P R E F A C E

Welcome to IFAS: TIFR Paper Solutions. This book is meticulously crafted to serve as a comprehensive resource for students preparing for the Tata Institute of Fundamental Research entrance examinations. TIFR has long been renowned for its commitment to excellence in research and education, and the journey to success in its rigorous exams requires a combination of deep understanding, problem-solving skills, and strategic preparation.

As authors, I recognize the challenges and aspirations of students aspiring to join the prestigious academic community fostered by TIFR. This book is not just a collection of solutions; it is a guide that aims to empower you with a nuanced approach to problem-solving and a deep understanding of the fundamental principles that underlie the questions in TIFR papers.

Key Features of This Book:

1. **Comprehensive Solutions:** Each problem solution is presented with clarity and precision, ensuring that you not only grasp the answer but also the methodology behind it.
2. **Conceptual Insights:** Beyond solutions, we provide conceptual insights, tips, and strategies to enhance your understanding of the subject matter. This approach is designed to empower you with the tools needed to tackle a wide range of problems.
3. **Topic-wise Organization:** The book is structured to align with the various topics covered in TIFR examinations. This organization aids in focused study sessions, allowing you to target specific areas of improvement.
4. **Exam-oriented Approach:** Our solutions are tailored to the format and difficulty level of TIFR exams, providing you with the confidence and competence to face the challenges presented by these prestigious tests.

Who Can Benefit From This Book?

- **Aspiring TIFR Applicants:** This book is an invaluable resource for those preparing to appear for TIFR exams, whether at the undergraduate or postgraduate level.
- **Physics Enthusiasts:** The solutions provided in this book are not just exam-centric; they are crafted to deepen the understanding of mathematical concepts for enthusiasts who are passionate about the subject.
- **Educators and Tutors:** Teachers and tutors can leverage this book as a supplementary resource to augment their teaching materials and offer additional problem-solving exercises to their students.

We hope this book serves as a guiding light on your journey towards mastering the challenges presented by TIFR examinations. Remember, success is not just about finding the right answers but understanding the beauty of the questions.

Best wishes for your academic endeavors!

A C K N O W L E D G E M E N T

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To the students who diligently contributed to solving and explaining the problems, your passion for learning and commitment to mastering the material are evident in the clarity and depth of the solutions presented in this book. Your hard work has been invaluable in creating a comprehensive resource for fellow learners.

I also acknowledge the collaborative spirit among the institute members who worked together to review, edit, and refine the solutions. Your collective expertise has significantly enhanced the quality of this publication.

This TIFR solution book stands as a testament to the collaborative spirit and academic excellence that defines our institute. I hope it serves as a valuable resource for current and future students pursuing a deeper understanding of the subjects covered.

Thank you all for your hard work, dedication, and unwavering commitment to the pursuit of knowledge.

I N D E X

TIFR - PHYSICS

CHAPTER NO.	CHAPTER NAME	PAGE NO.
1	*MATHEMATICAL METHODS IN PHYSICS*	*1*
2	*CLASSICAL MECHANICS*	*21*
3	*ELECTROMAGNETICS THEORY*	*48*
4	*QUANTUM MECHANICS*	*72*
5	*THERMODYNAMICS & STATISTICAL MECHANICS*	*94*
6	*ELECTRONICS*	*116*
7	*ATOMIC & MOLECULAR PHYSICS*	*131*
8	*CONDENSED MATTER PHYSICS*	*139*
9	*NUCLEAR PHYSICS*	*148*

CHAPTER-1 MATHEMATICAL METHODS IN PHYSICS

PREVIOUS YEAR EXAM QUESTIONS

VECTOR ALGEBRA & VECTOR CALCULUS

[TIFR 2012]

1. The function $f(x)$ represents the nearest integer less than x, e.g.

$$f(3.14) = 3$$

The derivative of this function (for arbitrary x) will be given in terms of the integers n as

$f'(x) =$

(a) 0 (b) $\sum_n \delta(x-n)$

(c) $\sum_n |x-n|$ (d) $\sum_n f(x-n)$

[TIFR-2013]

2. The value of the integral $\int_0^\infty dx\, x^9 \exp(-x^2)$ is

(a) 20160 (b) 12

(c) 18 (d) 24

[TIFR-2013]

3. The integral $\int_{-\infty}^{\infty} dx\delta(x^2-\pi^2)cosx$ evaluates to

(a) -1 (b) 0

(c) $1/\pi$ (d) $-1/\pi$

[TIFR-2014]

4. In spherical polar coordinates $\vec{r} = (r, \theta, \varphi)$ the delta function $\delta(\vec{r}_1 - \vec{r}_2)$ can be written as

(a) $\delta(r_1 - r_2)\delta(\theta_1 - \theta_2)\delta(\varphi_1 - \varphi_2)$

(b) $\frac{1}{r_1^2}\,\delta(r_1 - r_2)\delta(cos\theta_1 - cos\theta_2)\delta(\varphi_1 - \varphi_2)$

