# P R E F A C E

The idea for the book **"CUET PG MATHEMATICS PYQ cum PRACTICE BOOK"** arose from the need for a comprehensive resource to revise all the concepts required for entry-level Mathematical sciences competitive examinations, specifically CUET PG MATHEMATICS. This book may also prove beneficial for IIT JAM MA and other M.Sc. entrance examinations.

The book covers sorted and detailed solutions to previous year questions of "CUET PG MATHEMATICS" and includes numerous practice questions. The questions are sorted according to the updated syllabus of "CUET PG MATHEMATICS" and are organized into 13 chapters, namely:

1. REAL ANALYSIS
2. METRIC SPACE AND TOPOLOGY
3. COMPLEX ANALYSIS
4. GROUP THEORY
5. RING & FIELD THEORY
6. LINEAR ALGEBRA
7. INTEGRAL CALCULUS
8. VECTOR CALCULUS
9. ORDINARY DIFFERENTIAL EQUATIONS
10. PARTIAL DIFFERENTIAL EQUATION
11. CALCULUS OF VARIATION AND INTEGRAL EQUATION
12. NUMERICAL ANALYSIS
13. LINEAR PROGRAMMING

Each chapter features sub-topic-wise sorting, providing a systematic approach for targeting the CUET exam. The answers to each question were provided using standard books or the author's best knowledge.

The ultimate goal of this book is to offer readers the best revision of all the concepts required for the CUET PG Mathematics examination, making it a valuable resource for all other M.Sc. entrance examinations, including IIT JAM MA. Every effort has been made to ensure that the book is error-free, and I am open to readers' feedback, criticism, and suggestions for improving the content of this book.

# A C K N O W L E D G E M E N T

First and foremost, I would like to express my gratitude to the entire Institute for Advanced Studies (IFAS) student community for being a constant source of inspiration and encouragement throughout this writing journey.

I extend my sincere thanks to Mr. Radheshyam Choudhary and Mr. Dadasaheb Sondge for their invaluable assistance and critical suggestions, which played a pivotal role in completing this work. Special appreciation goes to Mr. Chandan Kumar Roy and Mr. Mahesh Shelke for their thoughtful reviews and assistance in structuring the content systematically. Your comments have been instrumental in enhancing the quality of this book.

I am also indebted to Mr. Manoj Gunjal, Mr. Vikas Yadav, Mr. Rohit Muranjan, Mr. Omkar Girkar, Mr. Gaurav Agnihotri, Mr. Md. Sohail, and Mr. Lokesh Pachlore for their meticulous proofreading and verification of answers. Additionally, heartfelt thanks to Miss Rutuja Vetal for her contribution to sorting and compiling questions by chapter and section.

A special acknowledgment is due to the entire IFAS team, where I not only continued my teaching journey but also gained valuable insights into the multifaceted process of creating this book.

This book is the result of a collaborative effort, and its realization wouldn't have been possible without the exceptional members of the IFAS publication team. Throughout the production, I had the pleasure of collaborating with dedicated and creative individuals from IFAS publications. I extend my special thanks to Mr. Vikendra Metha for translating our ideas into a captivating cover page design and text diagrams. Appreciation also goes to Mr. Chandar Singh and his team for their contributions to formatting and typesetting.

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# REAL ANALYSIS

CHAPTER 1

## Unit 1.1: Point Set Topology

### [CUET PG]

**[CUET PG 2022]**

1. Let A and B be two non-empty subsets of $R$. If $\underline{A}, A°$, and A' denote closure, interior and set of limit points of A respectively, then
   (1) $A° \cap \underline{B}$ is open (2) $A° \cap B'$ is open
   (3) $\underline{A} \cap B'$ is closed (4) $\underline{A} \cap B$ is closed

| Answer Key |
|---|
| 1 |
| 3 |

### [CU-CET]

**[CUCET 2016]**

1. Consider the set $S = \left\{x \in R : \frac{2x+1}{x+2} < 1\right\}$,where R is the set of reals. Determine which one of the following statements about S is correct.
   (1) S is bounded below but not above and $infS = -2$
   (2) S is bounded above but not below and $supS = 1$
   (3) S is bounded both below and above with $infS = -2, supS = 1$
   (4) S is neither bounded below nor above

**[CUCET 2016]**

2. Consider the set $S = \left\{\frac{mn}{1+m+n} : m, n \; are \; natural \; numbers\right\}$. Then determine which one of the following statements is correct.
   (1) S is a bounded set
   (2) S is bounded below with $infS = \frac{1}{3}$ but not bounded above
   (3) S is bounded above with $supS = 1$ but not below
   (4) S is neither bounded below nor above

**[CUCET 2016]**

3. Let $X = \{x \in R : x \neq 1, 2, \ldots, 100\}$ be a subset of R. Determine which one of the following statements is true
   (1) Integers 1, 2,...,100 are the only limit points of $X$
   (2) No limit point of $X$ lies between 1 and 100
   (3) X is a closed subset of R
   (4) X is an open subset of R

**[CUCET 2016]**

4. Let $X = \left\{\frac{1}{n} : n \in Z, n \neq 0\right\}$be a subset of R (Z is the set of all integers). Determine which one of the following properties of X is true.
   (1) X is a bounded set
   (2) X is an open subset of R
   (3) X is a closed subset of R
   (4) X has no limit point in R

**[CUCET 2016]**

5. The set Z of all integers
   (1) is an open subset of R
   (2) is a closed subset of R
   (3) is a compact subset of R
   (4) has infinitely many limit points in R

**[CUCET 2016]**

6. Determine which one of the following subsets of R is connected.
   (1) $(-\infty, 0) \cup (0, \infty)$
   (2) The set Q of all rational numbers
   (3) $\cup_{n=1}^{\infty}(-n, n)$
   (4) $R \; Z$

**[CUCET 2016]**

7. Which one of the following statements is true
   (1) Every closed interval in R is homeomorphic to R
   (2) Every open interval of the type $(a, b)$ is homeomorphic to R
   (3) Every interval of the type $[a, b)$ is homeomorphic to R
   (4) Every interval of the type $(a, b]$ is homeomorphic to R

**[CUCET 2017]**

8. The set $U = \left\{x \in R : sin \; sin \; x = \frac{1}{2}\right\}$ is
   (1) open
   (2) closed
   (3) both open and closed
   (4) neither open nor closed

**[CUCET 2021]**

9. Let $Q$ denotes the complement of the set of rational numbers $Q$. The set of all boundary points of $Q$ in the set of real numbers $R$ is

(1) $Q^c$ (2) $R$
(3) $\phi$ (4) $Q$

**[CUCET 2021]**

10. The set $U = \{x \in R \mid sinx = \frac{1}{2}\}$ is
(1) closed
(2) open
(3) neither open nor closed
(4) both open and closed

| Answer Key | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** |
| 3 | 2 | 4 | 1 | 2 | 3 | 2 | 2 | 2 | 1 |

**[PRACTICE QUESTIONS]**

1. Let $A$ be a closed subset of $\mathbb{R}, A \neq \phi, A \neq \mathbb{R}$. Then $A$ is
(1) the closure of the interior of A.
(2) a countable set.
(3) a compact set.
(4) not open.

