GATE Instrumentation Past Years Cut-off

YEAR	General	OBC	SC/ST/PH
2025	25.6	23.0	17.0
2024	32.7	29.4	21.8
2023	34.8	31.3	23.2
2022	42.4	38.1	28.2
2021	36	32.4	24
2020	34.6	31.1	23
2019	31.9	28.7	21.3
2018	37.1	33.3	24.7
2017	34.60	31.10	23
2016	31.6	28.4	21
2015	25.45	22.9	16.96
2014	25	22.5	16.67

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GATE – INSTRUMENTATION ENGINEERING

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Engineering Mathematics

CHAPTER **2**

Unit 2.1: Linear Algebra

Multiple Choice Questions (MCQ)

[2000: 1 Mark]

1. The rank of matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 6 & 8 \end{bmatrix}$ is

(A) 0 (B) 1

(C) 2 (D) 3

[2000: 1 Mark]

2. The necessary condition to diagonalize a matrix is that

(A) its all eigen values should be distinct

(B) its eigen vectors should be independent

(C) its eigen value should be real

(D) the matrix is non-singular

[2003: 1 Mark]

3. A system of equations represented by $AX = 0$, where X is a column vector of unknowns and A is matrix containing coefficients, has a nontrivial solution when A is

(A) nonsingular (B) singular

(C) symmetric (D) Hermetian

[2005: 1 Mark]

4. Let A be a 3×3 matrix with rank 2. Then $AX = 0$ has

(A) only the trival solution $X = 0$

(B) one independent solution

(C) two independent solutions

(D) three independent solutions

[2005: 1 Mark]

5. For a given 2 x 2 matrix A, it is observed that

$A\begin{bmatrix} 1 \\ -1 \end{bmatrix} = -\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ *and* $A\begin{bmatrix} 1 \\ -2 \end{bmatrix} = -2\begin{bmatrix} 1 \\ -2 \end{bmatrix}$

Then matrix A is

(A) $A = \begin{bmatrix} 2 & 0 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$

(B) $A = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}$

(C) $A = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}$

(D) $A = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix}$

[2007: 1 Mark]

6. Let A be an $n \times n$ real matrix such that $A^2 = I$ and y be an n-dimensional vector, Then the linear system of equations $Ax = y$ has

(A) no solution

(B) a unique solution

(C) more than one but finitely many independent solutions

(D) infinitely many independent solutions

[2007: 2 Mark]

7. Let $A = [a_{ij}], 1 \le i, j \le n$, with $n \ge 3$ and $a_{ij} = i, j$. Then the rank of A is

(A) 0 (B) 1

(C) $n - 1$ (D) n

[2009: 1 Mark]

8. Let $\mathbf{P} \neq \mathbf{0}$ be a 3×3 real matrix. There exist linearly independent vectors $\mathbf{x}$ and $\mathbf{y}$ such that $\mathbf{Px} = \mathbf{0}$ and $\mathbf{Py} = \mathbf{0}$. The dimension of the range space of $\mathbf{P}$ is

(A) 0 (B) 1

(C) 2 (D) 3

[2009: 2 Mark]

9. The eigenvalues of a (2×2) matrix $\mathbf{X}$ are -2 and -3. The eigenvalues of the matrix $(\mathbf{X} + \mathbf{1})^{-1}(\mathbf{X} + \mathbf{5I})$ are

(A) $-3, -4$ (B) $-1, -2$

(C) $-1, -3$ (D) $-2, -4$

[2009: 2 Mark]

10. The matrix $\mathbf{P} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ rotates a vector about the axis $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ by an angle of

(A) $30°$ (B) $60°$

(C) $90°$ (D) $120°$

[2010: 1 Mark]

11. A real $n \times n$ matrix $A = [a_{ij}]$ is defined as follows:

$a_{ij} = i$, if $i = j$

$= 0$, otherwise

The summation of all n eigenvalues of A is

(A) $n(n+1)/2$ (B) $n(n-1)/2$

(C) $\frac{n(n+1)(2n+1)}{6}$ (D) n^2

[2010: 2 Mark]

12. X and Y are non-zero square matrices of size $n \times n$. If $\mathbf{XY} = \mathbf{0}_{n \times k}$ then

(A) $|\mathbf{X}| = 0$ and $|\mathbf{Y}| \neq 0$ (B) $|\mathbf{X}| \neq 0$ and $|\mathbf{Y}| = 0$

(C) $|\mathbf{X}| = 0$ and $|\mathbf{Y}| = 0$ (D) $|X| \neq 0$ and $|Y| \neq 0$

[2011: 1 Mark]

13. The matrix $\mathbf{M} = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ has eigenvalues $-3, -3, 5$. An eigenvector corresponding to the eigenvalue 5 is $[1 \quad 2 \quad -1]^{\mathrm{T}}$. One of the eigenvectors of the matrix $\mathbf{M}^3$ is

(A) $[1 \quad 8 \quad -1]^{\mathrm{T}}$ (B) $[1 \quad 2 \quad -1]^{\mathrm{T}}$

(C) $[1 \quad \sqrt[3]{2} \quad -1]^{\mathrm{T}}$ (D) $[1 \quad 1 \quad -1]^{\mathrm{T}}$

[2012: 2 Mark]

14. Given that $A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, the value of A^3 is

(A) $15A + 12I$ (B) $19A + 30I$

(C) $17A + 15I$ (D) $17A + 21I$

[2013: 1 Mark]

15. The dimension of the null space of the matrix $\begin{bmatrix} 0 & 1 & 1 \\ 1 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix}$ is

(A) 0 (B) 1

(C) 2 (D) 3

[2013: 2 Mark]

16. One pair of eigenvectors corresponding to the two eigenvalues of the matrix $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ is

(A) $\begin{bmatrix} 1 \\ -j \end{bmatrix}, \begin{bmatrix} j \\ -1 \end{bmatrix}$ (B) $\begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

(C) $\begin{bmatrix} 1 \\ j \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 \\ j \end{bmatrix} \cdot \begin{bmatrix} j \\ 1 \end{bmatrix}$

[2014: 2 Mark]

17. A scalar valued function is defined as $f(\mathbf{x}) = \mathbf{x}^T\mathbf{A}\mathbf{x} + \mathbf{b}^T\mathbf{x} + c$, where $\mathbf{A}$ is a symmetric positive definite matrix with dimension $n \times n$; $\mathbf{b}$ and $\mathbf{x}$ are vectors of dimension $n \times 1$. The minimum value of $f(\mathbf{x})$ will occur when $\mathbf{x}$ equals

(A) $(A^TA)^{-1}b$ (B) $-(A^TA)^{-1}b$

(C) $-\left(\frac{A^{-1}b}{2}\right)$ (D) $\frac{A^{-1}b}{2}$

[2014: 2 Mark]

18. For the matrix $\mathbf{A}$ satisfying the equation given below, the eigenvalues are

$$[A]\begin{bmatrix} 1 & 2 & 3 \\ 7 & 8 & 9 \\ 4 & 5 & 6 \end{bmatrix}$$

(A) $(1, -j, j)$ (B) $(1,1,0)$

(C) $(1,1,-1)$ (D) $(1,0,0)$

[2015: 1 Mark]

19. Let $\mathbf{A}$ be an $n \times n$ matrix with rank $r(0 < r < n)$. Then $\mathbf{A}\mathbf{x} = \mathbf{0}$ has p independent solutions, where p is

(A) r (B) n

(C) $n - r$ (D) $n + r$

[2016: 1 Mark]

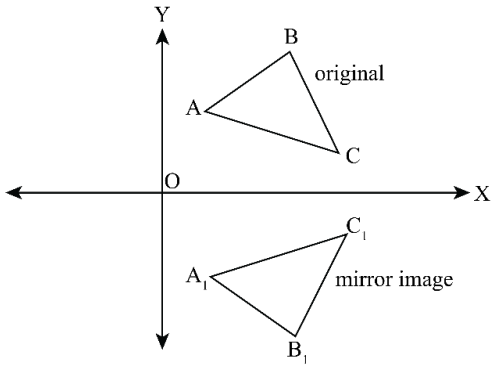
20. The vector that is NOT perpendicular to the vectors $(i + j + k)$ and $(i + 2j + 3k)$ is ____

(A) $(i - 2j + k)$ (B) $(-i + 2j - k)$

(C) $(0i + 0j + 0k)$ (D) $(4i + 3j + 5k)$

[2017: 1 Mark]

21. The figure shows a shape ABC and its mirror image $\mathbf{A_1}\,\mathbf{B_1}\mathbf{C_1}$ across the horizontal axis (X axis). The coordinate transformation matrix that maps ABC to $\mathbf{A_1}\,\mathbf{B_1}\mathbf{C_1}$ is



(A) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

[2017: 1 Mark]

22. The eigenvalues of the matrix $\mathbf{A} = \begin{bmatrix} 1 & -1 & 5 \\ 0 & 5 & 6 \\ 0 & -6 & 5 \end{bmatrix}$ are

(A) $-1,5,6$ (B) $1, -5 \pm j6$

(C) $1,5 \pm j6$ (D) 1, 5, 5

[2018: 1 Mark]

23. Let N be a 3 by 3 matrix with real number entries. The matrix N is such that $N^2 = 0$. The eigen values of N are

(A) 0,0,0 (B) 0,0,1

(C) 0,1,1 (D) 1,1,1

[2018: 2 Mark]

24. Consider the following system of linear equations:

$3x + 2ky = -2$

$kx + 6y = 2$

Here x and y are the unknowns and k is a real constant. The value of k for which there are infinite number of solutions is

(A) 3 (B) 1

(C) -3 (D) -6

[2019: 1 Mark]

25. $\vec{a}, \vec{b}, \vec{c}$ are three orthogonal vectors. Given that $\vec{a} = \hat{\imath} + 2\hat{\jmath} + 5\hat{k}$ and $\vec{b} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$, the vector $\vec{c}$ is parallel to

(A) $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ (B) $2\hat{\imath} + \hat{\jmath}$

(C) $2i - j$ (D) $4\hat{k}$

[2020: 1 Mark]

26. The unit vectors along the mutually perpendicular x, y and z axes are $\hat{\imath}, \hat{\jmath}$ and $\hat{k}$ respectively. Consider the plane $z = 0$ and two vectors $\vec{a}$ and $\vec{b}$ on that plane such that $\vec{a} \neq \alpha\vec{b}$ for any scalar α. A vector perpendicular to both $\vec{a}$ and $\vec{b}$ is .