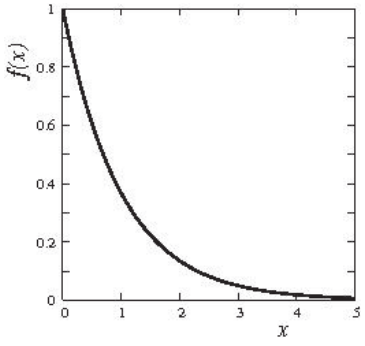
(c) $\frac{1}{|\vec{r}_1 - \vec{r}_2|^2}\,\delta(r_1 - r_2)\delta(cos\theta_1 - cos\theta_2)\delta(\varphi_1 - \varphi_2)$

(d) $\frac{1}{r_1^2 cos\theta_1}\delta(r_1 - r_2)\delta(cos\theta_1 - cos\theta_2)\delta(\varphi_1 - \varphi_2)$

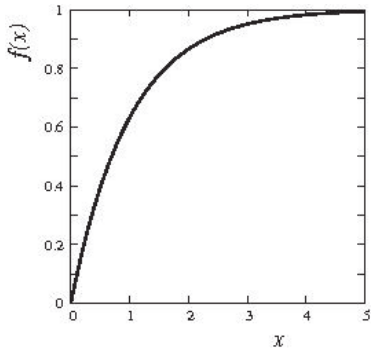
[TIFR-2014]

5. The solution of the integral equation $f(x) = x - \int_0^x dt\, f(t)$has the graphical form

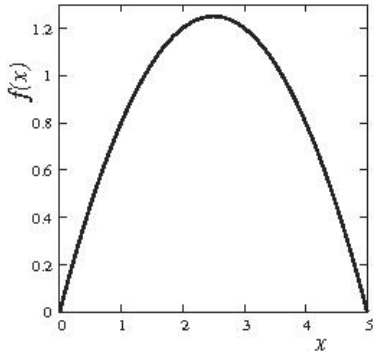
(a)



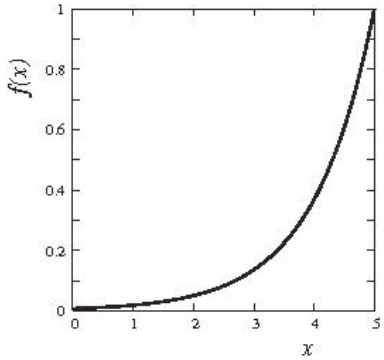
(b)



(c)



(d)



[TIFR-2015]

6. Which of the following vectors is parallel to the surface $x^2y + 2xz = 4$ at the point$(2, -2, 3)$?

(a) $-6\hat{\imath} - 2\hat{\jmath} + 5\hat{k}$ (b) $6\hat{\imath} + 2\hat{\jmath} + 5\hat{k}$

(c) $6\hat{\imath} - 2\hat{\jmath} + 5\hat{k}$ (d) $6\hat{\imath} - 2\hat{\jmath} - 5\hat{k}$

[TIFR-2016]

7. If x is a continuous variable which is uniformly distributed over the real line from $x = 0$ to $x \to \infty$ according to the distribution $f(x) = ex(-4x)$ then the expectation value of $\cos 4x$ is

(a) Zero (b) 1/2
(c) 1/4 (d) 1/16

[TIFR-2018]

8. Evaluate the integral
$\int_{-\infty}^{+\infty} dx \exp(-x^2) \cos(\sqrt{2}x)$

[TIFR-2018]

9. A fourth rank Cartesian tensor T_{ijkt} satisfies the following identities
(i) $T_{ijk\ell} = T_{ijk\ell}$
(ii) $T_{ijk\ell} = T_{ij\ell k}$
(iii) $T_{ijk\ell} = T_{k\ell ij}$
Assuming a space of three dimensions $(i.e. i, j, k = 1, 2, 3)$, what is the number of independent components of $T_{ijk\ell}$?

[TIFR-2019]

10. Consider the surface defined by $ax^2 + by^2 + cz + d = 0$, where a, b, c and d are constants. If $\hat{n}_1$ and $\hat{n}_2$ are unit normal vectors to the surface at the points $(x, y, z) = (1, 1, 0)$ and $(0, 0, 1)$ respectively and $\hat{m}$ is a unit vector normal to both $\hat{n}_1$ and $\hat{n}_2$, then $\hat{m}$
(a) $\frac{-a\hat{\imath}+b\hat{\jmath}}{\sqrt{a^2+b^2}}$ (b) $\frac{a\hat{\imath}+b\hat{\jmath}+c\hat{k}}{\sqrt{a^2+b^2+c^2}}$
(c) $\frac{b\hat{\imath}-a\hat{\jmath}}{\sqrt{a^2+b^2}}$ (d) $\frac{2a\hat{\imath}+2b\hat{\jmath}-c\hat{k}}{\sqrt{4a^2+4b^2+c^2}}$

[TIFR-2019]

11. An array T has elements T_{ijkl} where $i, j, k, l =$ 1,2,3,4. It is given that
$$T_{ijkl} = T_{ijkl} = T_{ijkl} = -T_{klij}$$
for all values of i, j, k, l. The number of independent components in this array is
(a) 55 (b) 256
(c) 45 (d) 1