2. Let A and B be two sets having $m$ and $n$ many elements respectively, where $m < n; m, n \in \mathbb{N}$. The number of injective functions possible from A to B is:
(1) $m^n$ (2) ${}^nP_m$
(3) ${}^nC_m$ (4) $n^m$

3. The set of rational numbers has:
(1) Least upper bound property
(2) Greatest lower bound property
(3) Archimedean property
(4) None of the above

4. Which of the following statement is not true?
(1) $\sqrt{2}$ is an irrational number
(2) $\sqrt{2}$ is not an algebraic number
(3) $\sqrt{2}$ is not a transcendental number
(4) $\sqrt{2}$ is not a rational number

5. Which set is uncountable?
(1) The set of positive primes
(2) The set of integers
(3) The set of rational numbers
(4) The set of irrational numbers in [0, 1]

6. An algebraic number is a root of a polynomial whose coefficients are rational. The set of algebraic number is
(1) finite
(2) countably infinite
(3) uncountable
(4) none of these

7. Which one of the following does not imply $= 0$ ?
(1) For all $\varepsilon > 0, 0 \leq \alpha < \varepsilon$
(2) For all $\varepsilon > 0, -\varepsilon < \alpha < \varepsilon$
(3) For all $\varepsilon > 0, \alpha < \varepsilon$
(4) For all $\varepsilon > 0 - \varepsilon, < \alpha < \varepsilon$

8. Let $S$ and $T$ be subsets of $\mathbb{R}$. Select the incorrect statement:
(1) $(\text{int}\, S) \cap (\text{int}\, T) = \text{int}(S \cap T)$
(2) $(\text{int}\, S) \cup (\text{int}\, T) \subset \text{int}(S \cup T)$
(3) $\overline{S}$ is closed in $\mathbb{R}$
(4) $\overline{T}$ is the largest closed set containing $T$

9. Choose the correct statements
(1) $\cup_{n=1}^{\infty} [1/n, 2] = [0,2]$
(2) $\cup_{n=1}^{\infty} [1/n, 2] = (0,2]$
(3) $\cap_{n=1}^{\infty} (1/n, 2] = (1,2]$
(4) $\cap_{n=1}^{\infty} \left(1 - \frac{1}{n}, 2\right] = (1,2)$

10. If $\mathbb{Q} \subset A \subset \mathbb{R}$, which of the following must be true?
(1) If $A$ is open, then $A = \mathbb{R}$
(2) If $A$ is closed, then $A = \mathbb{R}$
(3) If $A$ is uncountable, then $A$ is closed
(4) If $A$ is countable, then $A$ is closed

11. Which one of the following set is not countable?
(1) $\mathbb{N}^r$, where $r \geq 1$ and $\mathbb{N}$ is the set of natural numbers
(2) $\{0,1\}\mathbb{N}$, the set of all the sequences which takes values 0 and 1.
(3) $\mathbb{Z}$, set of integers
(4) $\sqrt{2}\mathbb{Q}, \mathbb{Q}$ is set of rational numbers.

12. Let $X$ be a countably infinite subset of $\mathbb{R}$ and $A$ be a countably infinite subset of $X$. Then the set $X \backslash A = \{x \in X | x \notin A\}$
(1) is empty
(2) is a finite set.
(3) can be a countably infinite set.
(4) can be an uncountable set

13. The subset $A = \{x \in \mathbb{Q}: x^2 < 4\}$ of $\mathbb{R}$ is
    (1) bounded above but not bounded below
    (2) bounded above and sup $A = 2$.
    (3) bounded above but does not have a supremum.
    (4) not bounded above.

14. The subset $A = \{x \in \mathbb{Q}: -1 < x < 0\} \cup$ of $\mathbb{R}$ is
    (1) bounded, infinite set and has a limit point in $\mathbb{R}$
    (2) unbounded, infinite set and has a limit point in $\mathbb{R}$
    (3) unbounded, infinite set and does not have a limit point in R
    (4) bounded, infinite set and does not have a limit point in $\mathbb{R}$

15. Let
    $S = \prod_{n=1}^{\infty}\left[-\frac{1}{2}, 1+\frac{1}{n}\right]$
    then $S$ equals [DU-2017]
    (1) [0,1] (2) (0,1]
    (3) (0,1) (4) [10,1)

16. Consider the sets
    $S = \left\{\frac{1}{n}: n \in \mathbb{N} \text{ and } n \text{ is prime}\right\} T = \{x^2: x \in \mathbb{R}\}$
    Then
    (1) $sup(S \cap T) = 1$
    (2) $\sup S = 1 \text{ and } \inf\ \mathrm{T} = 0$
    (3) $\sup S = \frac{1}{2} \text{and } \inf\ \mathrm{T} = 0$
    (4) $\inf(S \cup T) = \frac{1}{2}$

17. Which of the following sets is in one-to-one correspondence with $\mathbb{N}$
    (I) $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\right\}$
    (II) $\{\ldots 3, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
    (III) $\left\{\frac{P}{q}: p, q \in \mathbb{N}, q \neq 0\right\}$
    (IV) $\left\{\frac{P}{q}: p, q \in \mathbb{N}\right\}$
    (1) (I) and (II) (2) (I), (II) and (II)
    (3) (I) and (IV) (4) All of above

18. Let $X = \{(x, y) \in \mathbb{R}^2: x \in \mathbb{Q}, y \in \mathbb{R}\backslash\mathbb{Q}\}$. Where $\mathbb{Q}$ is the set of rationals. Then
    (1) $X$ is an open and dense subset of $\mathbb{R}^2$
    (2) $X$ is an open but not dense subset of $\mathbb{R}^2$
    (3) $X$ is not an open but a dense subset of $\mathbb{R}^2$
    (4) $X$ is neither an open nor a dense subset of $\mathbb{R}^2$

19. Let $S = \{x \in \mathbb{R}: 3 - x^3 > 0\}$. Then
    (1) $S$ is bounded above and 3 is the least upper bound of $S$.
    (2) $S$ is bounded above and does not have a least upper bound in $\mathbb{R}$
    (3) $S$ is bounded above and does not have a least upper bound in $Q$ , the set of rational members.
    (4) $S$ is not bounded above.

20. Let $P$ be the set of all the polynomials with rational coefficients and $S$ be the set of all sequences of natural numbers. Then which one of the following statement is true?
    (1) $S$ is countable but is not
    (2) Both the sets $P$ and $S$ are uncountable
    (3) Both the sets $P$ and $S$ are countable.
    (4) $P$ is countable but $S$ is not

21. Let $S = \cap_{n=1}^{\infty}\left[2 - \frac{1}{n}, 3 + \frac{1}{n}\right]$.
    Then $S$ equals
    (1) (2,3] (2) [2,3]
    (3) [2,3) (4) (2,3).