(A) $\hat{k}$ (B) $i - j$

(C) $-j$ (D) $\hat{\imath}$

[2020: 1 Mark]

27. A set of linear equations is given in the form $Ax = b$, where A is a 2×4 matrix with real number entries and $b \neq 0$. Will it be possible to solve for x and obtain a unique solution by multiplying both left and right sides of the equation by A^T (the super script T denotes the transpose) and inverting the matrix A^TA ? Answer is _____

(A) Yes, it is always possible to get a unique solution for any 2×4 matrix A .

(B) No, it is not possible to get a unique solution for any 2×4 matrix A .

(c) Yes, can obtain a unique solution provided the matrix A^TA is well conditioned

(D) Yes, can obtain a unique solution provided the matrix A is well conditioned

[2020: 2 Mark]

28. Consider the matrix $M = \begin{bmatrix} 1 & -1 & 0 \\ 1 & -2 & 1 \\ 0 & -1 & 1 \end{bmatrix}$, One of the eigenvectors of M is

(A) $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

(C) $\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

[2021: 1 Mark]

29. Consider the row vectors $v = (1, 0)$ and $w = (2, 0)$. The rank of the matrix $M = 2v^Tv + 3w^Tw$, where the superscript T denotes the transpose, is

(A) 1 (B) 2

(C) 3 (D) 4

[2023: 1 Mark]

30. Choose solution set S corresponding to the systems of two equations $x - 2y + z = 0$, $x - z = 0$ Note: $\because$ denotes the set of real numbers

(A) $S = \left\{ \alpha \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \middle| \ \alpha \in \Re \right\}$

(B) $S = \left\{ \alpha \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \middle| \ \alpha, \beta \in \Re \right\}$

(C) $S = \left\{ \alpha \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \middle| \ \alpha, \beta \in \Re \right\}$

(D) $S = \left\{ \alpha \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \middle| \ \alpha \in \Re \right\}$

[2024: 1 Mark]

31. A matrix M is constructed by stacking three column vectors v_1, v_2, v_3 as $M = [v_1 \quad v_2 \quad v_3]$. Choose the set of vectors from the following options such that $(M) = 3$.

(A) $v_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$, $v_3 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

(B) $v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $v_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$, $v_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

(C) $v_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $v_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$, $v_3 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

(D) $v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $v_2 = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$, $v_3 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$

[2025]

32. A $2n \times 2n$ matrix $A = [a_{ij}]$ has its elements defined as:

$$a_{ij} = \begin{cases} \beta & \text{if } (i + j) \text{ is odd,} \\ -\beta & \text{if } (i + j) \text{ is even,} \end{cases}$$

where n is any integer greater than 2 and β is any non-zero real number. The rank of A is:

(A) 1 (B) 2

(C) n (D) $2n$

Multiple Select Questions (MSQ)

1. Identify which one of the following is an eigenvector of the matrix $A = \begin{bmatrix} 1 & 0 \\ -1 & -2 \end{bmatrix}$

(A) $[-1 \quad 1]^T$ (B) $[1 \ -1]^T$

(C) $[3 - 1]^T$ (D) $[-2 \quad 1]^T$

2. The matrix $A = \begin{bmatrix} 4 & 3 \\ 9 & -2 \end{bmatrix}$ has eigenvalues -5 and 7. The eigenvector(s) is/are _____ [2022: 2 Mark]

(A) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ (B) $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$

(C) $\begin{bmatrix} 2 \\ -6 \end{bmatrix}$ (D) $\begin{bmatrix} 2 \\ 8 \end{bmatrix}$

[2022: 1 Mark]

3. Given $M = \begin{bmatrix} 2 & 3 & 7 \\ 6 & 4 & 7 \\ 4 & 6 & 14 \end{bmatrix}$, which of the following statement(s) is/are correct?

(A) The rank of M is 2

(B) The rank of M is 3

(C) The rows of M are linearly independent

(D) The determinant of M is 0

Numerical Answer Type Questions (NAT)

[2016: 2 Mark]

1. **Consider the matrix $\mathbf{A}-\begin{pmatrix}\mathbf{2} & \mathbf{1} & \mathbf{1}\\ \mathbf{2} & \mathbf{3} & \mathbf{4}\\ \mathbf{-1} & \mathbf{-1} & \mathbf{-2}\end{pmatrix}$ whose eigenvalues are $\mathbf{1}, \mathbf{-1}$ and 3. Then Trace of $(\mathbf{A^3 - 3A^2})$ is_____**

[2017: 1 Mark]

2. **If $\mathbf{v}$ is a non-zero vector of dimension $\mathbf{3 \times 1}$, then the matrix $\mathbf{A = vv^T}$ has a rank =_____**

[2017: 2 Mark]

3. **The angle between two vectors $x_1 - [2 \quad 6 \quad 14]^\top$ and $x_2 - [-12 \quad 8 \quad 16]^\top$ in radian is _____ .**

[2021: 1 Mark]

4. **The determinant of the matrix M shown below is _____.**

$$\mathrm{M}=\begin{bmatrix}1 & 2 & 0 & 0\\ 3 & 4 & 0 & 0\\ 0 & 0 & 4 & 3\\ 0 & 0 & 2 & 1\end{bmatrix}$$

[2021: 2 Mark]

5. **Given $A=\begin{pmatrix}2 & 5\\ 0 & 3\end{pmatrix}$. The value of the determinant $|A^4 - 5A^3 + 6A^2 + 2I| =$**

[2023: 2 Mark]

6. **The rank of the matrix A given below is one. The ratio $\frac{\alpha}{\beta}$ is _____ (rounded off to the nearest integer).**

$$A=\begin{bmatrix}1 & 4\\ -3 & \alpha\\ \beta & 6\end{bmatrix}$$

[2024: 1 Mark]

7. **A 3 × 3 matrix P with all real elements has eigenvalues $\frac{1}{4}$ 1, and –2. The value of $|P^{-1}|$ is _____ (rounded off to nearest integer).**

[2025: 1 Mark]

8. **If one of the eigenvectors of the matrix $A=\begin{bmatrix}-1 & -1\\ x & -4\end{bmatrix}$ is along the direction of $\begin{bmatrix}\alpha\\ 2\alpha\end{bmatrix}$, where α is any non-zero real number, then the value of x is ___2___ (in integer).**

Answer Key									
Multiple Choice Question (MCQ)									
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
C	B	B	B	D	A	C	B	C	D
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
A	C	B	B	B	A	C	A	C	D
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
D	C	A	C	C	A	B	D	A	A
31	**32**								
C	A								

Multiple Select Question (MSQ)		
1	**2**	**3**
A,C	A,C	A,D
Numerical Answer Type (NAT)		
1	**2**	**3**
-6.-6	1 TO 1	0.65 TO 0.80
4	**5**	**6**
4 TO 4	4 TO 4	-8 TO -8
7	**8**	
-2 TO -2	2	

EXPLANATIONS

Multiple Choice Questions (MCQ)

1. **Explanation: (C)**

Given the matrix:

$$A=\begin{bmatrix}1 & 2 & 3\\ 3 & 4 & 5\\ 4 & 6 & 8\end{bmatrix}$$

Perform row operations:

$R_2 \to R_2 - 3R_1$

$$\begin{bmatrix}1 & 2 & 3\\ 0 & -2 & -4\\ 4 & 6 & 8\end{bmatrix}$$

$R_3 \to R_3 - 4R_1$

$$\begin{bmatrix}1 & 2 & 3\\ 0 & -2 & -4\\ 0 & -2 & -4\end{bmatrix}$$

Notice that Row 2 and Row 3 are identical, so subtract Row 2 from Row 3:

$R_3 \to R_3 - R_2$

$$\begin{bmatrix}1 & 2 & 3\\ 0 & -2 & -4\\ 0 & 0 & 0\end{bmatrix}$$

The first two rows are nonzero.

The third row is zero.

The rank of a matrix is the number of linearly independent rows or columns in the matrix.

Since there are two nonzero rows, the rank of matrix A is 2.

Final Answer: 2 (Option (c)

2. **Explanation: (B)**

A matrix A of size $n \times n$ is diagonalizable if and only if it has n linearly independent eigenvectors.

