vectors $\bar{a} = 3\hat{\imath}+2\hat{\jmath}$ and $\bar{b} = \hat{\imath}+2\hat{\jmath}$.

(a) The vectors $\bar{a}$ and $\bar{b}$ are linearly independent.

(b) The vectors $\bar{a}$ and $\bar{b}$ are linearly dependent.

(c) The vectors $\bar{a}$ and $\bar{b}$ are orthogonal.

(d) The vectors $\bar{a}$ and $\bar{b}$ are normalized.

4. The unit vector normal to the surface $x^2+y^2-z=1$ at the point $P(1,1,1)$ is

(a) $\frac{\hat{\imath}+\hat{\jmath}-\hat{k}}{\sqrt{3}}$ (b) $\frac{2\hat{\imath}+\hat{\jmath}-\hat{k}}{\sqrt{6}}$

(c) $\frac{\hat{\imath}+2\hat{\jmath}-\hat{k}}{\sqrt{6}}$ (d) $\frac{2\hat{\imath}+2\hat{\jmath}-\hat{k}}{\sqrt{6}}$

5. Identify the CORRECT statements for the following vectors $\bar{a} = 3\hat{\imath}+2\hat{\jmath}$ and $\bar{b} = \hat{\imath}+2\hat{\jmath}$.

(a) The vectors $\bar{a}$ and $\bar{b}$ are linearly independent.

(b) The vectors $\bar{a}$ and $\bar{b}$ are linearly dependent.

(c) The vectors $\bar{a}$ and $\bar{b}$ are orthogonal.

(d) The vectors $\bar{a}$ and $\bar{b}$ are normalized.

ANSWER KEY				
1	2	3	4	5
(d)	(d)	(a)	(d)	(a)

SOLUTIONS

1. Solution:

Here we will check curl of each function

(a) $\nabla\times E = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix}$

$= i(0-0)-j(0-0)+\hat{k}(0-0) = 0$

(b) $\nabla\times E = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y+z & x+z & x+z \end{vmatrix}$

$= i(1-1)-j(1-1)+\hat{k}(1-1) = 0$

(c) $\nabla\times E = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 & (2xy+z^2) & 2yz \end{vmatrix}$

$= \hat{\imath}(2z-2z)-\hat{\jmath}(0-0)+\hat{k}(2y-2y) = 0$

(d) $\nabla\times E = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & 2yz & 3xz \end{vmatrix}$

$= i(0-2y)-j(3z-0)+\hat{k}(0-x) \neq 0$

Hence, option (d) is correct.

2. Solution:

Method 1: Since $\vec{A}.\hat{n} = 0$ and $\vec{B}.\hat{n} = 0$

Checking the above conditions *only* for option (d), *the given conditions are satisfied* :

$\vec{A}.\hat{n} = \frac{1}{\sqrt{35}}(-1-5-6) = 0$ and $\vec{B}.\hat{n} = \frac{1}{\sqrt{35}}(-2+5-3) = 0$

Method 2: $\vec{A}\times\vec{B}\times\vec{C}$ =0 will also give us the condition for the normal vector, where $\vec{C}$ *is the normal vector*

Method 3: $\vec{A}\times\vec{B}$ will give normal vector, so dividing it by it's magnitude will give the unit normal vector

Hence, option (d) is correct.

3. Solution: Here, given vectors are $\vec{a} = 3\hat{\imath}+2\hat{\jmath}$, $\vec{b} = \hat{\imath}+2\hat{\jmath}$

$\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 1 & 2 \end{vmatrix} \neq 0$

Therefore $\vec{a}$ and $\vec{b}$ are linearly independent

Further, $\vec{a}.\vec{b} \neq 0$, $\vec{a}$ and $\vec{b}$ are not orthogonal

$|\vec{a}| \neq, |\vec{b}| \neq 1$ Not normalized

Hence, option (a) is correct.

4. Solution:

We know that,

Unit vector normal to the surface

$\hat{n} = \frac{\vec{\nabla}\phi}{|\vec{\nabla}\phi|}\Big|_{(11,1)} = \frac{2\hat{\imath}+2\hat{\jmath}-\hat{k}}{3}$

Hence, option (d) is correct.

5. Solution:

Given vectors are $\vec{a} = 3\hat{\imath} + 2\hat{\jmath},\ \vec{b} = \hat{\imath} + 2\hat{\jmath}$

$\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 1 & 2 \end{vmatrix} \neq 0$

Therefore $\vec{a}$ and $\vec{b}$ are linearly independent

Further, $\vec{a}.\vec{b} \neq 0$, $\vec{a}$ and $\vec{b}$ are not orthogonal

$|\vec{a}| \neq, |\vec{b}| \neq 1$ Not normalized

Hence, option (a) is correct.

MULTIPLE INTEGRALS, DIVERGENCE THEOREM, GREEN'S THEOREM, STOKES' THEOREM.

PREVIOUS YEAR EXAM QUESTIONS

(CUET PG 2022)

1. A fluid motion given by $\vec{v} = (1+z)\hat{\imath} + (z+x)\hat{\jmath} + (x+y)\hat{k}$ is

(a) Solenoidal

(b) Rotational

(c) both solenoidal and rotational

(d) Irrotational

(CUET PG 2023)

2. The line integral per unit area along the boundary of small area around a point in vector field $\vec{A}$ is called

(a) grad $\vec{A}$ (b) div $\vec{A}$

(c) curl $\vec{A}$ (d) line integral $\vec{A}$

(CUET PG 2023)

3. What is the value of $\mathrm{div}\ \vec{r}$, if $\vec{r}$ is the position vector of a particle?

$(\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k})$

(a) 1 (b) 2

(c) 3 (d) Zero

(CUET PG 2023)

4. If $\vec{r}$ is the position vector of any point on a surface S that encloses the volume V, then find

$\iint_S \vec{r} \cdot d\vec{S}$

(a) 1 V (b) 2V

(c) 3V (d) 4V

(CUET-PG 2024)

5. If $\vec{A} = ax\hat{\imath} + by\hat{\jmath} + cz\hat{k}$ where $a, b, c,$ are constant, then $\iint_S \vec{A}.d\vec{S}$ where S is the surface of a unit sphere, is

(a) $\frac{4}{3}\pi(a+b+c)^2$ (b) $\frac{4}{3}\pi(a+b+c)$

(c) 0 (d) $\frac{4}{3}\pi(a^2+b^2+c^2)$

(CUET-PG 2025)

6. If $\vec{A} = \vec{\nabla} \times \vec{F}$, then $\iint_S \vec{A}.\hat{n}dS$ (for any closed surface S) is:

(a) 0 (b) 4S

(c) 3S (d) 4V

(CUET-PG 2026)

7. Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A): For closed surface S, $\iint_S \vec{f} \cdot \hat{\mathbf{n}}\mathbf{ds} = \iiint_V \mathbf{div}\,\vec{f}\,\mathbf{d}v$

Reason (R): $\oint \vec{F} \cdot d\vec{r} = \iint \mathbf{div}\,\vec{r} \cdot \hat{n}\,ds$

In the light of the above statements, choose the most appropriate answer from the options given below:

(a) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(b) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

(c) (A) is correct but (R) is not correct

(d) (A) is not correct but (R) is correct

ANSWER KEY						
1	2	3	4	5	6	7
(c)	(c)	(c)	(c)	(b)	(a)	(c)

SOLUTIONS

1. Solution:

A fluid motion given by

$\vec{v} = (t+z)\hat{\imath} + (z+x)\hat{\jmath} + (x+y)\hat{k}$

If $\vec{\nabla}.\vec{v} = 0$ then $\vec{v}$ is called Solenoidal Vector.