[TIFR-2020]

12. The limit $\lim_{x\to\infty} x \log\frac{x+1}{x-1}$ evaluates to
(a) 2 (b) 0
(c) ∞ (d) 1

[TIFR-2022]

13. Consider the two-dimensional polar integral
$$P = \int dr d\theta r^{19} e^{-r^2} \sin^8\theta \cos^{11}\theta$$
If the integration is over only the first quadrant $(0 \leq \theta \leq \pi/2)$, the value of P is
(a) 20160 (b) 88π
(c) 180 (d) 16π

[TIFR-2022]

14. The value of the integral
$$\int_{-\pi/2}^{+\pi/2} dx \cosh(kx^2) \ \sin^2 x$$
In the large-k limit, will be

(a) $\frac{1}{2k\pi} e^{k\pi^2/4}$ (b) $\frac{1}{k^2\pi^2} cosh\left(\frac{\pi^2}{4}\right)$
(c) $cosh\left(\frac{\pi^2}{4}\right)$ (d) $\frac{1}{k\pi} e^{k\pi^2/4}$

[TIFR-2023]

15. A surface is given by 4xy-2x+32-0
Which of the following is a vector normal to it at the point (2, 3, 1)
(a) $30\hat{\imath} - 8\hat{\jmath} + 9\hat{k}$ (b) $15\hat{\imath} - 4\hat{\jmath} + 9\hat{k}$
(c) $30\hat{\imath} + 8\hat{\jmath} - 9\hat{k}$ (d) $30\hat{\imath} - 8\hat{\jmath} - 9\hat{k}$

[TIFR-2023]

16. The value of the first derivative of the function $f(x) = \frac{2}{\sqrt{3}} e^{-\sqrt{3x^2|x|}}$
(a) 0 (b) 2
(c) $\frac{2}{\sqrt{3}}$ (d) Undefined

[TIFR 2024]

17. A surface is given by $2x^3z + 4y^2z + 3z^2 = 81$
Which of the following is a vector tangential to it at the point on the surface with coordinates (x, y, z) =(1,2,3)?
(a) $2i - 3j + 3k$ (b) $18i + 48j + 36k$
(c) $-3i + 2j + 6k$ (d) $-3i - 2j + 6k$

MATRICES

[TIFR-2010]

18. The matrix $\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ can be related by a similarity transformation to the matrix
(a) $\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & 1 & 0 \\ 1 & -1 & -1 \\ 0 & -1 & 2 \end{pmatrix}$
(c) $\begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & -1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$

[TIFR-2011]

19. Consider the matrix
$M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}$
A 3-dimensional basis formed by eigenvectors of M is

(a) $\begin{pmatrix}2\\1\\-1\end{pmatrix},\begin{pmatrix}1\\-1\\1\end{pmatrix}$ And $\begin{pmatrix}0\\1\\-1\end{pmatrix}$

(b) $\begin{pmatrix}1\\1\\-1\end{pmatrix},\begin{pmatrix}0\\1\\1\end{pmatrix}$ And $\begin{pmatrix}0\\1\\-1\end{pmatrix}$

(c) $\begin{pmatrix}0\\0\\0\end{pmatrix},\begin{pmatrix}0\\1\\1\end{pmatrix}$ And $\begin{pmatrix}0\\1\\-1\end{pmatrix}$

(d) $\begin{pmatrix}2\\1\\-1\end{pmatrix},\begin{pmatrix}0\\1\\1\end{pmatrix}$ And $\begin{pmatrix}-1\\1\\1\end{pmatrix}$

[TIFR-2011]

20. The trace of the real 4×4 matrix $\mathbb{U}=\exp(\mathbb{A})$, where

$$\mathbb{A}=\begin{pmatrix}0 & 0 & 0 & \frac{\pi}{4}\\ 0 & 0 & -\frac{\pi}{4} & 0\\ 0 & \frac{\pi}{4} & 0 & 0\\ -\frac{\pi}{4} & 0 & 0 & 0\end{pmatrix}$$

Is equal to

(a) $2\sqrt{2}$

(b) $\pi/4$

(c) $\exp(i\varphi)\ \ for\ \varphi=0,\pi$

(d) Zero

(e) $\pi/2$

(f) 2

[TIFR-2012]

21. The different 2×2 matrices A and B are found to have the same eigen-values. It is then correct to state that $A=SBS^{-1}$ where S can be a

(a) Traceless 2×2 matrix

(b) Hermitian 2×2 matrix

(c) Unitary 2×2 matrix

(d) Arbitrary 2×2 matrix

[TIFR-2014]

22. The product MN of two Hermitian matrices M and N is anti-Hermitian. It follows that

(a) $\{M,N\}=0$ (b) $[M,N]=0$

(c) $M^{+}=N$ (d) $M^{+}=N^{-1}$

[TIFR-2016]