Analyzing the Options:

(a) "All eigenvalues should be distinct" – Not necessarily true

If all eigenvalues are distinct, the matrix is always diagonalizable

However, a matrix can still be diagonalizable even if some eigenvalues are repeated, as long as there are enough linearly independent eigenvectors.

(b) "Its eigenvectors should be independent" – Correct
The matrix is diagonalizable if and only if it has n linearly independent eigenvectors.
If the geometric multiplicity of each eigenvalue equals its algebraic multiplicity, then the matrix is diagonalizable.

(c) "Its eigenvalues should be real" – Incorrect
A matrix can be diagonalizable even if it has complex eigenvalues.
Example: The rotation matrix $\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$ has complex eigenvalues i and $-i$, but it is still diagonalizable over complex numbers.

(d) "The matrix is non-singular" – Incorrect
A singular matrix (determinant = 0) can still be diagonalizable.
Example: $\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}$ is singular (det = 0) but diagonalizable.
Final
Answer: (b) Its eigenvectors should be independent

3. Explanation: (B)

The given system of equations is: $AX = 0$
where A is a coefficient matrix and X is a column vector of unknowns.
Understanding the Nature of Solutions
A trivial solution means $X = 0$ (the zero vector).
A nontrivial solution means there exists at least one nonzero vector $X \neq 0$ that satisfies $AX = 0$.
For the system $AX = 0$ to have a nontrivial solution, the matrix A must be singular.
Why?
If A is nonsingular (i.e., invertible, $\det(A) \neq 0$), then the only solution is $X = 0$ (trivial solution).
If A is singular (i.e., not invertible, $\det(A) = 0$), then the system has infinitely many solutions or at least one nontrivial solution.
Correct Answer: (b) singular

4. Explanation: (B)

The given system of equations is: $AX = 0$
where A is a 3×3 matrix with rank = 2.
According to the Rank-Nullity Theorem:
$\text{rank}(A) + \text{nullity}(A) = n$
where:
rank(A) = number of linearly independent rows (or columns) of A.
nullity(A) = dimension of the null space (solution space of $AX = 0$).
n = number of columns of A.
For our given 3×3 matrix A: $\text{rank}(A) = 2, n = 3$
Applying the Rank-Nullity Theorem:
$2 + \text{nullity}(A) = 3$
$\text{nullity}(A) = 1$
Interpretation
The nullity(A) = 1 means that the solution space of $AX = 0$ has one independent solution vector (one free variable).
This means that there is one linearly independent nontrivial solution.
Correct Answer: (b) one independent solution

5. Explanation: (D)

We are given that matrix A satisfies the following eigenvector equations:
$A\begin{bmatrix}1\\ -1\end{bmatrix} = -\begin{bmatrix}1\\ -1\end{bmatrix}$
$A\begin{bmatrix}1\\ -2\end{bmatrix} = -2\begin{bmatrix}1\\ -2\end{bmatrix}$
This means that $\begin{bmatrix}1\\ -1\end{bmatrix}$ is an eigenvector with eigenvalue $\lambda_1 = -1$, and $\begin{bmatrix}1\\ -2\end{bmatrix}$ is an eigenvector with eigenvalue $\lambda_2 = -2$.
Let:
$P = \begin{bmatrix}1 & 1\\ -1 & -2\end{bmatrix}$
be the matrix whose columns are the eigenvectors.
The diagonal matrix D is:
$D = \begin{bmatrix}-1 & 0\\ 0 & -2\end{bmatrix}$
Since A has these eigenvectors and eigenvalues, it must satisfy the diagonalization formula:
$A = PDP^{-1}$
Compute P^{-1}
Since:
$P = \begin{bmatrix}1 & 1\\ -1 & -2\end{bmatrix}$
The determinant of P is:
$\det(P) = (1)(-2) - (1)(-1) = -2 + 1 = -1$
The inverse of P is:
$P^{-1} = \frac{1}{\det(P)}\begin{bmatrix}-2 & -1\\ 1 & 1\end{bmatrix} = \begin{bmatrix}2 & 1\\ -1 & -1\end{bmatrix}$
Compute $A = PDP^{-1}$
$A = \begin{bmatrix}1 & 1\\ -1 & -2\end{bmatrix}\begin{bmatrix}-1 & 0\\ 0 & -2\end{bmatrix}\begin{bmatrix}2 & 1\\ -1 & -1\end{bmatrix}$
First Multiply PD:
$PD = \begin{bmatrix}1 & 1\\ -1 & -2\end{bmatrix}\begin{bmatrix}-1 & 0\\ 0 & -2\end{bmatrix} =$
$\begin{bmatrix}(-1)(1) + (0)(1) & (1)(0) + (1)(-2)\\ (-1)(-1) + (-2)(0) & (-1)(0) + (-2)(-2)\end{bmatrix}$
$= \begin{bmatrix}-1 & -2\\ 1 & 4\end{bmatrix}$
Now Multiply $(PD)P^{-1}$:
$A = \begin{bmatrix}-1 & -2\\ 1 & 4\end{bmatrix}\begin{bmatrix}2 & 1\\ -1 & -1\end{bmatrix}$

$$= \begin{bmatrix} (-1)(2)+(-2)(-1) & (-1)(1)+(-2)(-1) \\ (1)(2)+(4)(-1) & (1)(1)+(4)(-1) \end{bmatrix}$$
$$= \begin{bmatrix} -2+2 & -1+2 \\ 2-4 & 1-4 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix}$$

Comparing with the options, we find that:

$A = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix}$

which corresponds to option (d).

Final Answer: $(d)\begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix}$

6. Explanation: (A)

We are given that A is an $n \times n$ real matrix satisfying $A^2 = I$,

This means A is involutory, i.e., its own inverse:

$A^2 = I \implies A^{-1} = A$

From $A^2 = I$, applying A to both sides of $Ax = y$ gives:

$A(Ax) = Ay \implies x = Ay$

Thus, $x = Ay$ is a solution.

Hence, a solution always exists.

Since $A^{-1} = A$, A is invertible if and only if it is nonsingular, meaning $\det(A) \neq 0$. However, if A has eigenvalues ± 1, it may be singular.

$\lambda^2 = 1 \implies \lambda = \pm 1$

If A has only eigenvalues ± 1 with no zero eigenvalues, then A is invertible, and the system has a unique solution given by $x = Ay$.

If A has a zero eigenvalue, the system could have either no solution or infinitely many solutions, depending on y.

However, since we showed $x = Ay$ is always a solution, there must be a unique solution.

7. Explanation: (C)

Each element of A is given by:

$$A = \begin{bmatrix} 2 & 3 & 4 & \dots & (n+1) \\ 3 & 4 & 5 & \dots & (n+2) \\ 4 & 5 & 6 & \dots & (n+3) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ (n+1) & (n+2) & (n+3) & \dots & 2n \end{bmatrix}$$

Let's express each row:

The first row is $(2,3,4,\dots,n+1)$.

The second row is $(3,4,5,\dots,n+2)$.

The third row is $(4,5,6,\dots,n+3)$.

Each row is obtained by adding 1 to the previous row.

This suggests that the rows are linearly dependent.

We find a basis by noting that any row can be written as a linear combination of the first $n-1$ rows. This suggests that at most $n-1$ rows are independent.

Since there are n rows but they are linearly dependent, the rank of A is at most $n-1$.

8. Explanation: (B)

For our given 3×3 matrix P:

$\text{rank}(P) + \text{nullity}(P) = 3$.

We are given that there exist two linearly independent vectors in the null space of P.

This means $\text{nullity}(P) = 2$.

From the rank-nullity theorem:

$\text{rank}(P) = 3 - \text{nullity}(P) = 3 - 2 = 1$.

Thus, the dimension of the range space of P is 1.

9. Explanation: (C)

If X has an eigenvalue λ, then:

1. The matrix $X + cI$ shifts the eigenvalues by c, so the eigenvalues of $X + cI$ are $\lambda + c$.
2. The matrix $(X + cI)^{-1}$ has eigenvalues $\frac{1}{\lambda + c}$.
3. The product of two matrices transforms eigenvalues accordingly.

Eigenvalues of $X + I$:

Since X has eigenvalues -2 and -3,

The eigenvalues of $X + I$ (i.e., adding identity I) are:

$(-2+1, -3+1) = (-1, -2)$.

Eigenvalues of $(X + I)^{-1}$:

The inverse of a matrix in terms of eigenvalues means reciprocating each eigenvalue:

$\left(\frac{1}{-1}, \frac{1}{-2}\right) = (-1, -\frac{1}{2})$.

Eigenvalues of $X + 5I$:

The eigenvalues are shifted by 5:

$(-2+5, -3+5) = (3,2)$.

Eigenvalues of $(X + I)^{-1}(X + 5I)$:

This product transforms each eigenvalue as:

$\left(-1 \cdot 3, -\frac{1}{2} \cdot 2\right) = (-3, -1)$.

10. Explanation: (D)

Applying P to the standard basis vectors:

1. Effect on $e_1 = (1,0,0)$:
$P\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = e_2$.
2. Effect on $e_2 = (0,1,0)$:
$P\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = e_3$.
3. Effect on $e_3 = (0,0,1)$:
$P\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = e_1$.

Thus, the transformation cycles the coordinates:

$(x, y, z) \to (z, x, y)$.