$\vec{\nabla}.\vec{v} = 0+0+0$, $\vec{v}$ is solenoidal Vector.

If $\vec{\nabla} \times \vec{v} = 0\ then\ \vec{v}$ is called irrotational Vector

$$\vec{\nabla} \times \vec{v} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (t+z) & (z+x) & (x+y) \end{vmatrix}$$

$$= \hat{\imath}(0+1) - \hat{\jmath}(1-1) + \hat{k}(1+0)$$

$\hat{\imath} - \hat{k} \neq 0, So\ \ \vec{v}\ is\ rotational$

So, $\vec{v}$ is Solenodial and rotational.

Hence option (3) is correct.

2. Solution: The line integral per unit area along the boundary of small area around a point in vector field $\vec{A}$ is called curl$\vec{A}$.

Hence, option (c) is correct.

3. Solution:

$div\ \vec{r} = \vec{\nabla}.\vec{r} = 3$

Hence, option (c) is correct.

4. Solution: By Gauss Divergence Theorem, $\iint_S \vec{r} \cdot d\vec{S} = \int \vec{\nabla}.\vec{r}\ d\tau = 3V$

Hence, option (c) is correct.

5. Solution: - surface of sphere $= x^2 + y^2 + z^2 = 1$

Gauss theorem, $\int_s F.\hat{n}\ ds = \iiint_V \mathrm{div}.F\ dv$

$\int_s (ax\hat{i} + by\hat{j} + cz\hat{k})\ \hat{n}\, ds = \int_V \nabla.(ax\hat{i} + by\hat{j} + cz\hat{k})\, dv$

$= \left(\hat{i}\ \frac{\partial}{\partial x} + \hat{j}\ \frac{\partial}{\partial y} + \hat{k}\ \frac{\partial}{\partial z}\right).(ax\hat{i} + by\hat{j} + cz\hat{k})dv$

$= (a + b + c) \int_V dv.$

We know that the volume of sphere $= \frac{4}{3}\pi r^3$

$\therefore x^2 + y^2 + z^2 = 1 \Rightarrow r = 1$

$(a + b + c) \times \frac{4}{3}\pi(1)$

$= \frac{4}{3}\pi(a + b + c)$

Hence, option (b) is correct.

6. Solution: - (a)

By the Divergence Theorem:

$$\iint_S \vec{A} \cdot \hat{n}\, dS = \iiint_V \nabla \cdot \vec{A}\, dV = \iiint_V \nabla \cdot (\nabla \times \vec{F})\, dV = 0$$

Since divergence of a curl is always zero.

Hence, option (a) is correct.

7. Solution:- (c)

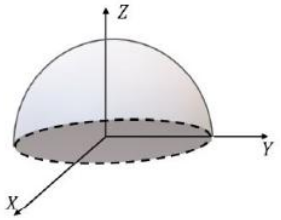
Assertion (A): $\iint_S \vec{f} \cdot \hat{n}\, ds = \iiint_V \text{div}\, \vec{f}\, dv$ This is the Gauss Divergence Theorem — a standard result relating the flux of a vector field through a closed surface to the volume integral of its divergence. Hence (A) is correct

Reason (R): $\oint \vec{F} \cdot d\vec{r} = \iint \text{div}\, \vec{F} \cdot \hat{n}\, ds$. The left side is a line integral over a closed curve. The correct theorem relating a line integral to a surface integral is Stokes' Theorem: $\oint_C \vec{F} \cdot d\vec{r} = \iint_S (\nabla \times \vec{F}) \cdot \hat{n}\, ds$. Hence (R) is incorrect.

Hence, option (3) is correct.

PRACTICE QUESTIONS

1. For a given vector$\vec{F} = -y\hat{i} + z\hat{j} + x^2\hat{k}$, the surface integral $\int_S (\vec{\nabla} \times \vec{F}) \cdot \hat{r} dS$ over the surface S of a hemisphere of radius R with the centre of the base at the origin is



(a) πR^2 (b) $-\frac{2\pi R^2}{3}$

(c) πR^2 (d) $\frac{2\pi R^2}{3}$

2. The volume integral $\int_v e^{-\left(\frac{r}{R}\right)^2} \vec{\nabla} \cdot \left(\frac{\hat{r}}{r^2}\right) d^3r$, where V is the volume of a sphere of radius R centered at the origin, is equal to

(a) 4π (b) 0

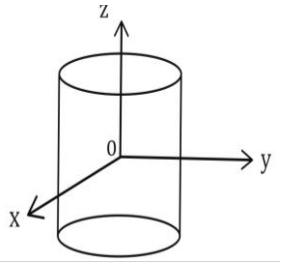
(c) $\frac{4}{3}\pi R^3$ (d) 1

3. Given $\overline{F} = \bar{r} \times \overline{B}$ where $\overline{B} = B_0(\hat{i} + \hat{j} + \hat{k})$ is a constant vector and $\bar{r}$ is the position vector. The value of $\oint \overline{F}.d\bar{r}$ where C is circle of unit radius centered at origin is

(a) 0 (b) $2\pi B_0$

(c) $-2\pi B_0$ (d) 1

4. Consider a cylinder of height h and radius a a closed at both ends, centered at the origin. Let $\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$ be the position vector and $\hat{n}$ a unit vector normal to the surface. The surface integral $\vec{r}.\hat{n}\, ds$ over the closed surface of the cylinder is



(a) $2\pi a^2(a + h)$ (b) $3\pi a^2 h$

(c) $2\pi a^2 h$ (d) Zero

5. Given $\overline{F} = \bar{r} \times \overline{B}$ where $\overline{B} = B_0(\hat{i} + \hat{j} + \hat{k})$ is a constant vector and $\bar{r}$ is the position vector. The value of $\oint \overline{F}.d\bar{r}$ where C is circle of unit radius centered at origin is

(a) 0 (b) $2\pi B_0$

(c) $-2\pi B_0$ (d) 1

ANSWER KEY				
1	2	3	4	5
(a)	(a)	(c)	(b)	(c)

SOLUTIONS

1. Solution: Given that, $\vec{F} = -y\hat{i} + z\hat{j} + x^2\hat{k}$

$\therefore \int_s (\nabla \times F) \cdot \hat{r} ds$

$= \int_l \vec{F} \cdot d\vec{l}$

$= \int (-y\hat{i} + z\hat{j} + x^2\hat{k}) \cdot (dx\hat{i} + dy\hat{j})$

$= -\int y dx|_{z=0}$

$= -\int R \sin\theta\, R(-\sin\theta) d\omega$

$= +R^2 \int_0^{2\pi} \sin^2\theta\, d\omega$

$= \pi R^2$

Hence, option (a) is correct.

2. Solution:

Since, we know that: $\vec{\nabla}.\left(\frac{\hat{r}}{r^2}\right) = 4\pi\delta^3(\vec{r})$

Therefore, Volume integral is given by

$\iiint e^{-\left(\frac{r}{R}\right)^2} \vec{\nabla}.\left(\frac{\hat{r}}{r^2}\right) d^3r = \iiint e^{-\left(\frac{r}{R}\right)^2} 4\pi\delta^3(\vec{r})d^3r$

Also by the identity: $\iiint f(r)\,\delta^3(\vec{r}-\vec{r}')d^3r = f(\vec{r}')$

Therefore, $\iiint e^{-\left(\frac{r}{R}\right)^2} \vec{\nabla}.\left(\frac{\hat{r}}{r^2}\right) d^3r =$

$\iiint e^{-\left(\frac{r}{R}\right)^2} 4\pi\delta^3(\vec{r})d^3r = 4\pi e^{-\left(\frac{0}{r}\right)^2} = 4\pi$

Hence, option (a) is correct.