23. If the eigenvalues of a symmetric 3×3 matrix A are 0, 1, 3 and the corresponding eigenvalues can be written as $\begin{pmatrix}1\\1\\1\end{pmatrix},\begin{pmatrix}1\\0\\-1\end{pmatrix},\begin{pmatrix}1\\-2\\1\end{pmatrix}$ respectively, then matrix A^4 is

(a) $\begin{pmatrix}41 & -81 & 40\\ -81 & 0 & -81\\ 40 & 81 & 41\end{pmatrix}$

(b) $\begin{pmatrix}-82 & -81 & 79\\ -81 & 81 & -81\\ 79 & -81 & 83\end{pmatrix}$

(c) $\begin{pmatrix}14 & -27 & 13\\ -27 & 54 & -27\\ 13 & -27 & 14\end{pmatrix}$

(d) $\begin{pmatrix}14 & -13 & 27\\ -13 & 54 & -13\\ 27 & -13 & 14\end{pmatrix}$

[TIFR-2017]

24. The matrix

$$\begin{pmatrix}100\sqrt{2} & x & 0\\ -x & 0 & -x\\ 0 & x & 100\sqrt{2}\end{pmatrix}$$

where $x>0$, is known to have two equal eigenvalues. Find the value of x.

[TIFR-2017]

25. A unitary matrix U is expanded in terms of a Hermitian matrix H, such that $U=e^{i\pi H/2}$

If we know that

$$H=\begin{pmatrix}1/2 & 0 & \sqrt{3}/2\\ 0 & 1 & 0\\ \sqrt{3}/2 & 0 & -1/2\end{pmatrix}$$

Then U must be

(a) $\begin{pmatrix}i & 1/2 & \sqrt{3}/2\\ 1/2 & i & 1/2\\ \sqrt{3}/2 & 1/2 & i\end{pmatrix}$

(b) $\begin{pmatrix}i/2 & 0 & \sqrt{3}/2\\ 0 & 1 & 0\\ i\sqrt{3}/2 & 0 & -i/2\end{pmatrix}$

(c) $\begin{pmatrix}1 & 0 & \sqrt{3}\\ 0 & 2 & 0\\ \sqrt{3} & 0 & -1\end{pmatrix}$

(d) $\begin{pmatrix}2i & 1 & \sqrt{3}/2\\ 1 & 2i & 0\\ \sqrt{3}/2 & 0 & 2i\end{pmatrix}$

[TIFR-2018]

26. If a 2×2 matrix M is given by

$$M=\begin{pmatrix}1 & \frac{1-i}{\sqrt{2}}\\ \frac{1+i}{\sqrt{2}} & 0\end{pmatrix}$$

Then $\det\exp M=$

(a) e^2e (b) e

(c) $2i\sin\sqrt{2}$ (d) $\exp\left(-2\sqrt{2}\right)$

[TIFR-2019]

27. The eigenvalues of a 3×3 matrix M are

$\lambda_1=2\ \ \lambda_2=-1\ \ \lambda_3=1$

And the eigenvectors are

Answer Key									
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
(b)	(b)	(d)	(b)	(b)	(c)	(b)	1.07	18	(c)
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
(d)	(a)	10.64	(d)	(a)	(a)	(a)	(d)	(b)	(a)
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
(c)	(a)	(c)	50	(b)	(b)	(d)	(a)	(a)	(a)
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
(a)	(a)	(b)	(a)	(c)	81	(a)	(c)	(d)	(a)
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
(b)	(a)	(a)	(b)	(c)	(a)	(c)	(a)	(c)	(d)
51	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
(d)	(a)	(a)	(c)	A^n	(a)	(a)	(c)	(d)	(a)
61	**62**	**63**	**64**	**65**	**66**	**67**			
(b)	(c)	(a)	(a)	(a)	(a)	(c)			

SOLUTIONS

1. Solution: (b)

Here given function is step function and derivative of step function is delta function so option 2nd is correct.

2. Solution: (b)

$\int_0^\infty x^9 e^{-x^2} dx$

Let $x^2 = t \quad 2xdx = dt$

$I = \frac{1}{2}\int_0^\infty t^4 e^{-t} dt = \frac{\lceil 5}{2} = \frac{1}{2} \times 4 \times 3 \times 2 = 12$

3. Solution: (d)

$\delta(x^2 - \pi^2) = \frac{1}{|\pi-(-\pi)|}[\delta(x-\pi) + \delta(x+\pi)]$

$= \frac{1}{2\pi}[\delta(x-\pi) + \delta(x+\pi)$

$\therefore \int_{-\infty}^{\infty} dx\delta(x^2 - \pi^2)\cos x = \frac{1}{2\pi}\int_{-\infty}^{\infty} dx\,[\delta(x-\pi) + \delta(x+\pi)]\cos x$

$= \frac{1}{2\pi}[\cos\pi + \cos(-\pi)] = \frac{1}{2\pi}(-1-1) = \frac{1}{\pi}$

Option (d) is correct

4. Solution: (b)

Let us begin with the relation

$\int f(\vec{r})\delta(\vec{r} - \vec{r})d\tau = f(\vec{r}) = f_1(r') = f_2(\theta')f_3(\Phi')$