This cyclic permutation corresponds to a rotation by 120° about the axis $(1,1,1)$. This is because:

The matrix permutes the coordinates in a way characteristic of a rotation by 120° in a symmetric coordinate frame.

In a counterclockwise sense, moving each coordinate in a cyclic fashion among three components typically corresponds to 120° rotation in 3D space.

11. Explanation: (A)

We are given an $n \times n$ real matrix $A = [a_{ij}]$, where

$a_{ij} = \begin{cases} i, & \text{if } i = j \text{ (diagonal elements)} \\ 0, & \text{otherwise (off-diagonal elements)} \end{cases}$

$\lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 3, \ldots, \lambda_n = n$

Thus, the eigenvalues of A are:

$\{1,2,3, \ldots, n\}$

The sum of all eigenvalues of a matrix equals the trace of the matrix, which is the sum of its diagonal elements:

$$\sum_{i=1}^{n} i = 1 + 2 + 3 + \cdots + n$$

Using the formula for the sum of the first n natural numbers:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

12. Explanation: (C)

Step 1: Determinant Properties

The determinant of a product of matrices satisfies:

$$|XY| = |X| \cdot |Y| = |0_{n \times n}| = 0$$

Thus,

$|X| \cdot |Y| = 0$

which means that **at least one of** $|X|$ **or** $|Y|$ **must be zero**.

1. **Option (A):** $|X| = 0$ **and** $|Y| \neq 0$
   if Y were invertible ($|Y| \neq 0$), multiplying both sides by Y^{-1} would give $X = 0$, which contradicts the fact that X is non-zero
2. **Option (B):** $|\mathbf{X}| \neq 0$ and $|\mathbf{Y}| = 0$
   If X were invertible ($|X| \neq 0$), multiplying both sides of $XY = 0$ by X^{-1} would give $Y = 0$, which contradicts the fact that Y is non-zero.
3. Option (C): $|X| = 0$ and $|Y| = 0$
   both X **and** Y **must be non-invertible**, meaning $|X| = 0$ and $|Y| = 0$
4. **Option (D):** $|X| \neq 0$ **and** $|Y| \neq 0$
   This is **not possible**,
   $|X| = 0$ and $|Y| = 0$ (Option C)

13. Explanation: (B)

Given the eigenvalues of M are -3, -3, and 5, the eigenvalues of M^3 are:

$\lambda_1^3 = (-3)^3 = -27, \lambda_2^3 = (-3)^3 = -27, \lambda_3^3 = 5^3 = 125$

So, the eigenvalues of M^3 are -27, -27, and 125.

The eigenvector corresponding to the eigenvalue 5 for M is given as:

$$\mathrm{v} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

Since the eigenvalues of M^3 are the cubes of the eigenvalues of M, the eigenvector corresponding to the eigenvalue 125 of M^3 is the same as the eigenvector corresponding to the eigenvalue 5 of M.

Thus, the eigenvector corresponding to the eigenvalue 125 of M^3 is:

$$\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

Therefore, one of the eigenvectors of M^3 is: $(B)\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$

14. Explanation: (B)

We are given the matrix:

$A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix}$

and the identity matrix:

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

We need to compute A^3 and express it in the form:

$A^3 = kA + mI$

for some constants k and m.

$A^2 = A \cdot A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix} \cdot \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} (-5)(-5) + (-3)(2) & (-5)(-3) + (-3)(0) \\ (2)(-5) + (0)(2) & (2)(-3) + (0)(0) \end{bmatrix}$

$A^2 = \begin{bmatrix} 25 - 6 & 15 + 0 \\ -10 + 0 & -6 + 0 \end{bmatrix} = \begin{bmatrix} 19 & 15 \\ -10 & -6 \end{bmatrix}$

Compute A^3

$A^3 = A \cdot A^2 = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix} \cdot \begin{bmatrix} 19 & 15 \\ -10 & -6 \end{bmatrix}$

$A^3 =$

$\begin{bmatrix} (-5)(19) + (-3)(-10) & (-5)(15) + (-3)(-6) \\ (2)(19) + (0)(-10) & (2)(15) + (0)(-6) \end{bmatrix}$

$= \begin{bmatrix} -95 + 30 & -75 + 18 \\ 38 + 0 & 30 + 0 \end{bmatrix} = \begin{bmatrix} -65 & -57 \\ 38 & 30 \end{bmatrix}$

We assume:

$A^3 = kA + mI$

$\begin{bmatrix} -65 & -57 \\ 38 & 30 \end{bmatrix} = k\begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix} + m\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} -65 & -57 \\ 38 & 30 \end{bmatrix} = \begin{bmatrix} -5k + m & -3k \\ 2k & m \end{bmatrix}$

Equating elements:

1. $-5k + m = -65$
2. $-3k = -57 \implies k = 19$
3. $2k = 38 \implies k = 19$ (consistent)
4. $m = 30$

Thus, $k = 19$ and $m = 30$, so:

$A^3 = 19A + 30I$

The correct answer is:$19A + 30I$

which corresponds to option (B).

15. Explanation: (B)

The given matrix is:

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix}$$

The rank of a matrix is the number of linearly independent rows (or columns).

Perform row reduction (Gaussian elimination):

Swap R_1 and R_2 to get a leading 1 in the first column:

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & -1 \end{bmatrix}$$

Add R_1 to R_3 to eliminate the first column in R_3:

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{bmatrix}$$

Add R_2 to R_3:

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Since there are two nonzero rows, the rank of A is 2.

The nullity of A is:

$$\text{Nullity} = \text{Number of Columns} - \text{Rank}$$
$$= 3 - 2 = 1$$

The dimension of the null space is 1.

So, the correct answer is: (B) 1

16. Explanation: (A)

We are given the matrix:

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Eigenvalues λ satisfy:

$$\det(A - \lambda I) = 0$$
$$\begin{vmatrix} 0-\lambda & -1 \\ 1 & 0-\lambda \end{vmatrix} = 0$$
$$\lambda^2 - (0) + 1 = 0$$
$$\lambda^2 + 1 = 0$$
$$\lambda = \pm j$$

(where $j = \sqrt{-1}$ is the imaginary unit).

Eigenvector for $\lambda = j$:

$$(A - jI)x = 0$$
$$\begin{bmatrix} -j & -1 \\ 1 & -j \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

From the first row:

$$-jx_1 - x_2 = 0 \Rightarrow x_2 = -jx_1$$

Choosing $x_1 = 1$, we get: $x_2 = -j$

Thus, one eigenvector is: $\begin{bmatrix} 1 \\ -j \end{bmatrix}$

Eigenvector for $\lambda = -j$:

$$(A + jI)x = 0$$
$$\begin{bmatrix} j & -1 \\ 1 & j \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

From the first row:

$$jx_1 - x_2 = 0 \Rightarrow x_2 = jx_1$$

Choosing $x_1 = 1$, we get: $x_2 = j$

Thus, the second eigenvector is: $\begin{bmatrix} 1 \\ j \end{bmatrix}$

The correct pair of eigenvectors: $\begin{bmatrix} 1 \\ -j \end{bmatrix}, \begin{bmatrix} 1 \\ j \end{bmatrix}$

This matches option (A): $\begin{bmatrix} 1 \\ -j \end{bmatrix}, \begin{bmatrix} j \\ -1 \end{bmatrix}$

Correct Answer: (A) $\begin{bmatrix} 1 \\ -j \end{bmatrix}, \begin{bmatrix} j \\ -1 \end{bmatrix}$

17. Explanation: (C)

We are given the scalar-valued function:

$f(x) = x^T Ax + b^T x + c$

Where:

A is a symmetric positive definite matrix of size $n \times n$,

b and x are column vectors of size $n \times 1$,

c is a scalar constant.

The gradient of the function $f(x)$ is given by:

$$\nabla f(x) = \frac{d}{dx}(x^T Ax + b^T x + c)$$

The term $x^T Ax$ is a quadratic form, and since A is symmetric, we can use the following result for the gradient of a quadratic form:

$\nabla(x^T Ax) = 2Ax$

The gradient of $b^T x$ is simply b, and the gradient of the constant c is zero.

Thus, the gradient of $f(x)$ is:

$\nabla f(x) = 2Ax + b$

To find the critical point (where the minimum occurs), set the gradient equal to zero:

$2Ax + b = 0$

$2Ax = -b$

$$x = -\frac{1}{2}A^{-1}b$$

The minimum value of $f(x)$ occurs when:

$$x = -\frac{1}{2}A^{-1}b$$

Thus, the correct answer is: (C) $-\frac{1}{2}A^{-1}b$.