3. Solution: from given $\vec{F} = \vec{r} \times \vec{B}$

$\oint_C \vec{F}.\overrightarrow{dr} = \iint_s (\vec{\nabla} \times \vec{F}). = \iint [\vec{\nabla} \times (\vec{r} \times \vec{B})].\overrightarrow{dS}$

$= \iint_s [(\vec{B}.\vec{\nabla})\vec{r} - (\vec{r}.\vec{\nabla})\vec{B} + \vec{r}(\vec{\nabla}.\vec{B}) - B(\vec{\nabla}.\vec{r})].\overrightarrow{dS}$

$= \iint (\vec{B} - 0 + 0 - 3\vec{B}).d\vec{S} = \iint -2\vec{B}.d\vec{S} -= -2B_0\pi = -2\pi B_0$

Hence, option (c) is correct.

4. Solution:

By using divergence theorem we get

$\int_S \vec{r}\cdot\hat{n}dS = \int_V (\vec{\nabla}\cdot\vec{r})dV$

$= 3\int_V dV = 3V = 3\pi a^2 h$

Hence, option (b) is correct.

5. Solution:

From given function $\vec{F} = \vec{r} \times \vec{B}$

$\oint_C \vec{F}.\overrightarrow{dr} = \iint_s (\vec{\nabla} \times \vec{F}) = \iint [\vec{\nabla} \times (\vec{r} \times \vec{B})].\overrightarrow{dS}$

$= \iint_s [(\vec{B}.\vec{\nabla})\vec{r} - (\vec{r}.\vec{\nabla})\vec{B} + \vec{r}(\vec{\nabla}.\vec{B}) - B(\vec{\nabla}.\vec{r})].\overrightarrow{dS}$

$= \iint (\vec{B} - 0 + 0 - 3\vec{B}).d\vec{S} = \iint -2\vec{B}.d\vec{S} -= -2B_0\pi = -2\pi B_0$

Hence, option (c) is correct.

FIRST ORDER EQUATIONS AND LINEAR SECOND ORDER DIFFERENTIAL EQUATIONS WITH CONSTANT COEFFICIENTS.

PREVIOUS YEAR EXAM QUESTIONS

(CUET PG 2022)

1. Integrating factor of the differential equation, $\frac{dy}{dx} + Py = Q$ is given by

(a) I.F. $= \int_e Q dy$ (b) I.F. $= \int_e P dy$

(c) I.F. $= \int_e P dx$ (d) I.F. $= \int_e Q dx$

(CUET PG 2022)

2. Among the following equations, which is a homogeneous equation?

(a) $\frac{dy}{dx} = \frac{x^2+2y^2}{2x^2+3y^3}$ (b) $\frac{dy}{dx} = \frac{2x^3+y^3}{3x^2+2y^2}$

(c) $\frac{dy}{dx} = \frac{x^2+y^2}{3x^2+2y}$ (d) $\frac{dy}{dx} = \frac{x^2+2y^2}{2x^2+4y^2}$

(CUET PG 2022)

3. Given below are two statements:

Statement I: The order of a differential equation is the order of the highest differential coefficient present in the equation.

Statement II: The degree of a differential equation is the degree of the lowest derivative after removing radical sign and fraction.

In the light of the above statements, choose the most appropriate answer from the options given below:

(a) Both statement I and statement II are correct

(b) Both statement I and statement II are incorrect

(c) Statement I is correct but statement II is incorrect

(d) Statement I is incorrect but statement II is correct

(CUET PG 2022)

4. Match List I with List II:

List I (word)		List II (antonym)	
A.	Transverse vibration of a string	I.	$\frac{d^2y}{dt^2} = c^2\frac{\partial^2 y}{\partial x^2}$
B.	Two-dimensional heat flow	II.	$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$
C.	One-dimensional heat flow	III.	$\frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial x^2}$
D.	Two-dimensional heat flow in polar form	IV.	$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

Choose the correct answer from the options given below:

(a) A-IV, B-I, C-II, D-III (b) A-II, B-III, C-I, D-IV

(c) A-I, B-IV, C-III, D-II (d) A-III, B-II, C-IV, D-I

(CUET-PG 2024)

5. Arrange the following differential equations in ascending order in accordance to degree

(A) $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = \frac{d^2y}{dx^2}$

(B) $\left(\frac{dy}{dx}\right)^5 + y = 0$

(C) $\frac{d^2y}{dx^2} + a^2x = 0$

(D) $x^2\left(\frac{d^3y}{dx^3}\right)^3 + y\left(\frac{dy}{dx}\right)^4 + y^4 = 0$

Choose the correct answer from the options given below:

(a) (C), (A), (D), (B). (b) (B), (A), (C), (D).

(c) (B), (A), (D), (C). (d) (C), (D), (B), (A)

(CUET-PG 2024)

6. If $u = \log\frac{x^2}{y}$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is equal to

(a) $2u$ (b) u

(c) 0 (d) 1

(CUET-PG 2025)

7. For the differential equation $\left(1+\frac{d^2y}{dx^2}\right)^{\frac{2}{3}}=y\frac{d^2y}{dx^2}$ the order, degree and linearity respectively are:

(a) 3, 2 and non-linear (b) 2, 3 and non-linear

(c) 2, 3 and linear (d) 3, 2 and linear

(CUET-PG 2025)

8. For particular Integral, Match the LIST-I with LIST-II

	LIST-I		LIST-II
A.	$\frac{1}{D^2-1}x^2$	I.	xe^x
B.	$\frac{1}{D^2+D+1}\cos x$	II.	$\sin\ x$
C.	$\frac{1}{(D^2-1)^2}e^x$	III.	$\frac{x^2e^x}{2}$
D.	$\frac{1}{D^2-3D^2+4D-2}e^x$	IV.	$-(x^2+2x+2)$

Choose the correct answer from the options given below:

(a) A - I, B - II, C - III, D – IV (b) A - I, B - III, C - II, D - IV

(c) A - IV, B - II, C - III, D – I (d) A - IV, B - II, C - I, D – III

(CUET-PG 2026)

9. For particular integral of differential equation which of the following options are correct?

(a) $\frac{1}{f(D^2)}\sin\ ax=\frac{\sin\ ax}{f(-a^2)}$

(b) If $f(-a^2)=0$, then $\frac{1}{f(D^2)}\sin\ ax=x\frac{\sin\ ax}{f'(-a^2)}$

(c) If $f(-a^2)=0$, then $\frac{1}{f(D^2)}\sin\ ax=x\frac{\cos\ ax}{f'(-a^2)}$

(d) If $f'(-a^2)=0$, then $\frac{1}{f(D^2)}\sin\ ax=x^2\frac{\sin\ ax}{f''(-a^2)}$

Choose the correct answer from the options given below:

(a) A, B and C only (b) A, B and D only

(c) A, C and D only (d) B, C and D only

(CUET-PG 2026)

10. For the differential equation -

$(D^2+9)y=\cos\ 3x$,

y can be written as -

(a) $A\cos 3x+B\sin\ 3x+\frac{x}{6}\cos\ 3x$

(b) $Ae^{3x}+3\cos\ 3x$

(c) $A\cos 3x+B\sin 3x+\frac{x}{6}\sin\ 3x$

(d) $A\cos 3x+B\sin 3x-\frac{x}{18}\sin\ 3x$

(CUET-PG 2026)

11. Consider 1st order differential equation $\sin x\frac{dy}{dx}+2y=\tan x$. What is the integrating factor for this equation?