Where A_1, A_2 and A_3 are to be found

$\iiint f(\vec{r})A_1\delta(r - r')A_2\delta(\theta - \theta')A_3\delta(\Phi - \Phi')r^2\sin\theta dr d\theta d\Phi = f(r', \theta', \Phi')$

$\Rightarrow \int_0^\infty f_1(r)\delta(r - r')A_1 r^2 dr \int_0^\pi f_2(\theta)\sin\theta\delta(\theta - \theta')A_2 d\theta \int_0^{2\pi} f_3(\Phi)\delta(\Phi - \Phi')d\Phi = f_1(r)f_2(\theta)f_3(\theta)$

$\Rightarrow A_1 = \frac{1}{r^2}, A_2 = \frac{1}{\sin\theta,}, A_3 = 1$

So, $\delta(\vec{r} - \vec{r}) = \frac{1}{r^2}\delta(\vec{r} - \vec{r})\frac{1}{\sin\theta}\delta(\theta - \theta')\delta(\Phi - \Phi')$

Now, we can use a general formula

$\delta(f(x) = \sum_j \frac{1}{|-\sin\theta|}\delta(x - x_j)$

Where f is a function having only first order roots x_j

So, $\delta(\cos\theta - \cos\theta') = \frac{1}{|-\sin\theta|}\delta(\theta - \theta') = \frac{1}{\sin\theta}\delta(\theta - \theta')$

So, $\delta(\vec{r} - \vec{r}) = \frac{1}{r^2}\delta(\vec{r} - \vec{r})\delta(\cos\theta - \cos\theta')\,\delta(\Phi - \Phi')$

5. Solution: (b)

Let $F(x) = x - \int_0^x f(t).dt$

$\frac{d}{dx}f(x) = 1 - f(x)$

$\frac{d}{dx}f(x) + f(x) = 1$

Integrating factor $= e^{\int dx} = e^x$

Solution of above equation is \

$f(x)e^x = \int e^x . dx$

$f(x)e^x = e^x + c$

$f(x) = 1 + ce^{-x}$.... (1)

Consider at $x = 0$

$f(x = 0) = 0$

$\therefore$ Put in equation (1)

$= 0 = 1 + c \Rightarrow c = -1$

$\therefore f(x) = 1 - e^{-x}$

If x is in. $f(x)$ is also

$\therefore$ Option (b) is correct

6. Solution: (c)

Vector perpendicular to the surface $x^2y + 2xz = 4$ at the point $(2, -2, 3)$ lies in direction of gradient ∇S

$\nabla S = (2xy + 2z)\hat{\imath} + x^2 j + 2xk$

$\nabla S|_{(2,-2,3)} = -2\hat{\imath} + 4j + 4k$

Vector parallel to surface will be normal to ∇S we have to check each option by taking dot product with ∇S we find that $6\hat{\imath} - 2j + 5k$ is perpendicular to ∇S.

7. Solution: (b)

$\langle \cos 4\, x \rangle = \frac{\int_0^\infty \cos 4\, x f\,(x) dx}{\int_0^\infty f(x) dx}$

$$\frac{\int_0^\infty \cos 4x e^{-4x} dx}{\int_0^\infty e^{-4x} dx}$$

$$I = \int_0^\infty \cos 4x e^{-4x} dx$$

$= \frac{e^{-4x}}{4} \sin 4x + \int e^{-4x} \sin 4x\, dx$ (Integration by parts)

$= 2I = \frac{e^{-4x}}{4} (\sin 4x - \cos 4x)$

$I = \frac{e^{-4x}}{8} (\sin 4x - \cos 4x)|_0^\infty = \frac{1}{8}$

$\int_0^\infty d^{-4x}\, dx = -\frac{\frac{e^{-4x}}{4}|_0^\infty 1}{4}$

$\langle \text{Cos } 4\, x \rangle == \frac{1}{8} - \frac{1}{4} = \frac{1}{2}$

8. Solution: (1.07)

$\int_{-\infty}^{\infty} cos(\sqrt{2x}) = 2\int_0^{\propto} e^{-x^2} cos\sqrt{2x}\, dx$

Consider integral $\int_{-\infty}^{\infty} e^{-x^2 dz = \int_C^{\cdot} f(z) dz}$

Where C is the rectangle with vertices at the points $O, R, R + ia$ and ia. Since, $f(z)$ has no poles within and on this contour, we have by Cauchy's theorem

$\int_C^{\cdot} f(z)dz = \int_0^R f(x)dx + \int_0^R f(R + iy)dx + \int_R^0 f(x + ia)dx + \int_a^0 f(iy)i\, dy$

$= I_1 + I_2 + I_3 + I_4$

$|I_2| = \left|\int_0^a exp(-(R + iy)^2)i\, dy\right|$

$\leq \int_0^a |exp(-R^2 + y^2 - 2iRy\,)i| dy$

$\leq \int_0^a e^{-R^2 e^{y^2 dy}} \to 0\ as\ R\ \to \infty$

$Hence, when\ R \to \infty, we\ get\ from\ equation\ (1)$

$\int_0^\infty e^{-x^2} dx + 0\int_\infty^0 exp\{-(x + ia)^2\}dx - i\int_0^\infty e^{y^2} dy = 0$