18. Explanation: (A)

The given matrix is:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 7 & 8 & 9 \\ 4 & 5 & 6 \end{bmatrix}$$

To find the eigenvalues, we need to solve the characteristic equation, which is:

$\det(A - \lambda I) = 0$

where A is the matrix and I is the identity matrix. So we calculate $A - \lambda I$:

$$A-\lambda I=\begin{bmatrix}1-\lambda & 2 & 3\\ 7 & 8-\lambda & 9\\ 4 & 5 & 6-\lambda\end{bmatrix}$$

Next, we calculate the determinant of the matrix $A-\lambda I$:

$$\det(A-\lambda I)=\det\begin{bmatrix}1-\lambda & 2 & 3\\ 7 & 8-\lambda & 9\\ 4 & 5 & 6-\lambda\end{bmatrix}$$

Expanding this determinant, we get:

$$\det(A-\lambda I)=(1-\lambda)\begin{vmatrix}8-\lambda & 9\\ 5 & 6-\lambda\end{vmatrix}-2\begin{vmatrix}7 & 9\\ 4 & 6-\lambda\end{vmatrix}+3\begin{vmatrix}7 & 8-\lambda\\ 4 & 5\end{vmatrix}$$

We now simplify the 2x2 determinants:

$$\begin{vmatrix}8-\lambda & 9\\ 5 & 6-\lambda\end{vmatrix}=(8-\lambda)(6-\lambda)-9\cdot 5$$

$$\begin{vmatrix}7 & 9\\ 4 & 6-\lambda\end{vmatrix}=7(6-\lambda)-9\cdot 4$$

$$\begin{vmatrix}7 & 8-\lambda\\ 4 & 5\end{vmatrix}=7\cdot 5-(8-\lambda)\cdot 4$$

After expanding and simplifying, we get a cubic equation:

$-\lambda^3+15\lambda+9=0$

Now, we solve the cubic equation:

$-\lambda^3+15\lambda+9=0$

This equation has three solutions, and one of them is $\lambda=0$, which corresponds to one of the eigenvalues.

The other two eigenvalues are found by solving the quadratic part of the equation, which gives us: $\lambda=1,1,-1$

The eigenvalues of the matrix are $\lambda=1,1,-1$, which matches option (C) (1, 1, -1).

This is the correct set of eigenvalues for the matrix A.

19. Explanation: (C)

The Rank-Nullity Theorem states that for an $n\times n$ matrix A,

Rank(A) + Nullity$(A)=n$

r + Nullity$(A)=n$

Nullity$(A)=n-r$

Thus, the number of independent solutions (nullity) is $n-r$.

20. Explanation: (D)

We need to determine which vector is not perpendicular to both given vectors:

$\mathrm{A}=\mathrm{i}+\mathrm{j}+\mathrm{k}=(1,1,1)$

$\mathrm{B}=\mathrm{i}+2\mathrm{j}+3\mathrm{k}=(1,2,3)$

A vector C is perpendicular to both A and B if:

$\mathrm{C}\cdot\mathrm{A}=0$

$\mathrm{C}\cdot\mathrm{B}=0$

A vector perpendicular to both A and B lies in the direction of their cross product:

$$\mathrm{A}\times\mathrm{B}=\begin{vmatrix}\mathrm{i} & \mathrm{j} & \mathrm{k}\\ 1 & 1 & 1\\ 1 & 2 & 3\end{vmatrix}$$

$$\mathrm{A}\times\mathrm{B}=\mathrm{i}\begin{vmatrix}1 & 1\\ 2 & 3\end{vmatrix}-\mathrm{j}\begin{vmatrix}1 & 1\\ 1 & 3\end{vmatrix}+\mathrm{k}\begin{vmatrix}1 & 1\\ 1 & 2\end{vmatrix}$$

$$=\mathrm{i}(1\times 3-1\times 2)-\mathrm{j}(1\times 3-1\times 1)+\mathrm{k}(1\times 2-1\times 1)$$

$$=\mathrm{i}(3-2)-\mathrm{j}(3-1)+\mathrm{k}(2-1)$$

$$=\mathrm{i}(1)-\mathrm{j}(2)+\mathrm{k}(1)$$

$$=(1,-2,1)$$

Thus, any vector perpendicular to both A and B is a scalar multiple of $(1,-2,1)$.

We compare the given options:

(A): $(1,-2,1)$

This is exactly the cross-product vector, so it is perpendicular.

(B): $(-1,2,-1)$

This is $-1\times(1,-2,1)$, meaning it is also perpendicular.

(C): $(0,0,0)$

The zero vector is perpendicular to everything.

(D): $(4,3,5)$

$$(4,3,5)\cdot(1,-2,1)=4(1)+3(-2)+5(1)$$

$$=4-6+5=3\neq 0.$$

Since the dot product is not zero, this vector is not perpendicular to both given vectors.

21. Explanation: (D)

When a point (x,y) is reflected across the X-axis, its new coordinates become $(x,-y)$

This transformation is represented by the following matrix equation:

$$\begin{bmatrix}x'\\ y'\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}$$

Thus, the transformation matrix for reflection across the X-axis is $\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$

22. Explanation: (C)

Step 1: Formulate the Characteristic Equation

The characteristic equation is given by:

$$\det\begin{bmatrix}1-\lambda & -1 & 5\\ 0 & 5-\lambda & 6\\ 0 & -6 & 5-\lambda\end{bmatrix}=0$$

Step 2: Calculate the Determinant

Expanding the determinant along the first column:

$$(1-\lambda)\cdot\det\begin{bmatrix}5-\lambda & 6\\ -6 & 5-\lambda\end{bmatrix}=0$$

$$(1-\lambda)\cdot[(5-\lambda)^2+36]=0$$

$(1-\lambda)=0\Rightarrow\lambda=1$, $(5-\lambda)^2+36=0$

$(5-\lambda)^2=-365-\lambda=\pm\sqrt{-36}=\pm 6i\lambda=5\pm 6i$

Thus, the eigenvalues of the matrix A are $1{,}5+6i$, and $5-6i$.

The correct answer is **(C) 1,5±j6**.

23. Explanation: (A)

The equation $N^2 = 0$ implies that N is a nilpotent matrix

A fundamental property of nilpotent matrices states that all of their eigenvalues must be zero. This is because if λ is an eigenvalue of N, then:

$Nv = \lambda v$

$N^2v = N(\lambda v) = \lambda(Nv) = \lambda^2 v.$

Since $N^2 = 0$,

$\lambda^2 v = 0.$

Since $v \neq 0$, it must be that $\lambda^2 = 0$, which implies $\lambda = 0$.

Since N is a 3×3 matrix and all of its eigenvalues must be zero, the correct choice is: 0,0,0

24. Explanation: (C)

For the given system of linear equations:

$3x + 2ky = -2$

$kx + 6y = 2$

The coefficient matrix is:

$A = \begin{bmatrix} 3 & 2k \\ k & 6 \end{bmatrix}$

For the system to have infinitely many solutions, the determinant must be zero:

$\begin{vmatrix} 3 & 2k \\ k & 6 \end{vmatrix} = 0$

$18 - 2k^2 = 0$

$2k^2 = 18$

$k^2 = 9$

$k = \pm 3$

3 and -3

Since -3 is an option, the correct answer is -3.

25. Explanation: (C)

Compute a · c

$(1)(x) + (2)(y) + (5)(z) = 0$

$x + 2y + 5z = 0$ (Equation 1)

Compute b · c

$(1)(x) + (2)(y) + (-1)(z) = 0$

$x + 2y - z = 0$ (Equation 2)

Subtract Equation 2 from Equation 1:

$(x + 2y + 5z) - (x + 2y - z) = 0$

$x + 2y + 5z - x - 2y + z = 0$

$6z = 0$

$z = 0$

Substituting $z = 0$ into Equation 2:

$x + 2y - 0 = 0$

$x + 2y = 0$

$x = -2y$

Thus, c is parallel to:

$-2\hat{\imath} + \hat{\jmath}$

or equivalently:

$2\hat{\imath} - \hat{\jmath}$

26. Explanation: (A)

Given that the vectors a and b lie in the plane $z = 0$, they must be of the form:

$\mathrm{a} = a_1\hat{\imath} + a_2\hat{\jmath}$

$\mathrm{b} = b_1\hat{\imath} + b_2\hat{\jmath}$

Since these vectors lie in the xy-plane, their z-components are zero.

$$\mathrm{a} \times \mathrm{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & 0 \\ b_1 & b_2 & 0 \end{vmatrix}$$

$\mathrm{a} \times \mathrm{b} = (a_1b_2 - a_2b_1)\hat{k}$

Since $\mathrm{a} \neq \alpha\mathrm{b}$, the determinant $a_1b_2 - a_2b_1$ is nonzero, meaning the cross product is a nonzero multiple of $\hat{k}$.

Thus, the required perpendicular vector is in the $\hat{k}$-direction, which matches option $\hat{k}$.

27. Explanation: (B)

We are given the system of linear equations:

$Ax = b$

$A^TAx = A^Tb$

$x = (A^TA)^{-1}A^Tb$

A is a 2×4 matrix, meaning it has more columns than rows.

A^TA is a 4×4 matrix.

The rank of A^TA is at most 2 (since the rank of a product cannot exceed the rank of the original matrix A, which has at most rank 2).

A square matrix is invertible if and only if it is full rank (i.e., has rank 4 in this case).

Since A^TA has at most rank 2, it cannot be full rank (rank 4), meaning it is not invertible in general.

Since A^TA is not necessarily invertible for all 2×4 matrices, we cannot always obtain a unique solution.