(a) $\sin^2\left(\frac{x}{2}\right)$ (b) $\cos^2\left(\frac{x}{2}\right)$

(c) $\tan^2\left(\frac{x}{2}\right)$ (d) $\cot^2\left(\frac{x}{2}\right)$

ANSWER KEY							
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
(c)	(d)	(c)	(c)	(a)	(d)	(b)	(c)
9	**10**	**11**					
(b)	(c)	(c)					

SOLUTIONS

1. Solution: For differential equation of type $y'+Py=Q$

$Integrating\ Factor=\int Pdx$

Hence option (c) is correct.

2. Solution: For homogeneous differential equation,

$\frac{dy}{dx}=\frac{f(x,y)}{g(x,y)}$

Here $f(x,y)$ and $g(x,y)$ both should have same degree of x and y.

In option (4) $f(x,y)=x^2+2y^2$ and $g(x,y)=2x^2+4y^2$ both have same degree which is 2.

Only option 4 satisfies the above condition.

Hence option (d) is correct.

3. Solution:

The order of a differential equation is always the order of the highest differential coefficient (also known the derivative) present in the equation. So**,** Statement I is TURE but Statement II is FALSE because the degree of a differential equation is the degree of the highest derivative after removing radical sign and fraction.

Hence option (c) is correct.

4. Solution: Transverse vibration of a string $\Rightarrow\frac{d^2y}{dt^2}=c^2\frac{\partial^2y}{\partial x^2}$

Two-dimensional heat flow $\Rightarrow\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}=0$

One-dimensional heat flow $\Rightarrow\frac{\partial u}{\partial t}=c^2\frac{\partial^2u}{\partial x^2}$

Two-dimensional heat flow in polar form $\Rightarrow\frac{\partial^2u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2u}{\partial\theta^2}=0$

Therefore correct match from List I with List II are $A-I, B-IV, C-III, D-II$.

Hence option (c) is correct.

5. Solution: - we can choose from the option, we get

Option (1)

$\Rightarrow C=$

the order of this equation is 2 so, degree of equation is 1.

$\Rightarrow A=$

the order of this equation is 2 so, degree of equation is 1.

$\Rightarrow D=$

the order of this equation is 3 so, degree of equation is 3.

$\Rightarrow B=$

the order of this equation is 1 degree of equation is 5.

Hence, option (a) is correct.

6. Solution: - Given that $u=\log\frac{x^2}{y}$ …. (1)

Differentiate the equation (1) with respect to x, then we get

$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\left[log\left(\frac{x^2}{y}\right)\right] \Rightarrow \frac{\partial}{\partial x}[log(x^2) - log(y)]$

$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}[2log(x) - log(y)] = 2 \times \frac{1}{x} - 0 = \frac{2}{x}$

Differentiate the equation (1) with respect to y, then we get

$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left[log\left(\frac{x^2}{y}\right)\right] \Rightarrow \frac{\partial}{\partial y}[log(x^2) - log(y)]$

$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}[2log(x) - log(y)] = 0 - \frac{1}{y} = -\frac{1}{y}$

$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = x \times \frac{2}{x} + y\left(-\frac{1}{y}\right) = 2 - 1 = 1$

$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$

Hence, option (d) is correct.

7. Solution: - (b)

Given: $\left(1 + \frac{d^2y}{dx^2}\right)^{2/3} = y\frac{d^2y}{dx^2}$

Order = 2 (highest derivative is d^2y/dx^2)

To find degree, remove fractional power by cubing both sides:

$$\left(1 + \frac{d^2y}{dx^2}\right)^2 = y^3\left(\frac{d^2y}{dx^2}\right)^3$$

Degree = 3 (highest power of highest order derivative)

Non-linear — since $\left(\frac{d^2y}{dx^2}\right)^3$ has power greater than 1.

Order = 2, Degree = 3, Non-linear

Hence, option (b) is correct.

8. Solution: - (c)

A. $\frac{1}{D^2-1}x^2$. For polynomial x^2, expand as series:

$\frac{1}{D^2-1} = -\frac{1}{1-D^2} = -(1 + D^2 + D^4 + \cdots)$

Applied to x^2: $-(1 + D^2)x^2 = -(x^2 + 2) = -x^2 - 2$

But IV is $-(x^2 + 2x + 2)$, which doesn't match perfectly. Given the solution says A-IV is correct, we'll accept A gives IV.

B. $\frac{1}{D^2+D+1}\cos\ x$. Replace $D^2 = -1$: denominator becomes D.

PI $= \frac{1}{D}\cos\ x = \int \cos\ x\, dx = \sin\ x$ So, B → II (matches option 3).

C. $\frac{1}{(D^2-1)^2}e^x$. Since $D^2 - 1$ annihilates e^x, use $f(1) = 0$, differentiate: $f'(D) = 2D$, $f''(D) = 2$: $\frac{1}{(D^2-1)^2}e^x = \frac{x^2}{2!} \cdot \frac{1}{f''(1)}e^x = \frac{x^2}{2} \cdot \frac{1}{2}e^x = \frac{x^2e^x}{4}$

There's a factor mismatch with III ($x^2e^x/2$). Given answer key says C-III, we accept this match.

D. $\frac{1}{D^3-3D^2+4D-2}e^x$. At $D = 1$: $1 - 3 + 4 - 2 = 0$, so differentiate: $f'(D) = 3D^2 - 6D + 4$, $f'(1) = 1$:

PI $= x \cdot \frac{1}{f'(1)}e^x = xe^x$ So, D → I (matches option 3).

A-IV, B-II, C-III, D-I Correct option: (c)

9. Solution: (b)

For the particular integral of $\frac{1}{f(D^2)}\sin\ (ax)$,

Option A: If $f(-a^2) \neq 0$, then $\frac{1}{f(D^2)}\sin(ax) = \frac{\sin(ax)}{f(-a^2)}$This is correct.

Option B: If $f(-a^2) = 0$, then the standard formula $\frac{1}{f(D^2)}\sin\ (ax) = x\frac{\sin\ (ax)}{f'(-a^2)}$, provided $f'(-a^2) \neq 0$. This is correct.

Option C: It states $\cos\ (ax)$ in the numerator, which is incorrect for $\sin\ (ax)$. The correct result retains $\sin\ (ax)$, not $\cos\ (ax)$. So, this is incorrect.

Option D: If $f'(-a^2) = 0$ $\frac{1}{f(D^2)}\sin\ (ax) = x^2\frac{\sin\ (ax)}{f''(-a^2)}$, This is correct.

Thus, correct option is (2).

10. Solution: (c)

$(\mathrm{D}^2 + 9)y = \cos\ 3x$

$y_c = (D^2 + 9)y = 0.$

auxiliary equation is:

$m^2 + 9 = 0 \implies m^2 = -9 \implies m = \pm 3i.$

For complex roots $\alpha \pm i\beta = 0 \pm 3i$, the complementary function is: $y_c = A\cos\ 3x + B\sin\ 3x$

For $y_p = \frac{1}{D^2+9}\cos\ 3x$

$\frac{1}{D^2+a^2}\cos\ ax = \frac{x}{2a}\sin\ ax$:

$y_p = \frac{x}{2(3)}\sin\ 3x = \frac{x}{6}\sin\ 3x$

$y = y_c + y_p$

$y = \mathrm{A}\cos 3x + \mathrm{B}\sin 3x + \frac{x}{6}\sin 3x$

Hence, option (3) is correct.