22. Let $S = = \cap_{n=1}^{\infty}\left[0, \frac{1}{n}\right]$ then
    (1) $\limsup a_n = +\infty, \liminf a_n = -1$
    (2) $\limsup a_n = +\infty, \liminf a_n = 0$
    (3) $\limsup a_n \neq +\infty, \liminf a_n = -\infty$
    (4) $\limsup a_n = 1, \liminf a_n = -1$

23. Let $S = \cap_{n=1}^{\infty}\left[0, \frac{1}{n}\right]$. Then which one of the following statement is true?
    (1) $\inf S > 0$
    (2) $\sup S = 1 \text{ and } \inf S = 0$
    (3) $\sup > 0$
    (4) $\sup S = \inf S = 0$

24. Let $S = \{x \in R \backslash x^2 \leq 5\} \cap \mathbb{Q}$ which of the following statements is true about S ?
    (1) Sis bounded above and $sup, S \in \mathbb{Q}$
    (2) S is bounded above and . $sup\ S$
    (3) S is a closed interval.
    (4) S is an open interval

25. The set of all real numbers $x$ for which there is some positive real number $y$ such that $x < y$ equal to
(1) $\mathbb{R}$
(2) The set of all negative real numbers
(3) $\{0\}$
(4)None of the above

26. Let $X \subset N$ be a nonempty finite set and$\subset$ $N$ be an infinite set. Define $A = \{x - y : x \in X \text{ and } y \in Y\}$. Then
(1) $\inf(A) = -\infty \text{ and } \sup(A) = \infty$.
(2) $inf(A) = -\infty \text{ and } sup(A) < \infty$.
(3) $inf(A) > -\infty \text{ and } \sup(A) = \infty$.
(4) $inf(A) = -\infty \text{ and } sup(A) < \infty$.

27. Let $X$ be $a$ nonempty set and $A, B \subset X. If (A \cup B) \setminus (A \cap B)$ is $a$ finite set, then,
(1) Both A and B are finite sets.
(2) Atleast one of A, B is a finite set
(3) X is a finite set
(4) None of the above

28. Let $X$ be a set with 2m+1 elements, where $m \geq 4$. The number of subsets of $X$ having at least $m + 1$ elements is
(1) $2^{2m} + 1$ (2) $2^{2m} - 1$
(3) $2^{2m}$ (4) $2^m + 1$

29. Let $A \Delta B$ denote the symmetric difference of two sets : $A \Delta B := (A \setminus B) \cup (B \setminus A)$ which of the following is true.
(1) $A \cap (B \Delta A) = \emptyset \; alway \; hold \; true$
(2) $(A \cup (A \Delta B) \cap B \neq B \; always \; hold \; true$ .
(3) $A \Delta B = (A \cup B) \cap (A^c \cap B^c) \; always \; holds \; true$
(4) None of the above is true

30. For subsets $T_1$ and $T_2$ of $\mathbb{R}$,
we define $T_1 + T_2 := \{t_1 + t_2 | t_1 \in T_1, t_2 \in T_2\}$.
Let
$S_1 := (\mathbb{Q} \cap [0,1]) + \{\sqrt{2}\}$ and
$S_2 := (\mathbb{Q} \cap [1,2]) + \{\sqrt{3}\}$. Let $S = S_1 \cup S_2$.
Which one of the following is true?
(1) There exists a one to one map from [0,1] to S
(2) There exists an onto map from S to [0,1]
(3) $\neq S_1 = \#S_2$
(4) None of the above

31. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous map and S = $\left\{\pi + \frac{(-1)^n}{2n+1} | n \in \mathbb{N}\right\} \cup (0,1)$. Which one of the following is necessarily true?
(1) $f((0,1))$ is an open set
(2) $f(\bar{S})$ is compact
(3) $f(S)$ is compact
(4) $f(S)$ is closed but not compact

32. Which of the following is a compact subset of $\mathbb{R}$ ?
(1) $\left\{-\frac{1}{n} : \epsilon \mathbb{N}\right\} \cup \{0\} \cup (0, -]$
(2) $\left\{-\frac{1}{n} : \epsilon \mathbb{N}\right\} \cup (0, -]$.
(3) $\{1,2,3,\} \cup \left\{6 + \frac{1}{n} : n \in \mathbb{N}\right\}$.
(4) $\left\{-\frac{1}{n} : \epsilon \mathbb{N}\right\} \cup (0,2]$

33. Which of the following sets has/have the same cardinality as $\mathbb{R}$?
A. The set of all real sequences
B. The set of all continuous functions from $\mathbb{R}$ to $\mathbb{R}$
C. The set of all functions from $\mathbb{R}$ to the two-element set $\{0,1\}$
(1) A only (2) B only
(3) A and B only (4) C only

34. Let $f : \mathbb{R}^3 \to \mathbb{R}$ be a continuous function such that $f(\alpha v) = f(v)$ for all $\alpha \in \mathbb{R}, \alpha > 0$ and all $v \in \mathbb{R}$. Consider the following statements:
A. The image of $f$ is compact.
B. The image of $f$ is uncountable.
Which of the following is correct?
(1) A is true but B is false.
(2) B is true but A is false.
(3) Both A and B are true.
(4) Both A and B are false.

35. for any pair of subsets $C$ and $D$ of $\mathbb{R}^2$,let $C^0$ denote the interior of $C$, and $C/D$ denote the set of points in $C$ which do not belong to $D$. Consider the following statements:
A. $\left(\frac{C}{D}\right)^0 = C^0/D^0$
B. $\left(\frac{C^0}{D}\right)^0 = C^0/D^0$
Which is the following is necessarily correct?

(1) A is true but B is false
(2) B is true but A is false
(3) Both A and B are true
(4) Both A and B are false.

36. Consider the subset $S := \{(x, y) \in \mathbb{R}^2 : y = x \text{ or } |x| + |y| = 1\}$ of $\mathbb{R}^2$. Consider the following assertions about S.
A. S is a reflexive relation on $\mathbb{R}$.
B. S is a transitive relation on $\mathbb{R}$.
C. There exists a surjective map from $\mathbb{R}$ onto S.
D. There exists an injective map from S into $\mathbb{Q}^3$.
Which of the above assertions is/are correct?
(1) A only (2) A and D only
(3) A and C only (4) A and B only

37. Let $S = \{a + b\sqrt{2} : a, b \in \mathbb{Z}\} \subset \mathbb{R}$.
Which of the following assertion about S is correct
(1) For any two maps $f : \mathbb{Q} \times \mathbb{Q} \to \mathbb{R}$ and $g : \mathbb{R} \to S$, their compositions $g\ 0\ f$ is never surjective
(2) There exists a surjective map from S onto $R^2/Z^2$
(3) There exists an injective map from $R/Q$ into S.
(4) For any two maps $f : S \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}^3$, their composition $g\ 0\ f$ has a countable range.