28. Explanation: (D)

We are given the matrix

$$M = \begin{bmatrix} 1 & -1 & 0 \\ 1 & -2 & 1 \\ 0 & -1 & 1 \end{bmatrix}$$

$Mv = \lambda v$

for some scalar λ.

option (D): $v = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

Compute Mv:

$$M\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \cdot 1 + (-1) \cdot 1 + 0 \cdot 1 \\ 1 \cdot 1 + (-2) \cdot 1 + 1 \cdot 1 \\ 0 \cdot 1 + (-1) \cdot 1 + 1 \cdot 1 \end{bmatrix} = \begin{bmatrix} 1 - 1 + 0 \\ 1 - 2 + 1 \\ 0 - 1 + 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Since

$M\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix} = 0 \cdot \begin{bmatrix}1\\1\\1\end{bmatrix}$,

the vector $\begin{bmatrix}1\\1\\1\end{bmatrix}$ is an eigenvector corresponding to the eigenvalue $\lambda = 0$.

Thus, the correct answer is:

(D) $\begin{bmatrix}1\\1\\1\end{bmatrix}$.

29. Explanation: (A)

We are given the row vectors:

$v = (1,0)$ and $w = (2,0)$

We need to calculate the rank of the matrix $M = 2v^T v + 3w^T w$, where v^T and w^T denote the transpose of the vectors v and w, respectively.

$v^T v$ is the product of a column vector $v^T = \begin{bmatrix}1\\0\end{bmatrix}$ and the row vector $v = \begin{bmatrix}1\\0\end{bmatrix}$:

$v^T v = \begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}$

$w^T w$ is the product of a column vector $w^T = \begin{bmatrix}2\\0\end{bmatrix}$ and the row vector $w = \begin{bmatrix}2\\0\end{bmatrix}$:

$w^T w = \begin{bmatrix}2\\0\end{bmatrix}\begin{bmatrix}2\\0\end{bmatrix} = \begin{bmatrix}4 & 0\\0 & 0\end{bmatrix}$

Now, we compute $M = 2v^T v + 3w^T w$:

$$M = 2\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix} + 3\begin{bmatrix}4 & 0\\0 & 0\end{bmatrix} = \begin{bmatrix}2 & 0\\0 & 0\end{bmatrix} + \begin{bmatrix}12 & 0\\0 & 0\end{bmatrix} = \begin{bmatrix}14 & 0\\0 & 0\end{bmatrix}$$

The matrix M is:

$M = \begin{bmatrix}14 & 0\\0 & 0\end{bmatrix}$

The rank of this matrix is 1, because it has only one non-zero row (or column).

Thus, the rank of the matrix M is: (A) 1.

30. Explanation: (A)

We are given the system of two equations:

1. $x - 2y + z = 0$
2. $x - z = 0$

We need to find the solution set S corresponding to this system.

From the second equation:

$x - z = 0 \Rightarrow x = z$

Substitute $x = z$ into the first equation:

$x - 2y + z = 0$

$z - 2y + z = 0 \Rightarrow 2z - 2y = 0 \Rightarrow z = y$

Thus, we have: $x = z = y$

We can express the solutions in terms of a single parameter α.

Let $x = y = z = \alpha$, where $\alpha \in \mathbb{R}$.

Thus, the solution set is:

$S = \{\alpha\begin{pmatrix}1\\1\\1\end{pmatrix} \mid \alpha \in \mathbb{R}\}$

Check the options

Option A: $S = \{\alpha\begin{pmatrix}1\\1\\1\end{pmatrix} \mid \alpha \in \mathbb{R}\}$

Correct — matches the form we derived.

Option B: $S = \{\alpha\begin{pmatrix}1\\1\\1\end{pmatrix} + \beta\begin{pmatrix}1\\0\\1\end{pmatrix} \mid \alpha, \beta \in \mathbb{R}\}$

Incorrect — has an extra term involving β.

Option C: $S = \{\alpha\begin{pmatrix}1\\1\\1\end{pmatrix} + \beta\begin{pmatrix}2\\1\\2\end{pmatrix} \mid \alpha, \beta \in \mathbb{R}\}$

Incorrect — includes another linear combination with an additional vector.

Option D: $S = \{\alpha\begin{pmatrix}1\\0\\1\end{pmatrix} \mid \alpha \in \mathbb{R}\}$

Incorrect — wrong solution vector.

Correct Answer: A

31. Explanation: (C)

To determine which set of vectors results in a matrix M with rank $k(M) = 3$, we need to check the linear independence of the column vectors. The rank of a matrix is the maximum number of linearly independent columns, and if the rank is 3, the three column vectors must be linearly independent.

Option A:

$v_1 = \begin{bmatrix}1\\0\\1\end{bmatrix}, v_2 = \begin{bmatrix}0\\-1\\0\end{bmatrix}, v_3 = \begin{bmatrix}1\\-1\\1\end{bmatrix}$

Form the matrix M:

$M = \begin{bmatrix}1 & 0 & 1\\0 & -1 & -1\\1 & 0 & 1\end{bmatrix}$

Perform row reduction:

$\begin{bmatrix}1 & 0 & 1\\0 & -1 & -1\\1 & 0 & 1\end{bmatrix} \rightarrow \begin{bmatrix}1 & 0 & 1\\0 & -1 & -1\\0 & 0 & 0\end{bmatrix}$

The rank is 2, as there are only 2 linearly independent rows. So, this option does not result in rank 3.

Option B:

$v_1 = \begin{bmatrix}1\\1\\1\end{bmatrix}, v_2 = \begin{bmatrix}-1\\0\\1\end{bmatrix}, v_3 = \begin{bmatrix}0\\0\\0\end{bmatrix}$

Form the matrix M:

$M = \begin{bmatrix}1 & -1 & 0\\1 & 0 & 0\\1 & 1 & 0\end{bmatrix}$

The third column is the zero vector, which is linearly dependent. So, the rank is 2, not 3.

Option C:

$v_1 = \begin{bmatrix}1\\0\\1\end{bmatrix}, v_2 = \begin{bmatrix}-1\\0\\1\end{bmatrix}, v_3 = \begin{bmatrix}1\\-1\\1\end{bmatrix}$

Form the matrix M:

$M = \begin{bmatrix}1 & -1 & 1\\0 & 0 & -1\\1 & 1 & 1\end{bmatrix}$

Perform row reduction:

$\begin{bmatrix}1 & -1 & 1\\0 & 0 & -1\\1 & 1 & 1\end{bmatrix} \to \begin{bmatrix}1 & -1 & 1\\0 & 0 & -1\\0 & 2 & 0\end{bmatrix}$

The matrix has 3 linearly independent rows, so the rank is 3. This option is correct.

Option D:

$v_1 = \begin{bmatrix}1\\1\\1\end{bmatrix}, v_2 = \begin{bmatrix}-1\\1\\-1\end{bmatrix}, v_3 = \begin{bmatrix}0\\-1\\0\end{bmatrix}$

Form the matrix M:

$M = \begin{bmatrix}1 & -1 & 0\\1 & 1 & -1\\1 & -1 & 0\end{bmatrix}$

Perform row reduction:

$\begin{bmatrix}1 & -1 & 0\\1 & 1 & -1\\1 & -1 & 0\end{bmatrix} \to \begin{bmatrix}1 & -1 & 0\\0 & 2 & -1\\0 & 0 & 0\end{bmatrix}$

The rank is 2, so this option does not result in rank 3.

The correct answer is (C).

32. Explanation: (A)

The elements of the matrix are defined based on the parity of the sum of their indices. Let's consider a few entries:

- If $i + j$ is even, $a_{ij} = -\beta$.
- If $i + j$ is odd, $a_{ij} = \beta$.

For $n = 3$, the matrix looks like this:

$A = \begin{bmatrix}-\beta & \beta & -\beta & \beta & -\beta & \beta\\ \beta & -\beta & \beta & -\beta & \beta & -\beta\\ -\beta & \beta & -\beta & \beta & -\beta & \beta\\ \beta & -\beta & \beta & -\beta & \beta & -\beta\\ -\beta & \beta & -\beta & \beta & -\beta & \beta\\ \beta & -\beta & \beta & -\beta & \beta & -\beta\end{bmatrix}$.

Let's call first row to be R_1.

Note that, all the odd numbered rows are identical to R_1 and even numbered rows are $-R_1$.

So, there is only one independent row.

So, the rank of the matrix A is 1.

Multiple Select Questions (MSQ)

1. Explanation: (A,C)

To determine which vector is an eigenvector of the matrix

$A = \begin{bmatrix}1 & 0\\-1 & -2\end{bmatrix}$

we check whether any given vector v satisfies the eigenvector equation: $Av = \lambda v$

where λ is a scalar (the eigenvalue) and v is a nonzero vector.

Option (a) $v = \begin{bmatrix}-1\\1\end{bmatrix}$

$Av = \begin{bmatrix}1 & 0\\-1 & -2\end{bmatrix}\begin{bmatrix}-1\\1\end{bmatrix}$

$= \begin{bmatrix}(1)(-1) + (0)(1)\\(-1)(-1) + (-2)(1)\end{bmatrix} = \begin{bmatrix}-1\\1-2\end{bmatrix} = \begin{bmatrix}-1\\-1\end{bmatrix}$

Since:

$\begin{bmatrix}-1\\-1\end{bmatrix} = (-1)\begin{bmatrix}-1\\1\end{bmatrix}$

this means v is an eigenvector with eigenvalue $\lambda = -1$.

Option (a) is correct.

We verify if any other option satisfies $Av = \lambda v$.

Option (b) $v = \begin{bmatrix}1\\-1\end{bmatrix}$**

$Av = \begin{bmatrix}1 & 0\\-1 & -2\end{bmatrix}\begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}1\\1\end{bmatrix}$

This is not a scalar multiple of $\begin{bmatrix}1\\-1\end{bmatrix}$, so not an eigenvector.