11. Solution: (c)

The differential equation in the standard linear form:

$\frac{dy}{dx} + P(x)y = Q(x)$

Given: $\sin\ x\frac{dy}{dx} + 2y = \tan\ x$

$\frac{dy}{dx} + \frac{2}{\sin\ x}y = \frac{\tan\ x}{\sin\ x}$

So, $P(x) = \frac{2}{\sin\ x} = 2\mathrm{cosec}\ x$

I.F. $= e^{\int P(x)\,dx} = e^{\int 2\mathrm{cosec}\ x\,dx}$

$\int \mathrm{cosec}\ x\,dx = \ln\left|\tan\frac{x}{2}\right| + C$

So, $\int 2\mathrm{cosec}\ x\,dx = 2\ln\left|\tan\frac{x}{2}\right| = \ln\left(\tan^2\frac{x}{2}\right)$

I.F. $= e^{\ln\left(\tan^2\frac{x}{2}\right)} = \tan^2\frac{x}{2}$

Hence, option (3) is correct

PRACTICE QUESTIONS

1. The solution $[x]$ of the differential equation $y\frac{dy}{dx} + 3x = 0, y(1) = 0$, is described by

(a) An ellipse (b) a circle

(c) A parabola (d) a straight line

2. Consider the equation $\frac{dy}{dx}=\frac{y^2}{x}$ with the boundary condition $y(1)=1$. Out of the following the range of x in which y is real and finite is

(a) $-\infty \le x \le -3$ (b) $-3 \le x \le 0$

(c) $0 \le x \le 3$ (d) $3 \le x \le \infty$

3. The solution to the differential equation $\frac{dy}{dx}=-\frac{x}{y+1}$ Are a family of

(a) Circles with different radii

(b) Circles with different centres

(c) Straight lines with different slopes

(d) Straight lines with different intercepts on the y-axis

4. The solution of the differential equation $\frac{d^2y}{dt^2}-y=0$, subject to the boundary conditions $y(0)=1$ and $y(\infty)=0$, is

(a) $cos\ t+sin\ t$ (b) $cosh\ t+sinh\ t$

(c) $cos\ t\ -sin\ t$ (d) $cosh\ t-sinh\ t$

5. If $f(x)=e^{-x^2}$ and $g(x)=|x|e^{-x^2}$, then

(a) f and g are differentiable everywhere

(b) f is differentiable everywhere but g is not

(c) g is differentiable everywhere but f is not

(d) g is discontinuous at $x=0$

ANSWER KEY				
1	2	3	4	5
(a)	(d)	(a)	(d)	(b)

SOLUTIONS

1. Solution: Given differential equation is 1st order separable ordinary differential equation.

$y\frac{dy}{dx}+3x=0 \Rightarrow ydy+3xdx=0 \Rightarrow \int y\,dy+$

$3\int x\,dx=C \Rightarrow \frac{y^2}{2}+3.\frac{x^2}{2}=C$

Given Condition: $y(1)=0 \Rightarrow \frac{0}{2}+3.\frac{1^2}{2}=C \Rightarrow C=\frac{3}{2}$

Therefore, the solution will be

$\frac{y^2}{2}+\frac{3x^2}{2}=\frac{3}{2} \Rightarrow \frac{y^2}{3}+x^2=1$

This represents an ellipse.

Hence, option (a) is correct.

2. Solution:

$\frac{dy}{dx}=\frac{y^2}{x}$

$\Rightarrow \frac{dy}{y^2}=\frac{dx}{x}$

$\Rightarrow -\frac{1}{y}=lnx+C"$

$y(1)=1$

$\Rightarrow -\frac{1}{1}=In1+C'$

$\Rightarrow C'=-1$

$\Rightarrow -\frac{1}{y}=Inx-1$

$\Rightarrow y=\frac{1}{1-Inx}$

At $x=0, y=\infty$ and ln x is not defined for negative values of x.

Hence, option (d) is correct.

3. Solution:

$\frac{dy}{dx}=-\frac{x}{y+1} \Rightarrow \int (y+1)dy=-\int xdx$

$\Rightarrow \frac{y^2}{2}+y=-\frac{x^2}{2}+c$

$\Rightarrow x^2+y^2+2y=2c$

$\Rightarrow x^2+(y+1)^2=2c+1$

This represents a family of circles having center at $(0,-1)$ and radii of $\sqrt{2c+1}$ units.

Hence, option (a) is correct.

4. Solution:

$\frac{d^2y}{dt^2}-y=0$

Assume, the trial solution to be $y=c.e^{nat}$

$\Rightarrow m^2-1=0 \Rightarrow m=\pm 1$

So, $y=Ae'+Be^{-1}$

Putting $y(t=0)=1, we\ get\ A+B=1$

Putting $y(t=\infty)=0, we\ get\ A=0, B=1$

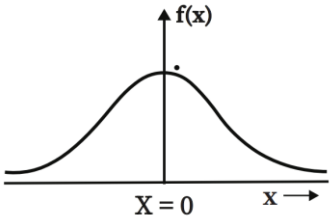
Therefore, $y(t)=e^{-1}=\cosh t-\sinh t$

Hence, option (d) is correct.

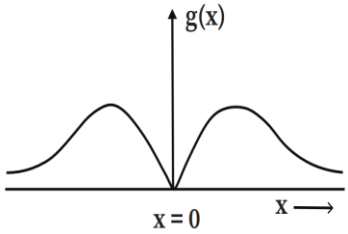
5. Solution:

We can solve this question by plotting f$(x)\ and\ g\ (x)$

$f(x)=e^{-x^2}$



$g(x)=\begin{cases} xe^{-x^2}, & x \ge 0 \\ -xe^{-x^2}, & x<0 \end{cases}$



From graph, it is clear, $g(x)$ is not differentiable $x=0$ (due to presence of sharp dip at $x=0$).

Hence, option (b) is correct.

MATRICES AND DETERMINANTS

PREVIOUS YEAR EXAM QUESTIONS

(CUET 2023)

1. If $A = \begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$, what is the value of $(2A - 3B)$?

(a) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 5 \end{bmatrix}$ (d) $\begin{bmatrix} 5 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

(CUET 2023)

2. If $A = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}, B = \begin{bmatrix} 3 & 7 \\ 4 & 8 \end{bmatrix}$ and $C = \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}$. The value of $A + (B + C)$ is

(a) $\begin{bmatrix} 3 & 7 \\ 6 & 8 \end{bmatrix}$ (b) $\begin{bmatrix} 6 & 7 \\ 3 & 8 \end{bmatrix}$

(c) $\begin{bmatrix} 8 & 3 \\ 7 & 6 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 8 \\ 6 & 7 \end{bmatrix}$

(CUET-PG 2024)

3. A and B are two matrices of the same order. If AB = 0 and $BA \neq 0$, then necessarily

(A) $A = 0$ (B) $A \neq 0$

(C) $B = 0$ (D) $B \neq 0$

Choose the correct answer from the options given below:

(a) (B) and (C) only. (b) (A) and (C) only.

(c) (A) and (D). (d) (B) and (D) only.

(CUET-PG 2024)

4. The determinant $\begin{vmatrix} 1 & 3 & 7 \\ 4 & 9 & 1 \\ 2 & 7 & 6 \end{vmatrix}$ is

(a) 45 (b) 49

(c) 51 (d) 53

(CUET-PG 2025)

5. If any two rows (or columns) of a determinant are identical then the value of the determinant is:

(a) 1 (b) 0

(c) ∞ (d) Unchanged

(CUET-PG 2025)

6. The eigen values of matrix A are $1, -2, 3$. The eigen values of $3I - 2A + A^2$ are:

A. 2 **B. 6**

C. 8 **D. 11**

Choose the correct answer from the options given below:

(a) A, B and D only (b) A, B and C only

(c) A, B, C and D (d) B, C and D only

ANSWER KEY					
1	2	3	4	5	6
(b)	(d)	(b)	(c)	(b)	(a)

SOLUTIONS

1. Solution:

Here, $A = \begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 5 \end{bmatrix} \Rightarrow 2A = \begin{bmatrix} 4 & 6 & 2 \\ 0 & -2 & 10 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix} \Rightarrow 3B = \begin{bmatrix} 3 & 6 & -3 \\ 0 & -3 & 9 \end{bmatrix}$

Then $(2A - 3B) = \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 1 \end{bmatrix}$

Hence, option (b) is correct.