38. Consider the following
A. Union of any collection of open sets in $\mathbb{R}^n$ is open
B. Intersection of any collection of open sets in $R^n$ is open. Then
(1) B is true and A is false.
(2) A is true and B is false.
(3) Both A and B are true.
(4) both A and B are false.

39. Suppose $\mathcal{R}$ Relation defined on $\mathbb{N}_{by}(a, b) \in \mathcal{R}$- iff $a - b$ is a multiple for 4 or a multiple of 6. Consider the statements:
A. $\mathcal{R}$ is symmetric.
B. $\mathcal{R}$ is transitive.
C. There is an infinite subset $U \subset \mathbb{N}$ or $\mathbb{N}$ such that $\mathcal{R}$ defined on $U$ is an equivalence relation. Then the number of correct statements above is
(1) 1 (2) 2
(3) 3 (4) 0

| Answer Key | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** |
| 4 | 2 | 3 | 2 | 4 | 2 | 3 | 4 | 2 | 2 |
| **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** |
| 2 | 3 | 2 | 2 | 1 | 3 | 4 | 3 | 2 | 4 |
| **21** | **22** | **23** | **24** | **25** | **26** | **27** | **28** | **29** | **30** |
| 2 | 2 | 4 | 3 | 4 | 2 | 4 | 3 | 4 | 3 |
| **31** | **32** | **33** | **34** | **35** | **36** | **37** | **38** | **39** | |
| 2 | 1 | 3 | 1 | 1 | 4 | 4 | 2 | 2 | |

## EXPLANATIONS

### [CUET PG]

**1. Explanation:**

Let $A$ & $B$ be two non-empty subject of $\mathbb{R}$.

Let

$A = Q \qquad B = Q$

$\bar{A} = \mathbb{R} \qquad A^o = \phi \qquad B' = \mathbb{R}$

$\bar{B} = \mathbb{R}$

1. $\bar{A} \cup B° = \mathbb{R} \cap Q = Q$

   Neither open nor closed (4) incorrect.

2. $A = [a\ b] B = [a, b]$

   $A° = (a, b) \qquad B° = (a, b)$

   $\bar{A} = [a, b] \qquad \bar{B} = [a, b]$

   $A° \cup B' = [a, b]$- closed set.

   So (2) incorrect.

   $A = [0, 2] \qquad B = [1, 3]$

   $A° = (0, 2) \qquad \bar{B} = [1, 3]$

   $B' = [1, 3]$

(1) $A° \cap \bar{B} = [1, 2)$

Option A incorrect.

(2) $A° \cup B = (0, 3]$

2 incorrect.

3 correct.

### [CU-CET]

**1. Explanation:**

$S = \left\{x \in \mathbb{R} \frac{2x+1}{x+2} < 1\right\}$

Take $x = 1$

$\frac{2+1}{3} = 3 < 1$ which not satisfy given condition

Take, $x = 0 \qquad \frac{1}{2} < 1$

$x = -1 \qquad \frac{-1}{1} < 1$

$x = -3 \qquad \frac{-6+1}{-1} = 5 < 1$

which not satisfy given condition

$-2 < x < 1$

∴ S is bounded both below & above with $infs = -2$

$sups = 1$

∴ option (3) is correct

**2. Explanation:**

$S = \left\{\frac{mn}{m+n+1}, m, n \in \mathbb{N}\right\}$

$x = \frac{mn}{m+n+1}$

$\Rightarrow x + 1 = \frac{(m+1)(n+1)}{1+m+n}$

$= \frac{(m+1)}{m}\frac{(n+1)}{n}$

$= \frac{x+1}{x}$

Now, note that for positive integer k,

we have $1 < \frac{k+1}{k} \leq 2$

We can thus infer $\frac{x+1}{x}$ takes value bet. 1 and 4

i.e. $1 < \frac{x+1}{x} \leq 4$

$\Rightarrow 0 < \frac{1}{x} \leq 3$

$\Rightarrow x \in \left[\frac{1}{3}, \infty\right)$

We see that $infs = {}^{1}/_{3}$ & $sups = \infty$

⇒ option (2) is correct

**3. Explanation:**

Let $X = \{x \in \mathbb{R}, x \neq 1,2, \ldots, 100\}$ be subset of $\mathbb{R}$

$X = R - \{1,2, \ldots, 100\}$

$X^c = \{1,2,3, \ldots 100\}$

$(X^c)' = \phi$

$X^c$ is not closed

⇒ X is open

⇒ option (4) is correct

**4. Explanation:**

$X = \left\{\frac{1}{n} n \in \mathbb{Z}\, n \neq 0\right\} \leq \mathbb{R}$

Given set X has upper bound 1 & lower bound 0

∴ Hence given set is bounded set

⇒ option (1) is correct

**5. Explanation:**

$(n, n+1)$ if is open in $\mathbb{R}$ for any $n \in \mathbb{Z}$

$\therefore U = \bigcup_{n\in\mathbb{Z}}(n, n+1)$ is open in $\mathbb{R}$

(any union of open set is open)

Thus $\mathbb{R}|U$ is closed in $\mathbb{R}$

(a compliment of an open set is a closed set)

But $\mathbb{R}|U = \mathbb{Z}$

Thus $\mathbb{Z}$ is closed subset of $\mathbb{R}$

⇒ option (2) is correct

**6. Explanation:**

$\cup_{n=1}^{\infty}(-n, n)$ is subset of $\mathbb{R}$ is connected

⇒ option (3) is correct

**7. Explanation:**

Every open interval of type (a,b) is homomorphic to $\mathbb{R}$

⇒ option (2) is correct

**8. Explanation:**

$U = \left\{x \in \mathbb{R}: \sin x = \frac{1}{2}\right\}$

$\sin x : \mathbb{R} \to [-1,1]$ is continuous function

Now since $\left\{\frac{1}{2}\right\}$ is closed in $[-1,1]$

$U = \sin^{-1}\left(\frac{1}{2}\right)$

We get that

U is closed in $\mathbb{R}$

⇒ option (2) is correct

**9. Explanation:**

Set of boundary point of $\mathbb{Q}^C$ are $\mathbb{Q}^C$

**10. Explanation:**

Let $U = \left\{x \in \mathbb{R} \mid \sin x = \frac{1}{2}\right\}$

$\sin x = \frac{1}{2}$

$\Rightarrow x = \frac{\pi}{6} + 2\pi n \qquad n \in \mathbb{N} \cup \{0\}$

Neither open nor closed.