$\sqrt{\frac{\pi}{2}} - e^{a^2}\int_0^\infty e^{-x^2}(cos\ 2ax - i\ sin\ 2ax)dx - i\ \int_0^\infty e^{y^2} dy = 0$

$e^{a^2}\int_0^\infty e^{-x^2}(cos2ax - isin\ 2\ ax)dx = \sqrt{\frac{\pi}{2}} - i\int_0^\infty e^{y^2} dy = 0$

Equating real part

$e^{a^2}\int_0^\infty e^{-x^2} cos2ax\ dx = \sqrt{\frac{\pi}{2}}$

Or,

$\int_0^\infty e^{-x^2}\ cos\ 2ax = \sqrt{\frac{\pi}{2}} e^{-a^2}$

Put $a = \frac{1}{\sqrt{2}}$

$\int_{-\infty}^\infty e^{-x^2} cos\ \sqrt{x}\ dx = 2\int_0^\infty e^{-x^2} cos\sqrt{x}\, dx = \sqrt{\pi} e^{-\frac{1}{2}} = \sqrt{\frac{\pi}{e}} = 1.07$

9. Solution: (18)

The three condition (i), (ii), (iii) impose symmetries on the tensor. Let's analyze them

(i) $T_{ijk\ell} = T_{ijk\ell}$ Indicator that the tensor is symmetric under the exchange of indices i and j

(ii) $T_{ijk\ell} = T_{ijk\ell}$ Indicates that tensor is symmetric under the exchange of indices k and l

(iii) $T_{ijk\ell} = T_{ijk\ell}$ Indicates that tensor is symmetric under the exchange of indices i and j, k and l

For a symmetric tensor the number of independent components.

For the indices I and j. We have $3 \times 3 = 9$ components. However, due to the symmetry under the exchange of I and J, we only need to consider the unique combinations, which gives us $\frac{1}{2} \times 9 = 4.5$

Similarly, for the indices k and l we have $3 \times 3 = 9$ component again, due to the symmetric under the exchange of k and ℓ we only need to consider the unique combinations,

Which gives us $\frac{1}{2} \times 9 = 4.5$

Finally for the indices i, j, k, l we have $3 \times 3 \times 3 \times 3 = 81$ components

Due to the symmetries (iii) we need to consider the combinations, which gives as $\frac{1}{31} \times 81 = 9$

So, dthe total number of independent component is

$$4.5 + 4.5 + 9 = 18$$

Therefore, there are 18 independent components of the fourth-rank Cartesian tensor $T_{ijk\ell}$ satisfying the given conditions.

10. Solution: - (c)

Equation of surface is given by

$S = ax^2 + by^2 + cz + d = 0$

$\nabla S - 2a\hat{i} + 2b\hat{j} + c\hat{k}$

$\nabla S|_{(1,1,0)} = 2a\hat{i} + 2b\hat{j} + \widehat{ck}$

$\nabla S|_{(0,0,1)} = \widehat{ck}$

$\widehat{n_1} = \frac{\nabla S}{|\nabla S|} = \frac{a\hat{i}+2\widehat{bj}+c\hat{k}}{\sqrt{4a^2+4b^2+c^2}}$

$\widehat{n_2} = \frac{\nabla S}{|\nabla S|} = \hat{k}$

Unit vector $\hat{m}$ will lie in direction of $\widehat{n_1} \times \widehat{n_2}$

$\widehat{n_1} \times \widehat{n_2} = \frac{a\hat{i}+2\widehat{bj}+c\hat{k}}{\sqrt{4a^2+4b^2+c^2}} \times \hat{k}$

$= \frac{-2a\hat{j}+2\widehat{bi}}{\sqrt{4a^2+4b^2+c^2}}$

Unit vector along $\widehat{n_1} \times \widehat{n_2}$, $\hat{m}\frac{a\hat{i}+2\widehat{bj}}{\sqrt{(a^2+b^2)}}$

11. Solution: (d)

The given array $T_{ijk\ell}$ is a fourth -rank Cartesian tensor with indices i, j, k, l each taking values in the range 1 to 4.

The symmetry condition $T_{ijkl} = T_{ijlk} = T_{klij} = -T_{klij}$ indicator that the tensor is completely antisymmetric with respect to the exchange of any pair of Indices'. In an antisymmetric tensor, swapping any two indices leads to a sign change,

For a completely antisymmetric fourth-rank tensor in 4 dimensions, the number of independent components can be found using the formula.

$\therefore$ Number of independent components $= \frac{1}{4!} \times (4!)^2 = 1$

$\therefore$ Option (d) is correct.