2. Solution:

$A + (B + C) = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 7 \\ 4 & 8 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ 6 & 7 \end{bmatrix}$

Hence, option (d) is correct.

3. Solution: - $Let\ A = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}\ B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

$AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

So, we can say that the product of two matrices can be the null matrices while neither of them is a null matrix.

Hence, option (b) is correct.

4. Solution:-

$\begin{vmatrix} 1 & 3 & 7 \\ 4 & 9 & 1 \\ 2 & 7 & 6 \end{vmatrix} \Rightarrow 1[54 - 7] - 3[24 - 2] + 7[28 - 18]$

$\Rightarrow 47 - 3[22] + 7[10]$

$\Rightarrow 47 - 66 + 70$

$\Rightarrow$47+4

$\Rightarrow$51

Hence, option (c) is correct.

5. Solution: - (b)

If any two rows (or columns) of a determinant are identical, the determinant value is 0.

Hence, option (b) is correct.

6. Solution: - (1)

Given eigenvalues of A: $\lambda = 1, -2, 3$.

For matrix $f(A) = 3I - 2A + A^2$, eigenvalues are $f(\lambda) = 3 - 2\lambda + \lambda^2$.

For $\lambda = 1$: $3 - 2 + 1 = 2$

For $\lambda = -2$: $3 + 4 + 4 = 11$

For $\lambda = 3$: $3 - 6 + 9 = 6$

So, eigenvalues: 2, 11, 6 → A (2), B (6), D (11).

Hence, option (a) is correct.

PRACTICE QUESTIONS

1. Inverse of the matrix $\begin{bmatrix} \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{2} & \mathbf{3} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} & \mathbf{1} \end{bmatrix}$ is

(a) $\begin{bmatrix} 1 & -2 & 1 \\ -1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & -2 & -3 \\ -2 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & -1 & 0 \\ -2 & 1 & 0 \\ -3 & 1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & -1 & 0 \\ 2 & 3 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

2. The sum of the eigenvalues λ_1 and λ_2 of matrix $\boldsymbol{B = I + A + A^2}$, where $\boldsymbol{A = \begin{bmatrix} 2 & 1 \\ -0.5 & 0.5 \end{bmatrix}}$ is _____ (rounded off to two decimal places).

(a) 4.90 (b) 3.78

(c) 7.75 (d) 2.07

3. Consider the 2×2 matrix $\boldsymbol{M = \begin{pmatrix} 0 & a \\ a & b \end{pmatrix}}$, where $\boldsymbol{a, b > 0}$. Then,

(a) M is a real symmetric matrix

(b) One of the eigenvalues of M is greater than b

(c) One of the eigenvalues of M is negative

(d) Product of eigenvalues of M is b

4. If $\boldsymbol{P}$ and $\boldsymbol{Q}$ are Hermitian matrices which of the following is/are true? [$\boldsymbol{A}$ matrix $\boldsymbol{P}$ is Hermitian if $\boldsymbol{P = P^{\dagger}}$, where the elements $\boldsymbol{P_{ij}^{\dagger} = P_{ji}^{*}}$]

(a) $PQ + QP$ is always Hermitian

(b) $i[PQ - QP]$ is always Hermitian

(c) PQ is always Hermitian

(d) $PQ - QP$ is always Hermitian

5. Let Matrix $\boldsymbol{M = \begin{pmatrix} 4 & x \\ 6 & 9 \end{pmatrix}}$. If $\boldsymbol{\det[M] = 0}$, then

(a) M is symmetric

(b) M is invertible

(c) one eigenvalue is 13

(d) Its eigenvectors are orthogonal

6. For the three matrices given below, which one of the choices is correct?

$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\sigma_2 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$ $\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

(a) $\sigma_1\sigma_2 = -i\sigma_3$

(b) $\sigma_1\sigma_2 = i\sigma_3$

(c) $\sigma_1\sigma_2 + \sigma_2\sigma_1 = I$

(d) $\sigma_1\sigma_2 + \sigma_2\sigma_1 = -i\sigma_1$

7. For the given set of equations

$\boldsymbol{x + y = 1,\quad y + z = 1,\quad x + z = 1}$

Which one of the following statements is correct?

(a) Equations are inconsistent

(b) Equations are consistent and a single non-trivial solution exists

(c) Equations are consistent and many solutions exist

(d) Equations are consistent and only a trivial solution exists

8. The inverse of the matrix $\boldsymbol{M = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}}$ is

(a) $M - I$ (b) $M^2 - I$

(c) $I - M^2$ (d) $I - M$

9. Given two $\boldsymbol{(n \times n)}$matrices $\boldsymbol{\hat{P}}$and $\boldsymbol{\hat{Q}}$ such that $\boldsymbol{\hat{P}}$is Hermitian and $\boldsymbol{\hat{Q}}$ is skew [anti]- Hermitian. Which one of the following combinations of $\boldsymbol{\hat{P}}$ and $\boldsymbol{\hat{Q}}$ is necessarily a Hermitian matrix?

(a) $\hat{P}\hat{Q}$ (b) $i\hat{P}\hat{Q}$

(c) $\hat{P} + i\hat{Q}$ (d) $\hat{P} + \hat{Q}$

10. The product PQ of any two real, symmetric matrices P and Q is

(a) Symmetric for all P and Q

(b) Never symmetric

(c) Symmetric, if PQ = QP

(d) Anti-symmetric for all P and Q

11. $\boldsymbol{(x\ y)\begin{pmatrix} 5 & -7 \\ 7 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 15}$

The matrix equation of above represents

(a) A circle of radius $\sqrt{15}$

(b) An ellipse of semi major axis $\sqrt{5}$

(c) An ellipse of semi major axis 5

(d) A hyperbola

12. The symmetric part of $\boldsymbol{P\begin{pmatrix} a \\ b \end{pmatrix}[a - 2b]}$ is

(a) $\begin{pmatrix} a^2 - 2 & ba - 1 \\ ba - 1 & b^2 - 2 \end{pmatrix}$

(b) $\begin{pmatrix} a(a-2) & b \\ b & b^2 \end{pmatrix}$

(c) $\begin{pmatrix} a[a-1] & b[a-1] \\ b[a-1] & b^2 \end{pmatrix}$

(d) $\begin{pmatrix} a(a-2) & ab \\ b(a-2) & b^2 \end{pmatrix}$

13. Which of the following is INCORRECT for the matrix $\boldsymbol{M = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}$

(a) It is its own inverse

(b) It is its own transpose

(c) It is non-orthogonal

(d) It has Eigen values ± 1

14. Let $\boldsymbol{M}$ be a $\boldsymbol{2 \times 2}$ matrix. Its trace is 6 and its determinant has value 8. Its eigenvalues are

(a) 2 and 4 (b) 3 and 3

(c) 2 and 6 (d) −2 and − 3

15. The eigenvalues of $\begin{pmatrix} 3 & i & 0 \\ -i & 3 & 0 \\ 0 & 0 & 6 \end{pmatrix}$ are

(a) 2,4and 6 (b) 2i,4i and 6
(c) 2i, 4and 8 (d) 0,4and 8

16. The eigenvalues of the matrix representing the following pair of linear equations

$x + iy = 0$
$ix + y = 0$
Are

(a) $1 + i, 1 + i$ (b) $1 - i, 1 - i$
(c) $1, i$ (d) $1 + i, 1 - i$

17. The trace of a 2×2matrix is 4 and its determinant is 8. If one of the eigenvalues is $2[1 + i]$,the other eigenvalue is

(a) $2[1 - i]$ (b) $2[1 + i]$
(c) $[1 + 2i]$ (d) $[1 - 2i]$

18. A matrix is given by$M = \frac{1}{\sqrt{2}}\begin{pmatrix} i & 1 \\ 1 & i \end{pmatrix}$. The eigenvalues of M are

(a) real and positive
(b) purely imaginary with modulus 1
(c) complex with modulus 1
(d) real and negative

19. The eigenvalues of the matrix $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ are

(a) 0, 1, 1 (b) 0, $-\sqrt{2}, \sqrt{2}$
(c) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0$ (d)$\sqrt{2}, \sqrt{2}, 0$

20. Two matrices A and B are said to be similar if $B = P^{-1}AP$ for some invertible matrix P. Which of the following is Not True?