**[PRACTICE QUESTIONS]**

**1. Explanation:**

$A$ is closed $A \subseteq \mathbb{R}\ A \neq \phi, A \neq \mathbb{R}$

$A^0 -$ interior of A $\overline{A} -$ closure of A

(1) $A = \overline{A}^{\circ}$

$A = \{1\} \cup [2,3]\ A \subseteq \mathbb{R}$ closed $A \neq \phi\ A \neq \mathbb{R}$

$A^{\circ} = (2,3)\ \overline{A}^{\circ} = [2,3]$

$A = \{1\} \cup [2,3] \neq [2,3] = \overline{A}^{\circ} \Rightarrow A \neq \overline{A}^{\circ}$

(2) $A = [1,2]\ A \neq \phi\ A \neq \mathbb{R}$ closed uncountable

(3) $A = [0, \infty)\ A \neq \phi\ A \neq \mathbb{R}$

Let $B \subseteq \mathbb{R}\ B$ is closed if $B^c$ is open $A^c = (-\infty, 0) \to$ open set in $\mathbb{R}$.

$A \neq$ compact.

A is compact in $\mathbb{R}^n$ iff A is closed & bounded $(A \subseteq \mathbb{R}^n)$.

(4) $\therefore A$ is not open

Suppose $A$ is open

$\mathbb{R} = A \cup A^c\ A \subseteq \mathbb{R}, A \neq \phi\ A$ – closed

$A \neq \phi \Rightarrow A^c \neq \mathbb{R}$

$A^c \neq \phi\ A^c \neq \mathbb{R}, A^c -$ open

$A \neq \phi\ A \neq \mathbb{R}\ A$ – open

$A$ & $A^c$ forms a seperation for $\mathbb{R}$.

$\therefore \Rightarrow \mathbb{R}$ is not connected.

$A$ contradiction

⇒ A not a open

⇒ option (4) is correct.

**2. Explanation:** A and B be two set having m,n many element respectively, where $\mathrm{m} < \mathrm{n},\ \mathrm{m}, \mathrm{n} \in \mathbb{N}$. The number of injective function possible form A to B is

$\mathrm{F} : \mathrm{A} \to \mathrm{B}$

$|\mathrm{A}| = \mathrm{m} \qquad |\mathrm{B}| = \mathrm{n} \Rightarrow \mathrm{m} < \mathrm{n}$

Total injective (one – one) function is ${}^{\mathrm{n}}\mathrm{P}_{\mathrm{m}}$

⇒ option (2) is correct

**3. Explanation:** The sett of rational numbers has Archimedean property

⇒ option (3) is correct.

**4. Explanation:**

$\therefore \sqrt{2}$ is an irrational number.

$\therefore \sqrt{2}$ is an algebraic number not transcender number.

$\therefore$ option (2) is correct.

**5. Explanation:**

⇒ The set of positive primes is countably infinite set ⇒ countable

⇒ Set of integer is countable

⇒ The set of rational number is also countable

⇒ The set of irrational number in $[0,1]$ which is uncountable

⇒ Option (4) is correct

**6.** Let $\alpha$ be an algebraic number, then by definition, there exist a polynomial, having all coefficient rational, such that $p(\alpha) = 0$. In other words, all roots of a polynomial having all coefficient rational are algebenic number.

Let for a fixed integer $n$, put

$P_n = \{a_0 x^n + a_1 x^{n-1} + \cdots + a_{n-1} x + a_n; a_i \in \mathbb{Q}$ for all $i = 1,2,3 \ldots n\}$

Then the mapping,

$(a_0\ x^n + a_1\ x^{n-1} + \cdots + a_{n-1}\ x + a_n\ ) \to (a_0, a_1, \ldots a)$

is $P_n$ to $\mathbb{Q}^{n+1}$

Since, $\mathbb{Q}$ is countable, so is countable product $\mathbb{Q}^{n+1}$.

So set of all polynomials, having rational coefficient is $\cup_{n=0}^{\infty} P_n$, which is a countable union of countable sets, and so countable.

Since, a polynomial of degree $n$, have at most $n$ roots, so the set of all possible roots of all polynomials with rational coefficients is a countable union of finite sets, hence at most countable.

So, the set of all algebraic numbers are countable.

Hence option (2) is correct

Since, $\varepsilon > 0$ was choosen arbitrary. Let $a = \frac{\varepsilon}{2} > 0\ as\ \varepsilon$ and $\frac{\varepsilon}{2} <\ \varepsilon$,

**7. Explanation:**

$\therefore$ for all $\varepsilon > 0$, we have $a\ < \varepsilon$, and $a\ \neq 0$.

Hence option (3) is correct

**8. Explanation:** Statement in option (D) is incorrect, as $\overline{T}$ is the smallest closed set which contain $T$

For example let $T = (0,1)$ is set in $\mathbb{R}$ and here $\overline{T} = [0,1]$ and $T \subset \overline{T}$

But this is not the largest closed set as $S = [10,8]$ is also closed set containing $T$.

(we take [0,8] as an example)

Hence option (4) is correct.

$$a_n = \begin{cases} 2^n + n^2 & , n \leq 100 \\ 3 + (-1)^n \frac{n^2}{2^n + 1} & , n > 100 \end{cases}$$

Note that to check the convergence or divergence of a sequence, if we remove some finite number of terms, it wont have any effect on the convergence or divergence.

For large,$n, n^2 < 2^n + 1$ , and $\lim_{n\to\infty} \frac{n^2}{2^n+1} \left(\frac{\infty}{\infty} \text{from}\right)$

Applying L'Hopital Rule

$\lim_{n\to\infty} \frac{n^2}{(\log 2).2^n} \left(\frac{\infty}{\infty} \text{from}\right)$

$\lim_{n\to\infty} \frac{n^2}{2^n.(\log 2)^2} \left(\frac{\infty}{\infty} \text{from}\right)$

$\lim_{n\to\infty} a_n = 3$

i.e., $\langle a_n \rangle$ is convergent sequence, hence cauchy.

Hence option (1) is correct

**9. Explanation:**

For option (A): Note that for all $n \in \mathbb{N}, \frac{1}{n} > 0$

Therefore, $0 \notin \bigcup_{n=1}^{\infty}\left[\frac{1}{n}, 2\right]$

$\Rightarrow 0 \notin \bigcup_{n=1}^{\infty}\left[\frac{1}{n}, 2\right]$

$\bigcup_{n=1}^{\infty}\left[\frac{1}{n}, 2\right] \neq [0,2]$

For option (B): Let $x \in (0,2]$ be arbitrary, then $0 < x$, then there exist a natural number $n$ such that $0 < \frac{1}{n} < x$,

for all $x > 0$, we have $n \in \mathbb{N}$ such that $(x, 2] \subset \left(\frac{1}{n}, 2\right]$

Since $x > 0$ was arbitrary

we have $(0,2] \subseteq \bigcup_{n=1}^{\infty}\left(\frac{1}{n}, 2\right]$ .....(1)

If $y \in \bigcup_{n=1}^{\infty}\left(\frac{1}{n}, 2\right]$ is arbitrary, then $y \in \left(\frac{1}{n}, 2\right]$ fore some $n \in \mathbb{N}$

i.e. $y > \frac{1}{n} > 0$

$\therefore \quad \bigcup_{n=1}^{\infty}\left(\frac{1}{n}, 2\right] \subseteq (0,2]$ .....(2)