Option (c) $v = \begin{bmatrix}3\\-1\end{bmatrix}$**

$Av = \begin{bmatrix}1 & 0\\-1 & -2\end{bmatrix}\begin{bmatrix}3\\-1\end{bmatrix} = \begin{bmatrix}3\\-3+2\end{bmatrix} = \begin{bmatrix}3\\-1\end{bmatrix}$

This is equal to v itself, meaning it satisfies $Av = (1)v$, so it is an eigenvector with $\lambda = 1$.

Option (c) is also correct.

Option (d) $v = \begin{bmatrix}-2\\1\end{bmatrix}$**

$Av = \begin{bmatrix}1 & 0\\-1 & -2\end{bmatrix}\begin{bmatrix}-2\\1\end{bmatrix} = \begin{bmatrix}-2\\2-2\end{bmatrix} = \begin{bmatrix}-2\\0\end{bmatrix}$

This is not a scalar multiple of $\begin{bmatrix}-2\\1\end{bmatrix}$, so not an eigenvector.

Final Answer: (a) and (c)

2. Explanation: (A,C)

We are given the matrix:

$A = \begin{bmatrix}4 & 3\\9 & -2\end{bmatrix}$

with eigenvalues $\lambda_1 = -5$ and $\lambda_2 = 7$. We need to determine the corresponding eigenvectors.

Eigenvectors satisfy:

$(A - \lambda I)v = 0$

where I is the identity matrix.

Eigenvector for $\lambda = -5$

$A - (-5)I = A + 5I = \begin{bmatrix}4+5 & 3\\9 & -2+5\end{bmatrix} = \begin{bmatrix}9 & 3\\9 & 3\end{bmatrix}$

$\begin{bmatrix}9 & 3\\9 & 3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$

This gives the equation:

$9x + 3y = 0$ or, $x = -\frac{1}{3}y$

Choosing $y = 3$, we get $x = -1$, so an eigenvector is:

$\begin{bmatrix} -1 \\ 3 \end{bmatrix}$

which is a scaled version of Option (C) $[2, -6]$.

Thus, Option C is correct.

Eigenvector for $\lambda = 7$

$A - 7I = \begin{bmatrix} 4-7 & 3 \\ 9 & -2-7 \end{bmatrix} = \begin{bmatrix} -3 & 3 \\ 9 & -9 \end{bmatrix}$

$\begin{bmatrix} -3 & 3 \\ 9 & -9 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

This gives:

$-3x + 3y = 0$ or, $x = y$

Choosing $x = 1, y = 1$, we get:

$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

which matches Option (A) $[1,1]$.

Thus, Option A is correct.

Correct Answer: (A) $[1,1]$ and (C) $[2, -6]$

3. Explanation: (A,D)

We are given the matrix:

$M = \begin{bmatrix} 2 & 3 & 7 \\ 6 & 4 & 7 \\ 4 & 6 & 14 \end{bmatrix}$

$\det(M) = \begin{vmatrix} 2 & 3 & 7 \\ 6 & 4 & 7 \\ 4 & 6 & 14 \end{vmatrix}$

Expanding along the first row:

$\det(M) = 2\begin{vmatrix} 4 & 7 \\ 6 & 14 \end{vmatrix} - 3\begin{vmatrix} 6 & 7 \\ 4 & 14 \end{vmatrix} + 7\begin{vmatrix} 6 & 4 \\ 4 & 6 \end{vmatrix}$

$\det(M) = (2 \times 14) - (3 \times 56) + (7 \times 20)$

$= 28 - 168 + 140 = 0$

Since $\det(M) = 0$, the matrix is singular and does not have full rank (rank < 3).

Thus, option (D) is correct.

The rank of a matrix is the number of linearly independent rows (or columns). Since the determinant is zero.

To check if the rank is 2, we see if there exists a nonzero 2×2 minor. From our previous calculations:

$\begin{vmatrix} 6 & 4 \\ 4 & 6 \end{vmatrix} = 20 \neq 0$

Since there exists at least one nonzero 2×2 determinant, the rank of M is at least 2.

However, because the determinant of the full 3×3 matrix is zero, we conclude that the rank is exactly 2.

Thus, option (A) is correct.

Since the rank of the matrix is 2 ,

The correct statements are:

(A) The rank of M is 2, (D) The determinant of M is 0

Numerical Answer Type Questions (NAT)

1. Explanation: (-6,-6)

We are given the matrix:

$A = \begin{pmatrix} 2 & 1 & 1 \\ 2 & 3 & 4 \\ -1 & -1 & -2 \end{pmatrix}$

The eigenvalues of A are $1, -1, 3$.

Using the properties of eigenvalues, we know that if λ is an eigenvalue of A, then $\lambda^3 - 3\lambda^2$ is an eigenvalue of $A^3 - 3A^2$.

For $\lambda = 1$, the eigenvalue of $A^3 - 3A^2$ is:

$1^3 - 3(1^2) = 1 - 3 = -2$

For $\lambda = -1$, the eigenvalue of $A^3 - 3A^2$ is:

$(-1)^3 - 3((-1)^2) = -1 - 3 = -4$

For $\lambda = 3$, the eigenvalue of $A^3 - 3A^2$ is:

$3^3 - 3(3^2) = 27 - 27 = 0$

The trace of a matrix is the sum of its eigenvalues.

$\text{Tr}(A^3 - 3A^2) = (-2) + (-4) + 0 = -6$

2. Explanation: (1 to 1)

The matrix $A = vv^T$ is a rank-1 matrix if v is a non-zero vector. This is because the matrix A can be expressed as the outer product of two vectors, which always produces a matrix with at most rank 1.

To see this more clearly, observe that every row of the matrix A is a scalar multiple of the vector v^T. Hence, all rows of A are linearly dependent, meaning that the rank of A is 1.

Since v is non-zero, the rank of $A = vv^T$ is 1.

3. Explanation: (0.65 to 0.80)

Given:

$x_1 = \begin{bmatrix} 2 \\ 6 \\ 14 \end{bmatrix}, x_2 = \begin{bmatrix} -12 \\ 8 \\ 16 \end{bmatrix}$

The dot product is:

$x_1 \cdot x_2 = (2)(-12) + (6)(8) + (14)(16)$

$= -24 + 48 + 224 = 248$

$\|x_1\| = \sqrt{2^2 + 6^2 + 14^2} = \sqrt{4 + 36 + 196} = \sqrt{236}$

$\|x_2\| = \sqrt{(-12)^2 + 8^2 + 16^2} = \sqrt{144 + 64 + 256}$

$= \sqrt{464}$

$\cos\theta = \dfrac{248}{\sqrt{236} \cdot \sqrt{464}}$

$\sqrt{236} \approx 15.36, \sqrt{464} \approx 21.56$

$\cos\theta = \dfrac{248}{15.36 \times 21.56} = \dfrac{248}{331.24} \approx 0.7485$

$\theta = \cos^{-1}(0.7485)$

$\theta \approx 0.72$ radians

4. Explanation: (4 to 4)

The determinant of the given 4×4 matrix:

$$M = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 3 & 4 & 0 & 0 \\ 0 & 0 & 4 & 3 \\ 0 & 0 & 2 & 1 \end{bmatrix}$$

The matrix has a block diagonal form:

$$M = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$$

where:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$$

For a block diagonal matrix, the determinant is the product of the determinants of the diagonal blocks:

$$\det(M) = \det(A) \cdot \det(B)$$

$$\det(A) = (1 \times 4) - (2 \times 3) = 4 - 6 = -2$$

$$\det(B) = (4 \times 1) - (3 \times 2) = 4 - 6 = -2$$

$$\det(M) = (-2) \times (-2) = 4$$

Correct Answer: 4

5. Explanation: (4 to 4)

We are given the matrix:

$$A = \begin{bmatrix} 2 & 5 \\ 0 & 3 \end{bmatrix}$$

We need to compute the determinant of:

$$|A^4 - 5A^3 + 6A^2 + 2I|$$

Compute A^2

$$A^2 = A \cdot A = \begin{bmatrix} 2 & 5 \\ 0 & 3 \end{bmatrix} \cdot \begin{bmatrix} 2 & 5 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} (2 \times 2 + 5 \times 0) & (2 \times 5 + 5 \times 3) \\ (0 \times 2 + 3 \times 0) & (0 \times 5 + 3 \times 3) \end{bmatrix} = \begin{bmatrix} 4 & 25 \\ 0 & 9 \end{bmatrix}$$

Compute A^3

$$A^3 = A \cdot A^2 = \begin{bmatrix} 2 & 5 \\ 0 & 3 \end{bmatrix} \cdot \begin{bmatrix} 4 & 25 \\ 0 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} (2 \times 4 + 5 \times 0) & (2 \times 25 + 5 \times 9) \\ (0 \times 4 + 3 \times 0) & (0 \times 25 + 3 \times 9) \end{bmatrix} = \begin{bmatrix} 8 & 95 \\ 0 & 27 \end{bmatrix}$$