12. Solution: (a)

$x \log \frac{x+1}{x-1} = x \log \frac{x+1/x}{x-1/x}$

$= x\left[\log\left(1+\frac{1}{x}\right) - \log\left(1-\frac{1}{x}\right)\right]$

$= x\left[\left(\frac{1}{x} - \frac{1}{2x^2} + \frac{1}{3x^2}\ldots\right) - \left(-\frac{1}{x} - \frac{1}{2}x^2 - \frac{1}{3x^2}\ldots\right]\right.$

$= x\left[\frac{2}{x} + \frac{2}{3x^3}\ldots..\right]$

$= \left[2 + \frac{2}{3x^2} + \cdots\right]$

So $\lim_{x\to\infty} x \log \frac{x+1}{x-1} = 2$

13. Solution: (10.64)

Let $I = \int_0^{\pi/2} \sin^8\theta \cos^{11}\theta \, d\theta$

We have

$\int_0^{\pi/2} \sin^8\theta \cos^9\theta \, d\theta = \frac{\left|\frac{P+1}{2}\right|\frac{q+1}{2}}{2\left|\frac{P+q+2}{2}\right.}$

$\therefore \int_0^{\pi} \sin^8\theta \cos^{11}\theta \, d\theta = \frac{\left|\frac{8+1}{2}\right|\frac{11+1}{2}}{2\left|\frac{11+8+1}{2}\right.} = \frac{\left|\frac{9}{2}\right|\frac{12}{2}}{2\left|\frac{21}{2}\right.}$

$= \frac{120\times26}{130945815}$

$\therefore \int_0^{\pi/2} \sin^8\theta \cos^{11}\theta \, d\theta = 5.8650 \times 10^{-5}$

$\int_0^{\infty} r^{19} \, e^{-r^2} dr = \frac{1}{2}\frac{\left|\frac{19+1}{2}\right.}{(2)^{10}} = \frac{1}{2} \times 9! = 181440$

$\therefore I = \int dr \, d\theta \, r^{19} \, e^{-r^2} \sin^8\theta \cos^{11}\theta$

$= 5.8650 \times 10^{-5} \times 18440$

$p = 10.6414$

14. Solution: (d)

$I = \int_{-\frac{\pi}{2}}^{\pi/2} dx \cos h\, kx^2 \sin^2 x.dx =$

$2\int_0^{\frac{\pi}{2}} dx \cos h\, kx^2 \sin^2 x.dx$

It is given k is very large so $\cos kx^2 = \frac{\exp kx^2}{2}$

The integration

$I = 2\int_0^{\pi/2} dx \, \frac{\exp kx^2}{kx} \sin^2 x.dx$

$= \int_0^{\frac{\pi}{2}} dx \; 2\, kx \exp kx^2 \left(\frac{\sin^2 x}{2kx}\right).dx$

$= \int_0^{\frac{\pi}{2}} dx \; 2kx \exp kx^2 \left(\frac{\sin^2 x}{2kx}\right).dx$

Using integration by parts

$I = \frac{\exp kx^2}{k}\left(\frac{sin\, x}{x}\right)\sin x \; |_0^{\frac{\pi}{2}} -$

$\int_0^{\pi/2} \frac{d}{dx}\left(\frac{\sin^2 x}{kx}\right)\exp kx^2.dx]$

$I = \frac{1}{k\pi}\exp kx^2 dx$ will contain $\frac{1}{k^2}$ term which will be zero at large k

$\therefore I = \frac{1}{k\pi}\exp\left(\frac{k\pi^2}{4}\right)$

$\therefore$ Option (4) is correct

CHAPTER-3 ELECTROMAGNETICS THEORY

TIFR - PREVIOUS YEAR EXAM QUESTIONS

COULOMB'S LAW

[TIFR-2012]

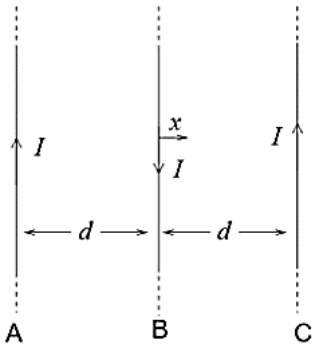
1. Three equal charges Q are successively brought from infinity and each is placed at one of the three vertices of an equilateral triangle. Assuming the rest of the universe as a whole to be neutral, the energy E_0 of the electrostatic field will increase, successively to $E_0+\Delta_1, E_0+\Delta_1+\Delta_2$, $E_0+\Delta_1+\Delta_2+\Delta_3$ Where $\Delta_1:\Delta_2:\Delta_3=$

(a) $1:2:3$ (b) $1:1:1$

(c) $0:1:1$ (d) $0:1:2$

[TIFR-2012]

2. Consider three identical infinite straight wires A, B and C arranged in parallel on a plane as shown in the figure



The wire carry equal currents I with directions as shown in the figure and have mass per unit length m. If the wires A and C are held fixed and the wire B is displaced by a small distance x from its position, then it (B) will execute simple harmonic motion with a time period.