From (1) and (2),

we have $\bigcup_{n=1}^{\infty}\left(\frac{1}{n}, 2\right] = (0,2]$

For option (C) Note that for all $n \in \mathbb{N}, 1 - \frac{1}{n} < 1$,

Also, $\lim_{n\to\infty}\left(1 - \frac{1}{n}\right) = 1$

Therefore, $1 \in \bigcup_{n=1}^{\infty}\left(1 - \frac{1}{n}, 2\right]$, But $1 \notin (1,2]$

Hence, $\bigcup_{n=1}^{\infty}\left[1 - \frac{1}{n}, 2\right]$ is arbitrary

For Option (D) Let $x \in [1,2]$ be arbitrary then $1 \leq x \leq 2$ . Since $\forall\, n \in \mathbb{N}, 1 - \frac{1}{n} < 1 \leq x$,

$\Rightarrow x \in \left[1 - \frac{1}{n}, 2\right] \forall\, n \in \mathbb{N}$

Since, $x$ was arbitrary, therefore $[1,2] \subseteq \bigcup_{n=1}^{\infty}\left[1 - \frac{1}{n}, 2\right]$ ....(3)

Let $y \in \bigcup_{n=1}^{\infty}\left[1 - \frac{1}{n}, 2\right]$ is arbitrary

Then, $y \in \left[1 - \frac{1}{n}, 2\right] \forall\, n \in \mathbb{N}$

Also, $\lim_{n\to\infty}\left(1 - \frac{1}{n}\right) = 1$, then $y \geq 1$.

So, $y \in [1,2]$

Since $y$ was choose arbitrary, there fore $\bigcup_{n=1}^{\infty}\left[1 - \frac{1}{n}, 2\right] \subseteq [1,2]$ ....(4)

From (3) and (4),

We have, $\bigcap_{n=1}^{\infty}\left[1 - \frac{1}{n}, 2\right] = [1,2]$

Hence option (2) and (4) are correct.

**10. Explanation:** For option (A) and (C) Let us consider a set $A = \left(-\infty, \sqrt{2}\right) \cup \left(\sqrt{2}, \infty\right)$

Clearly, A is open, as union of two open intervals

Also, note that $A \neq R$ as $\sqrt{2} \in \mathbb{R}$ But as $\sqrt{2} \notin A$ and is uncountable set.

Therefore, $Q \subset A \subset \mathbb{R}$, But $A \neq R$ and $A$ is uncountable but not closed.

It contradicts both (A) and (C)

Also, option (D) is wrong, as if we consider $A = \mathbb{Q} \cup \{\sqrt{2}, x\}$ clearly $Q \subset A \subset \mathbb{R}$. Also, $A$ is countable, as union of two countable set. But $A$ is not closed in $\mathbb{R}$

Also, option (2) is correct, as given that $Q \subset A$ then $\overline{Q} \subset \overline{A}$ Then $\overline{Q} \subseteq \overline{A}$ . But $\overline{Q} = \mathbb{R}$ and $\overline{A} = A$ (As $A$ is closed)

Then we have $\mathbb{R} \subseteq A$, this given $A = \mathbb{R}$.

Hence option (2) is correct

**11. Explanation:**

For option (A) Since $\mathbb{N}'$ is the countable product of countable sets. Since countable product of countable sets is countable. So, $\mathbb{N}'$ is countable

(C) the set of integers is countable

(D) Since, $\mathbb{Q}$ is countable and $\sqrt{2} \in \mathbb{Q}$ then $\sqrt{2}\mathbb{Q}$ is also countable.

(B) $\{0,1\}^{\mathbb{N}}$, is the set of all sequences which takes the value either 0 and 1.

Suppose that this set is countable then there exist bijection with $\mathbb{N}$.

Let, $\{0,1\}^{\mathbb{N}} = \{x_{1n}, x_{2n}, x_{3n} \ldots.\}$, where each $x_{in}$, is a sequence and $f(i) = x_{in}$

Let us consider a sequence $X$, such that

$X_1 \neq X_{11}$(if $X_{11} = 0$, we define $X_1 = 1$ or vice versa)

$X_2 \neq X_{22}$ (same as above)

: :

: :

$X_n \neq X_{nn}$

Hence, $\langle x_n \rangle$ is a sequence having value either O or 1 but not in the range set of $f$. Hence $\{0,1\}^{\mathbb{N}}$ is Uncountable

Hence, option (2) is correct.

**12. Explanation:** For example,

Consider, $X = \mathbb{Z}$ and $A = \mathbb{N}$

Clearly, both are countably infinite subset of $\mathbb{R}$, then

$X \backslash A = \{x \in X | x \in A\}$

$= \{x \in \mathbb{Z} | x \notin \mathbb{N}\}$

$= \{0, -1, -2, \ldots\}$ is countable infinite set.

Also $X \backslash A \subset X$, then $X$ cannot be uncountable.

**13. Explanation:** consider the finite series

$\sum u_n = x + \frac{2^2x^2}{\lfloor 2} + \frac{3^3x^3}{\lfloor 3} + \cdots$

Hence, $u_n = \frac{n^n.x^n}{\lfloor 3}$ and $u_{n+1} = \frac{(n+1)^{n+1}x^{n+1}}{\lfloor n+1}$

$\therefore \frac{u_{n+1}}{u_n} = \frac{(n+1)^{n+1}.x^{n+1}.\lfloor n}{\lfloor n+1 .n^n.x^n} = \frac{(n+1)^n}{n^n}.x$

$= \left(1+\frac{1}{n}\right)^n.x$

Therefore, $\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n.x = ex$

By D'Alembert ratio test given infinite series is convergent $ex < 1$ i.e. $x < \frac{1}{e}$

Hence option (1) is correct

(Note that because of options, we didn't check the convergence for $ex = 1$, otherwise we need to check.)

**14. Explanation:**

$A = \{x \in \mathbb{Q}: -1 < x < 0\} \cup \mathbb{N}$

$= \left((-1,0) \cap \mathbb{Q}\right) \cup \mathbb{N}$

Clearly, $A$ is unbounded set, as N is unbounded.

Clearly, $A$ is infinite set as both natural number and rational between-1 and 0, are infinite in numbers.

Clearly, N have no limit point, But the set of limit points of $(-1, \mathbb{Q}) \cap \mathbb{Q}$ is the whole interval [-1, 0] in $\mathbb{R}$.

Hence option (2) is correct.