Compute A^4

$$A^4 = A \cdot A^3 = \begin{bmatrix} 2 & 5 \\ 0 & 3 \end{bmatrix} \cdot \begin{bmatrix} 8 & 95 \\ 0 & 27 \end{bmatrix}$$

$$= \begin{bmatrix} (2 \times 8 + 5 \times 0) & (2 \times 95 + 5 \times 27) \\ (0 \times 8 + 3 \times 0) & (0 \times 95 + 3 \times 27) \end{bmatrix} = \begin{bmatrix} 16 & 370 \\ 0 & 81 \end{bmatrix}$$

Compute $A^4 - 5A^3 + 6A^2 + 2I$

$$A^4 - 5A^3 + 6A^2 + 2I = \begin{bmatrix} 16 & 370 \\ 0 & 81 \end{bmatrix} - 5\begin{bmatrix} 8 & 95 \\ 0 & 27 \end{bmatrix} + 6\begin{bmatrix} 4 & 25 \\ 0 & 9 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 16 & 370 \\ 0 & 81 \end{bmatrix} - \begin{bmatrix} 40 & 475 \\ 0 & 135 \end{bmatrix} + \begin{bmatrix} 24 & 150 \\ 0 & 54 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} (16 - 40 + 24 + 2) & (370 - 475 + 150 + 0) \\ (0 - 0 + 0 + 0) & (81 - 135 + 54 + 2) \end{bmatrix} = \begin{bmatrix} 2 & 45 \\ 0 & 2 \end{bmatrix}$$

$$\det \begin{bmatrix} 2 & 45 \\ 0 & 2 \end{bmatrix} = (2 \times 2) - (45 \times 0) = 4$$

Correct Answer: 4

6. Explanation: (-8 to -8)

We are given the matrix:

$$A = \begin{bmatrix} 1 & 4 \\ -3 & \alpha \\ \beta & 6 \end{bmatrix}$$

and we are told that the rank of A is 1. This means that all the rows of the matrix must be linearly dependent.

For the rank of A to be 1, all rows must be scalar multiples of each other. That is, there exists some constants k_1 and k_2 such that:

1. $(-3, \alpha) = k_1(1,4)$
2. $(\beta, 6) = k_2(1,4)$

From the first equation:

$-3 = k_1(1), \alpha = k_1(4).$

$k_1 = -3, \alpha = -3 \times 4 = -12.$

From the second equation:

$\beta = k_2(1), 6 = k_2(4).$

$k_2 = \frac{6}{4} = \frac{3}{2}, \beta = \frac{3}{2} \times 1 = \frac{3}{2}.$

$\frac{\alpha}{\beta} = \frac{-12}{3/2} = -12 \times \frac{2}{3} = -8.$

Correct Answer: -8

7. Explanation: (-2 to -2)

We are given a 3×3 matrix P with eigenvalues $\frac{1}{4}$, 1, and -2, and we are asked to find the value of $|P^{-1}|$, rounded to the nearest integer.

The determinant of the inverse of a matrix P is the reciprocal of the determinant of P:

$$|P^{-1}| = \frac{1}{|P|}$$

The determinant of a matrix is the product of its eigenvalues. Given the eigenvalues of P are $\frac{1}{4}$, 1, -2, we can compute the determinant of P as:

$$|P| = \left(\frac{1}{4}\right) \times 1 \times (-2) = -\frac{1}{2}$$

Now, using the property that $|P^{-1}| = \frac{1}{|P|}$, we find:

$$|P^{-1}| = \frac{1}{-\frac{1}{2}} = -2$$

The value of $|P^{-1}|$ is -2.

8. Explanation: (2)

If $\begin{bmatrix} \alpha \\ 2\alpha \end{bmatrix}$ is an eigenvector, then $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ is also an eigenvector because $\begin{bmatrix} \alpha \\ 2\alpha \end{bmatrix} = \alpha \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

$$\therefore \begin{bmatrix} -1 & -1 \\ x & -4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -3 \\ x - 8 \end{bmatrix}$$

Now, $\begin{bmatrix} -3 \\ x - 8 \end{bmatrix}$ is scalar multiple of $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

$$\therefore \frac{1}{-3} = \frac{2}{x-8}$$

$$\therefore x = 2$$

Unit 2.2 Calculus

Multiple Choice Questions (MCQ)

[1999: 1 Mark]

1. $\lim_{x\to 0} \frac{1}{10}\frac{1-e^{-j5x}}{1-e^{-jx}} =$ _____.

(A) 0 (B) 1.1

(C) 0.5 (D) 1

[2001: 1 Mark]

2. $\lim_{x\to \pi/4} \frac{sin2\left(x-\frac{\pi}{4}\right)}{x-\frac{\pi}{4}} =$ _____.

(A) 0 (B) 1/2

(C) 1 (D) 2

[2004: 2 Mark]

3. Vectors $\vec{x}_1 = 5i - 2j + k \; and \; \vec{x}_2 = 4i - j + 2k$ are transformed into $\vec{y}_1 = A\vec{x}_1$ and $\vec{y}_2 = A\vec{x}_2$ using an orthogonal matrix $A = \frac{1}{3}\begin{bmatrix} 2 & 1 & 2 \\ -2 & 2 & 1 \\ 1 & 2 & -2 \end{bmatrix}$

The cosine of the angle between $\vec{y}_1$ and $\vec{y}_2$ is

(A) $\frac{8}{\sqrt{70}}$ (B) $\frac{\sqrt{3}}{2}$

(C) $\frac{1}{\sqrt{70}}$ (D) $\frac{1}{2}$

[2005: 1 Mark]

4. If a vector $\vec{R}(t)$ has a constant magnitude, then

(A) $\vec{R} \cdot \frac{d\vec{R}}{dt} = 0$ (B) $\vec{R} \times \frac{d\vec{R}}{dt} = 0$

(C) $\vec{R} \cdot \vec{R} = \frac{d\vec{R}}{dt}$ (D) $\vec{R} \times \vec{R} = \frac{d\vec{R}}{dt}$

[2005: 2 Mark]

5. A scalar field is given by $f = x^{2/3} + y^{2/3}$, where x and yare the Cartesian coordinates. The derivative of f along the line $y = x$ directed away from the origin, at the point (8, 8) is

(A) $\frac{\sqrt{2}}{3}$ (B) $\frac{\sqrt{3}}{2}$

(C) $\frac{2}{\sqrt{3}}$ (D) $\frac{3}{\sqrt{2}}$

[2005: 1 Mark]

6. If $f = a_0x^n + a_1x^{n-1}y + \ldots\ldots + a_{n-1}xy^{n-1} + a_ny^n$, where, a, (i = 0 to n) are constants, then

$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$ *is*

(A) $\frac{f}{n}$ (B) $\frac{n}{f}$

(C) nf (D) $n\sqrt{f}$

[2005: 1 Mark]

7. The value of the integral $\int_{-1}^{1} \frac{1}{x^2} dx$ is

(A) 2 (B) does not exist

(C) -2 (D) ∞

[2005: 2 Mark]

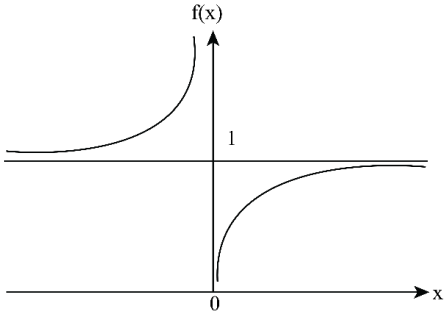
8. Given a real-valued continuous function $\mathbf{f(t)}$ defined over $[0, 1]$, $\lim_{t\to 0} \frac{1}{t}\int_0^1 \mathbf{f}(x)dx$ is

(A) ∞ (B) 0

(C) $f(1)$ (D) (0)

[2005: 2 Mark]

9. The plot of a function f(x) is shown in the following figure. A possible expression for the function f(x) is



(A) $exp(|x|)$ (B) $exp\left(\frac{-1}{x}\right)$

(C) $exp(-1)$ (D) $exp\left(\frac{1}{x}\right)$

[2006: 2 Mark]

10. The function $y = sin\phi \; (\phi > 0)$ is approximated as $y = \phi$, where is in radian. The maximum value of ϕ for which the error due to the approximation is within ± 2% is

(A) 0.1 rad (B) 0.2 rad

(C) 0.3 rad (D) 0.4 rad

[2007: 2 Mark]

11. The polynomial $p(x) = x^5 + x + 2$ has

(A) all real roots

(B) 3 real and 2 complex roots

(C) 1 real and 4 complex roots

(D) all complex roots

[2007: 2 Mark]

12. For real x, the maximum value of $\frac{e^{\sin x}}{e^{\cos x}}$ is

(A) 1 (B) e

(C) $e^{\sqrt{2}}$ (D) ∞

[2007: 2 Mark]

13. Consider the function $f(x) = |x|^3$, where x is real. Then the function $f(x)$ at $x = 0$ is

(A) continuous but not differentiable

(B) once differentiable but not twice

(C) twice differentiable but not thrice

(D) thrice differentiable

[2007: 2 Mark]

14. The value of the integral $\int_0^\infty \int_0^\infty e^{-x^2}e^{-y^2}dxdy$ is

(A) $\sqrt{\pi}/2$ (B) $\sqrt{\pi}$

(C) π (D) $\pi/4$

[2008: 1 Mark]

15. Given $y = x^2 + 2x + 10$, the value of $\frac{dy}{dx}\Big|_{x=1}$ is equal to

(A) 0 (B) 4

(C) 12 (D) 13