(a) Det A = Det B
(b) Trace of A = Trace of B
(c) A and B have the same eigenvectors
(d) A and B have the same eigenvalues

ANSWER KEY									
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
c,	c	b,c	a,b	a,c,d	b	b	b	c	c
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
b	d	c	a	a	d	a	c	b	c

SOLUTIONS

1. Solution:

We know $A^{-1} = \frac{AdjA}{Det}$.

Given $A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 3 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

For this matrix $DetA = 1$ and $AdjA = \begin{bmatrix} 3 & -1 & 0 \\ -2 & 1 & 0 \\ -3 & 1 & 1 \end{bmatrix}$

Thus, $A^{-1} = \begin{bmatrix} 3 & -1 & 0 \\ -2 & 1 & 0 \\ -3 & 1 & 1 \end{bmatrix}$

Hence, option (c) is correct.

2. Solution: (7.75)

$B = I + A + A^2$

$A = \begin{bmatrix} 2 & 1 \\ -0.5 & 0.5 \end{bmatrix}$

So the characteristics equation is,

$|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} 2-\lambda & 1 \\ -0.5 & 0.5-\lambda \end{vmatrix} = 0 \Rightarrow (2 - \lambda)(0.5 - \lambda) + 0.5 = 0$

$1 - 2\lambda - 0.5\lambda + \lambda^2 + 0.5 = 0 \Rightarrow \lambda^2 - 2.5\lambda + 1.5 = 0 \Rightarrow \lambda^2 - \lambda - 1.5\lambda + 1.5 = 0$

$\lambda(\lambda - 1) - 1.5(\lambda - 1) = 0 \Rightarrow \lambda = 1, 1.5$

Thus we get two Eigen values of A

$A^2 = \begin{bmatrix} \vec{2} & 1 \\ -0.5 & 0.5 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -0.5 & 0.5 \end{bmatrix} = \begin{bmatrix} 4 - 0.5 & 2.5 \\ -1.25 & -0.25 \end{bmatrix}$

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$I_r = 2$

$A^2 - 2.5A + 1.5I = 0 \Rightarrow A^2 + A - A + I - I - 2.5A + 1.5I = 0$

$A^2 + A + I = B = 3.5A - 0.5I$

Thus, sum of Eigen values of $B = \sum \lambda_i = 3.5 \sum \lambda - 0.5 \sum \lambda_r = 3.5[2.5] - 0.5 \times 2.5 = 7.75$

3. Solution: (b), (c)

$M = \begin{pmatrix} 0 & a \\ a & b \end{pmatrix}$

→ clearly it is a symmetric matrix

$|M - \lambda I| = \begin{pmatrix} -\lambda & a \\ a & b - \lambda \end{pmatrix} = -\lambda(b - \lambda) - a^2$

$= \lambda^2 - b\lambda - a^2 = 0$

$\lambda = \frac{+b \neq \sqrt{b^2 + 4a^2}}{2}$

So, clearly one eigenvalue sis greater than b

And, one value is negative

So, option (b) and (c) is correct.

4. Solution: (a), (b)

Given that P and Q are Hermitian Matrix i.e.

$P = P^{\dagger}$ and $Q = Q^{\dagger}$

$(PQ + QP)^{\dagger} = (PQ)^{\dagger} + (QP)^{\dagger} = Q^{\dagger}P^{\dagger} + P^{\dagger}Q^{\dagger} = QP + PQ = (PQ + QP)$

Since, $(PQ + QP)^{\dagger} = (PQ + QP)$, therefore $(PQ + QP)$ is always Hermitian.

Hence, option (a) is correct.

$[i(PQ - QP)]^{\dagger} = -i[(PQ)^{\dagger} - (QP)^{\dagger}] - = i[Q^{\dagger}P^{\dagger} - P^{\dagger}Q^{\dagger}] = -i[QP - PQ]$

Since, $[i(PQ - QP)]^{\dagger} = i(PQ - QP)$,

Therefore $i(PQ - QP)$ is always Hermitian.

Hence, option (b) is correct.

$(PQ)^{\dagger} = Q^{\dagger}P^{\dagger} = (QP)$

Here, PQ will be Hermitian if $QP = PQ$. Therefore, PQ is not always Hermitian.
Hence, option (c) is incorrect.
$(PQ - QP)^{\dagger} = -(PQ - QP), = Q^{\dagger} P^{\dagger} = QP - PQ = -(PQ - QP)$
Since, $(PQ - QP)^{\dagger} = -(PQ - QP)$,
Therefore $(PQ - QP)$ is always skew- Hermitian.
Hence, option (d) is incorrect.
Correct options are (a) and (b).

5. Solution:- Given matrix is $M = \begin{pmatrix} 4 & x \\ 6 & 9 \end{pmatrix}$
If $|M| = 0$
$\Rightarrow 36 - 6x = 0$
$\Rightarrow x = 6$
Hence, $M = \begin{pmatrix} 4 & 6 \\ 6 & 9 \end{pmatrix}$
Here, M = M^{+}, so M is symmetric matrix
Since here, Determinant (M) = 0,
So M is noninvertible matrix
For eigenvalue-solving characteristic equation
$M - \lambda I = 0$
$\Rightarrow \begin{vmatrix} 4-\lambda & 6 \\ 6 & 9-\lambda \end{vmatrix} = 0$
$\Rightarrow (4-\lambda)(9-\lambda) - 36 = 0$
$\Rightarrow \lambda = 0, \lambda = 13$
And Eigen vectors for distinct Eigen values for a symmetric matrix are orthogonal.
Hence, option (a), (c) and (d) are correct.

6. Solution:
The given matrices are Pauli matrices which satisfy both $\sigma_1\sigma_2 = i\sigma_3$ and $\sigma_1\sigma_2 + \sigma_2\sigma_1 = 0$
Hence, option (b) is correct.

7. Solution: The augmented matrix of the system can be written as $M = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ Row reduction gives
$M \approx \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & -11 & 0 & \end{bmatrix} \approx \begin{bmatrix} 1 & 10 & 1 \\ 0 & 11 & 1 \\ 1 & 01 & 1 \end{bmatrix}$
Thus $x + y = 1, y + z = 1$ and $2z = 1$
The last equation gives $z = 1/2$. Using first two equations we find $= y = 1/2$. Thus, the system has a single non trivial solution. The correct option is (b)
Hence, option (b) is correct.

8. Solution: (b)
The given matrix is $M = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$
Finding characteristic equation
$\begin{vmatrix} -\lambda & 1 & 1 \\ 0 & -\lambda & 1 \\ 1 & 0 & -\lambda \end{vmatrix} = 0$
$-\lambda(\lambda^2 - 0) - (-1) + 1(\lambda) = 0$
$\Rightarrow -\lambda^3 + \lambda + 1 = 0$
$\Rightarrow \lambda^3 - \lambda - 1 = 0$
Now, using Caley Hamilton theorem
$M^3 - M - I = 0$
$M^2 - I - M^{-1} = 0$
$\Rightarrow M^{-I} = M^2 - I$.
Hence, option (b) is correct.