**15. Explanation:** Since, $S = \prod_{n=1}^{\infty}\left[-\frac{1}{n}, 1+\frac{1}{n}\right]$

$= \prod_{n=1}^{\infty}\left[\frac{-1}{n}, 1+\frac{1}{n}\right]$

Let $x \in [0,1]$ is arbitrary, then for all $n \in \mathbb{N}$,

we have $\frac{-1}{n} < 0 \le x \le 1 < 1+\frac{1}{n}$

$\Rightarrow x \in \left[\frac{-1}{n}, 1+\frac{1}{n}\right]$ for all $n \in \mathbb{N}$

Since x was arbitrary. Therefore

$[0,1] \subseteq \cap_{n=1}^{\infty}\left[\frac{-1}{n}, 1+\frac{1}{n}\right]$

Now let $y \in \cap_{n=1}^{\infty}\left[\frac{-1}{n}, 1+\frac{1}{n}\right]$

$\Rightarrow y \in \left[\frac{-1}{n}, 1+\frac{1}{n}\right]$ for all $\backslash n \in \mathbb{N}$

$\Rightarrow \frac{-1}{n} \le y \le 1+\frac{1}{n} \forall\, n \in \mathbb{N}$

Since, $\lim_{n\to\infty}\left(\frac{-1}{n} = 0\right) = 0$ and $\lim_{n\to\infty}\left(\frac{-1}{n}\right) = 1+0 = 1$

Then by squeeze, theorem

$0 \le y \le 1$ i.e. $y \in [0,1]$

Since y was arbitrary, we have

$\cap_{n=1}^{\infty}\left[\frac{-1}{n}, 1+\frac{1}{n}\right] \subseteq [0,1]$

From (1) and (2), we have

$\cap_{n=1}^{\infty}\left[\frac{-1}{n}, 1+\frac{1}{n}\right] = [0,1]$

Hence option (1) is correct

**16. Explanation:** Given that

$S = \left\{\frac{1}{n}: n \in \mathbb{N} \text{ and } n \text{ is prime}\right\}$

$= \left\{\frac{1}{2}, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \ldots\right\}$

Carty, $p \ge 2$ for every prime $p \in \mathbb{N}$.

$\therefore \frac{1}{P} \le \frac{1}{2}$ for every prime $p \in \mathbb{N}$.

$\Rightarrow Sup(S) = \frac{1}{2}$

$\left(\text{as} \frac{1}{2} \text{ is an upper bound of S and element of S}\right)$

Also, $T = \{x^2: x \in \mathbb{R}\}$

Clearly, $x^2 \ge 0 \forall\, x \in \mathbb{R}$

Hence, is a lower bound of $T$ and $0 \in T (\text{as } x = 0 \in \mathbb{R})$, therefore $\inf T = 0$

Here option (3) is correct.

**17. Explanation:**

(I) Let, us define $f: N \to I$ as $f(n) = \frac{1}{n}$ clearly $f$ is one-one correspondence.

(II) $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$ is the set of all integers and integers are countable hence there must exist a one-one correspondence with natural number.

(III) $\left\{\frac{P}{Q}: p, q \in \mathbb{N}\right\}$ is the set of all Rational numbers, and set of rational number is countable hence there must exist a one-one correspondence with natural number.

(IV) Clearly, $\left\{\frac{P}{Q}: p, q \in \mathbb{N}\right\}$ is a subset of $\mathbb{Q}$

Since, $\mathbb{Q}$ is countable, so being a subset of countable set, (IV) is also countable.

Since it is infinite set, so there must exist a one-one correspondence with natural number.

Hence option (4) is correct.

**18. Explanation:** $X = \{(x,y) \in R^2; x \in \mathbb{Q}\, y \in \mathbb{R}\backslash\mathbb{Q}\}$

Now we have to check whether $X$ is open in $\mathbb{R}^2$ or dense in $\mathbb{R}^2$.

Open: $(\alpha, \beta) \in X$ be arbitrary point, then every neighbourhood of $(\alpha, \beta)$ in $\mathbb{R}^2$ contain a point where $\alpha \notin \mathbb{Q}$ and $\beta \notin \mathbb{Q}^C$ so $(\alpha, \beta)$ need not be interior point of $X, so\ X$ is not open

Dense: Let $(a,b) \in \mathbb{R}^2$ Then every neighbourhood of $(a,b)$ in $\mathbb{R}^2$ contains point where $a \in \mathbb{Q}, b \in \mathbb{Q}^2$ and $a \in \mathbb{Q}^c, b \in \mathbb{Q}$

$i.e.$ neighbourhood of $(a,b) \in X$ and $(a,b) \in X^C$

$i.e\ (a,b)$ is a boundary point of $\mathbb{R}^2$

So, $\overline{X} = \mathbb{R}^2$,therefore $X$ is dense in $\mathbb{R}^2$

Hence option, (3) is correct.

**19. Explanation:** Given that,

$S = \{x \in \mathbb{R}: 3 - x^2 > 0\}$

$= \{x \in \mathbb{R}: 3 > x^2\}$

$= \{x \in \mathbb{R}: |x| < \sqrt{3}\}$

$= (-\sqrt{3}, \sqrt{3})$

Clearly, $S$ is bounded in $\mathbb{R}$, also $\sup S = \sqrt{3} \notin \mathbb{Q}$

Hence option (3) is correct.

**20. Explanation:** Note that any polynomial is of the form $p(x) = a_0 + a_1x + a_2x^2 + \cdots$ where each $a_1 \in \mathbb{Q}$

Since rational number is countable and countable product of countable sets is always countable.

So the set $P$ is countable.

Also $S$ the set of all sequences of natural number is uncountable set

Let A be the set of all sequences of 0 and 1. Then by Using cantor's diagonal argument this set is uncountable. So $S$ being superset of this set is always uncountable.

Hence option (4) is correct

**21. Explanation:**

$S = \bigcap_{n=1}^{\infty}\left[2 - \frac{1}{n}, 3 + \frac{1}{n}\right]$

Since, $\lim_{n\to\infty} \frac{1}{n} = 0$ ,, then $\lim_{n\to\infty}\left(2 - \frac{1}{n}\right) = 2$, and $2 - \frac{1}{n} < 2 \forall n$

Also, $\lim_{n\to\infty} \frac{1}{n} = 0$, then $\lim_{n\to\infty}\left(3 + \frac{1}{n}\right) = 3$ and $3 + \frac{1}{n} > 3 \forall n$

$\therefore S = \bigcap_{n=1}^{\infty}\left[2 - \frac{1}{n}, 3 + \frac{1}{n}\right] = [2,3]$

Hence option (2) is correct

**22. Explanation:** Note that $\sin\left(\frac{n\pi}{2}\right) = \begin{cases} 1 \ ; \ n = 4m+1 \\ 0 \ ; \ n = 4m+2 \\ -1 \ ; \ n = 4m+1 \\ 0 \ ; \ n = 4m \end{cases}$

for same $m \in \mathbb{Z}$

Therefore we have three subsequence as,

$X_n = 1 \forall n = 2m, \ m \in \mathbb{Z}$

$X_n = n \ \forall \ n = 4m+1, \ m \in \mathbb{Z}$

$X_n = \frac{1}{n} \forall n = 4m+3, \ m \in \mathbb{Z}$

Since, $\lim_{n\to\infty} X_{4n+1} = \lim_{n\to\infty}(4m+1) = +\infty$

$\lim_{n\to\infty} X_{4n+1} = \lim_{n\to\infty}(4m+1) = 0$

$\therefore \limsup a_n = +\infty \ and \ \liminf a_n = 0$

Hence option (2) is correct.

**23. Explanation:** Note that $\frac{1}{n} > 0 \ \forall \ n$ and $\lim_{n\to\infty} \frac{1}{n} = 0$

$\therefore \bigcap_{n=1}^{\infty}\left[0, \frac{1}{n}\right] = \{0\} = S$

Hence $\sup S = \inf S = 0$

Hence option (4) is correct

**24. Explanation:** Given : $x^2 \leq 5$

$\Rightarrow x^2 - 5 \leq 0$

$\Rightarrow (x - \sqrt{5})(x + \sqrt{5}) = 0$

i.e. $-\sqrt{5} \leq x \leq \sqrt{5}$

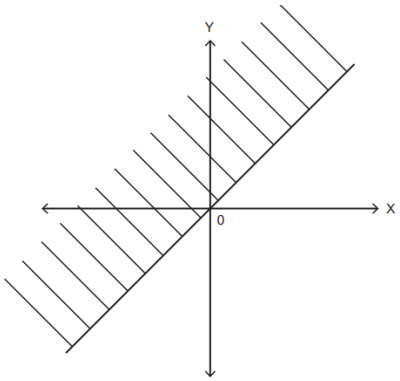
$\Rightarrow x \in [-\sqrt{5}, \sqrt{5}]$

$\Rightarrow$ S is closed interval

$\Rightarrow$ option (3) is correct

**25. Explanation:** Given: $x < y$

$\Rightarrow x - y < 0$




Which is neither R nor set of all negative real numbers

$\Rightarrow$ all options are wrong

**26. Explanation:** Given: $X \subset \mathbb{N}$ is a non-empty finite set and $Y \subset \mathbb{N}$ is an infinite set

Here, $A = \{x - y : x \in X \ and \ y \in Y\}$

Let $X = \{1,2,3,4,5\}$

And $Y = 2\mathbb{N}$

$A = x - y = \{\ldots -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$

$= \mathbb{Z}$

$\therefore \inf(A) = -\infty$ and $\sup(A) < \infty$

$\Rightarrow$ option (2) is correct

**27. Explanation:**

$A, B \subset X$

Let $X = \mathbb{Z}$

And $A = B = \mathbb{N}$

Now, $(A \cup B)\backslash(A \cap B) = (\mathbb{N} \cup \mathbb{N})\backslash(\mathbb{N} \cap \mathbb{N})$

$= \mathbb{N}\backslash\mathbb{N}$

$= \phi$

⇒ finite set

⇒ option (1), (2) and (3) are wrong and option (4) is correct

**28. Explanation:** Since total number of subsets of

$X = 2^{2m+1}$

$\therefore 2^{2m+1} = {}^{2m+1}C_0 + {}^{2m+1}C_1 + \cdots + {}^{2m+1}C_m + {}^{2m+1}C_{m+1} + {}^{2m+1}C_{m+2} + \cdots + {}^{2m+1}C_{2m+1}$

Since ${}^nC_r = {}^nC_{n-r}$

∴ The subsets having $m+1$ elements

$2^{2m+1} =$

$2\underbrace{({}^{2m+1}C_{m+1} + {}^{2m+1}C_{m+2} + \cdots + {}^{2m+1}C_{2m+1})}_{\alpha}$

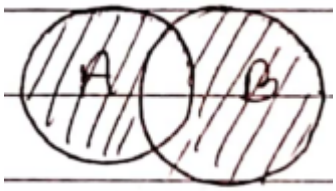
$2^{m+1} = 2\alpha$

$\therefore \alpha = \frac{2^{2m+1}}{2} = 2^{2m+1-1} = 2^{2m}$

⇒ option (3) is correct

**29. Explanation:**

Given: $A\Delta B = (A\backslash B) \cup (B\backslash A)$

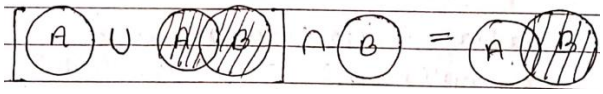



(A) $A \cap (B\Delta A) = A \cap [(A\backslash B) \cup (B\backslash A)]$

$\neq \phi$

⇒ option (A) is wrong

(B) $\big(A \cup (A \,\Delta\, B)\big) \cap B = B$




⇒ option (B) is wrong

(c) $A \,\Delta\, B = (A \cup B) \cap (A^c \cap B^c)$

$= (A \cup B) \cap (A \cap B)^c$

⇒ option (C) is wrong

⇒ option (4) is correct

**30. Explanation:**

a) Let $F : [0,1] \to S = S_1 \cup S_2$

here [0, 1] is uncountable set and $S = S_1 \cup S_2$ which is countable.

∴ There does not exist a map from uncountable set to countable set

⇒ option (a) and (b) are wrong

Here $S_1 = \{t_1 + t_2 \,|t_1 \in Q_n[0,1], t_2 = \sqrt{2}\}$

And $S_2 = \{t_1 + t_2 | t_1 \in Q_n[0,1], t_2 = \sqrt{3}\}$

Both are countable

If F is one-one → every element of [0,1] goes to different element of S

c) Define $F : S_1 \to S_2$ s.t

$$F(x) = \begin{cases} x, & \forall\ x \in Q_n[0,1] \\ \sqrt{3}, & x = \sqrt{2} \end{cases}$$

⇒ F is bijective

**31. Explanation:**

Given $F : \mathbb{R} \to \mathbb{R}$ is a continuous map and $S = \left\{\pi + \frac{(-1)^n}{2n+1} | \ n \in \mathbb{N}\right\} \cup (0,1)$

⇒ S is bounded

⇒ $\bar{S}$ is closed and bounded

⇒ $\bar{S}$ is compact and bounded

And under continuous map, compact set goes to compact set

⇒ $F(\bar{S})$ is compact

And $F : (0,1) \to \mathbb{R}$ S.t $F(x) = 0$ and singleton set is closed

⇒ Its image is also closed

**32. Explanation:**

We know that, A subset of $\mathbb{R}$ is compact iff it is closed and bounded

A) WkT, $\left\{\frac{1}{n}\right\}$ is bdd and {0} is closed as every singleton set is closed and $[0, \pi]$ is closed

⇒ (A) is compact

C) $\{1,2,3\}$ and $[4,5]$ are closed set but $6 \notin \left\{6 + \frac{1}{n} : n \in \mathbb{N}\right\}$

⇒ It is not closed

⇒ It is not compact

B) as $0 \notin [0,1]$ it is not closed and $0 \notin \left\{-\frac{1}{n}; n \in \mathbb{N}\right\}$

∴ The set is not closed

⇒ It is not compact

D) $1 \notin (1,2]$ and $1 \notin \left\{1 - \frac{1}{n}; n \in \mathbb{N}\right\}$

∴ It is not closed

⇒ It is not compact

**33. Explanation:**

$|\mathbb{R}| = z^x = c$

(A) Set of all real sequence