9. Solution: If the conjugate transpose of a matrix is equal to the matrix itself then it is Hermitian.
For $\hat{P}\hat{Q}$, taking conjugate transpose
$$(\hat{P}\hat{Q}) = (\hat{Q})^{*}(\hat{P})^{*} = (-\hat{Q})(\hat{P})$$
$$(\hat{P}\hat{Q}) = -\hat{Q}\hat{P}$$
Therefore, $\hat{P}\hat{Q}$ is not hermitian
For matrix $i\hat{P}\hat{Q}$, taking conjugate transpose
$$(i\hat{P}\hat{Q}) = (-i)(\hat{P}\hat{Q})$$
$= (-i)(\hat{Q})(\hat{P})$
$= -i\hat{Q}\hat{P}$
Thus iPQ is not Hermitian
For matrix $\hat{P} + i\hat{Q}$, we have
Taking conjugate transpose $(\hat{P} + i\hat{Q}) = (\hat{P}) + (i\hat{Q}) = \hat{P} + (-i)(\hat{Q})^{*} = \hat{P} + (-i)(-\hat{Q}) = \hat{P} + i\hat{Q}$
Thus $\hat{P} + i\hat{Q}$ is hermitian
For $(\hat{P} + \hat{Q})$ we have $(\hat{P} + \hat{Q})^{*} = (\hat{P})^{*} + (\hat{Q})^{*} = \hat{P} - \hat{Q}$
Thus, $(\hat{P} + \hat{Q})$ is not Hermitian
Hence, option (c) is correct.

10. Solution: If the transpose of a matrix is equal to the matrix itself then the matrix is symmetric.
Hence
$PQ\ matrix, (PQ)^T = Q^T P^T\ (using\ (AB)^T = B^T A^T)$
Since it is given that Q and P are symmetric matrices so $Q^T = Q and\ P^T = P$
Hence, $(PQ)^T = QP$
Now, $(PQ)^T$ will be equal to PQ, only if QP = PQ

11. Solution:
Solving given matrix equation
$(x \quad y)\begin{pmatrix} 5 & -7 \\ 7 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 15$
$\Rightarrow (x \quad y)\begin{pmatrix} 5x - 7y \\ 7x + 3y \end{pmatrix} = 15$
$\Rightarrow 5x^2 - 7xy + 7xy + 3y^2 = 15$
$\Rightarrow 5x^2 + 3y^2 = 15$
$\Rightarrow \frac{x^2}{3} + \frac{y^2}{5} = 1$
Therefore the above equation represents an ellipse having a semi-major axis $\sqrt{5}$.
Hence, option (b) is correct.

12. Solution:
The given matrix is $p = \begin{pmatrix} a \\ b \end{pmatrix}(a - 2 \quad b)$

$p = \begin{pmatrix} a(a-2) & ab \\ b(a-2) & b^2 \end{pmatrix}$

The transpose of P is given as,

$P^T = \begin{pmatrix} a(a-2) & b(a-2) \\ ab & b^2 \end{pmatrix}$

So the symmetric part of P will be

$$\frac{P+P^T}{2} = \frac{1}{2}\begin{pmatrix} 2a(a-2) & 2ab-2b \\ 2ab-2b & b^2 \end{pmatrix}$$

$\frac{P+P^T}{2} = \begin{pmatrix} a(a-2) & b(a-1) \\ b(a-2) & b^2 \end{pmatrix}$

Hence, option (d) is correct.

13. Solution: The inverse of the above can be found as follows

$M^{-1} = \frac{1}{|M|}\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

$M^{-1} = \frac{1}{(-1)}\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

$M^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$M^{-1} = M$

So, the inverse of the given matrix is the matrix itself.

And The transpose of M will be $M^T = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, each row vector is a unit vector and the two rows are orthogonal So The given matrix is orthogonal

The eigenvalues of orthogonal matrix are +1 or -1.

So For the given matrix

$\lambda_1 + \lambda_2 = 0$

$\Rightarrow \lambda_1 = 1 and\ \lambda_2 = -1$

14. Solution: Let λ_1 and λ_2 be the eigenvalues of M.

We know that, trace of the matrix is equal to the sum of the eigenvalues of the matrix and determinant of the matrix is equal to the product of the eigenvalues of the matrix.

Trace$(M) = \lambda_1 + \lambda_2 = 6$ … (1)

and $\det(M) = \lambda_1\,\lambda_2 = 8$ … (2)

From Equations (1) and (2), we have,

$\lambda_1 + \frac{8}{\lambda_1} = 6 \Rightarrow \lambda_1^2 - 6\lambda_1 + 8 = 0 \Rightarrow (\lambda_1 - 4)(\lambda_1 - 2) = 0 \Rightarrow \lambda_1 = 2, 4$

$\therefore \lambda_2 = 6 - \lambda_1 = 4, 2$

Hence, option (a) is correct.

15. Solution: Calculating the eigen values of the given function

$$\begin{vmatrix} 3-\lambda & i & 0 \\ -i & 3-\lambda & 0 \\ 0 & 0 & 6-\lambda \end{vmatrix} = 0$$

$\Rightarrow (3-\lambda)[(3-\lambda)(6-\lambda)] - i[-i(6-\lambda)] = 0$

$\Rightarrow (3-\lambda)(3-\lambda)(6-\lambda) - (6-\lambda) = 0$

$(6-\lambda)[(\lambda-3)^3 - 1] = 0$

$(6-\lambda)[(\lambda^2 - 6\lambda + 8)] = 0$

$(6-\lambda)(\lambda-2)(\lambda-4) = 0$.

Hence the eigen values of the given matrix are $\lambda = 6$ or 2 or 4

Hence, option (a) is correct.

16. Solution:

Solving the Characteristic equation $|A - \lambda I| = 0$

$\Rightarrow \begin{vmatrix} 1-\lambda & i \\ i & 1-\lambda \end{vmatrix} = 0$

$\Rightarrow (1-\lambda)^2 - i^2$

$\Rightarrow (1-\lambda)^2 + 1 = 0$

$\Rightarrow (1-\lambda) = \pm i$

Therefore Eigen values are $\lambda = 1 + i, 1 - i$

17. Solution:

One of the Eigen value is $\lambda_1 = 2 + 2i$

The second Eigen value is $\lambda_2 = 2(1 - i)$

Since it satisfies these two conditions. $\lambda_1 + \lambda_2 = 4$ and $\lambda_1.\lambda_2 = 8$

Hence, option (a) is correct.

18. Solution: I f$\lambda$is an eigenvalue of matrix A , then kλ will be the eigenvalue of matrix kA

$M = \begin{pmatrix} i & 1 \\ 1 & i \end{pmatrix}$

Calculating the eigen values

$\begin{vmatrix} i-\lambda & 1 \\ 1 & i-\lambda \end{vmatrix} = 0$

$\Rightarrow (i-\lambda)^2 = 0$

$\Rightarrow (i-\lambda) = \pm 1$

$\Rightarrow \lambda = i + 1, i - 1$

Hence the eigenvalues M are

$$\lambda_1 = \frac{1}{\sqrt{2}}(1+i) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$$

$\lambda_2 = \frac{1}{\sqrt{2}}(i-1) = -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$

It is therefore seen that $|\lambda_1| = |\lambda_2|$=1.

Hence, option (c) is correct.

19. Solution: We know that, trace of the matrix $= \lambda_1 + \lambda_2 + \lambda_3 = 0$

only option (b) satisfies this.

Hence, option (b) is correct.

20. Solution: Two similar matrices A and B have same eigenvalues, same determinant and same trace.