(a) $2\pi\sqrt{\frac{\pi}{\pi\mu_0}}\left(\frac{d}{I}\right)$

(b) $2\pi\sqrt{\frac{2\pi m}{\mu_0}}\left(\frac{d}{I}\right)$

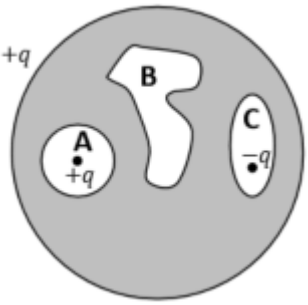
(c) $2\pi\sqrt{\frac{\pi m}{\mu_0}}\left(\frac{d}{I}\right)$

(d) $2\pi\sqrt{\frac{m}{2\pi\mu_0}}\left(\frac{d}{I}\right)$

(e) $2\pi\sqrt{\frac{m}{\mu_0}}\left(\frac{d}{I}\right)$

[TIFR-2015]

3. A solid spherical conductor encloses 3 cavities, a cross-section of which are as shown in the figure. A net charge $+q$ resides on the outer surface of the conductor. Cavities A and C contains point charges $+q$ and $-q$, respectively. The net charges on the surface of these cavities are



(a) $A=-q, B=0, C=-q$

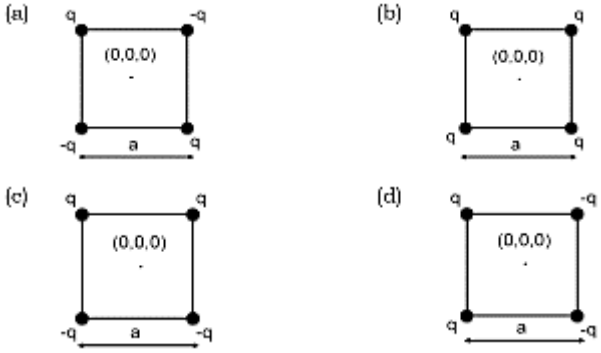
(b) $A=-q, B=q, C=0$

(c) $A=-q, B=q, C=0$

(d) $A=-q, B=0, C=+q$

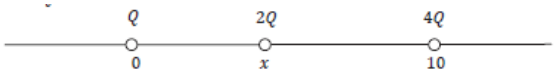
[TIFR-2015]

4. The electrostatic potential $\varphi(r)$ of a distribution of point charges has the form $\varphi(r)\propto r^{-3}$ at a distance r from the origin$(0,0,0)$, where $r\gg a$. Which of the following distributions can give rise to this potential?



[TIFR-2016]

5. Three positively charged particles lie on a straight line at position $0, x$ and 10 as indicated in the figure below. Their charges are $Q, 2Q$ and $4Q$ cm respectively



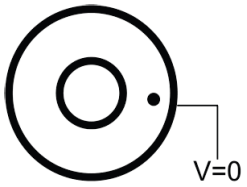
If the charges at $x = 0$ and $x = 10$ are fixed and the charge at x is movable the system will be in equilibrium when $x =$

(a) 8 (b) 2

(c) 20/3 (d) 10/3

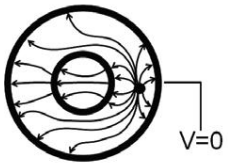
[TIFR-2017]

6. Two long hollow conducting cylinders, each of height h, are placed concentrically on the ground, as shown in the figure (top view). The outer cylinder is grounded, while the inner cylinder is insulated. A positive charge (the black dot in the figure) is placed between the cylinders at a height $h/2$from the ground.

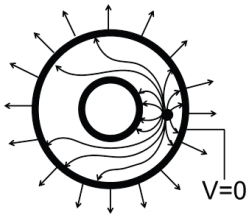


Which of the following figures gives the most accurate representation (top view) of the lines of force?

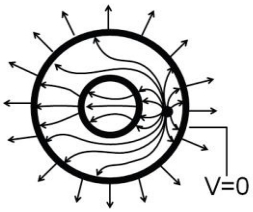
(a)



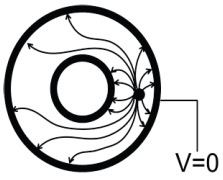
(b)



(c)



(d)



[TIFR-2017]

7. A common model for the distribution of charge in a hydrogen atom has a point-like proton of charge $+q_0$ at the centre and an electron with a static charge density distribution

$$\rho(r) = -\frac{q_0}{\pi a^3} e^{-2r/a}$$

Where a is a constant. The electric field $\vec{E}$ at $r = a$ due to this system of charge will be

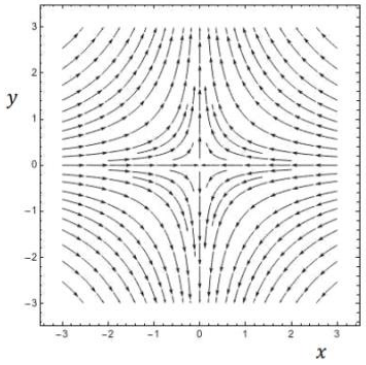
(a) $-\frac{5q_0}{4\pi\epsilon_0 e^2 a^2}\hat{r}$ (b) $-\frac{5q_0}{4\pi\epsilon_0 e a^2}\hat{r}$

(c) $\frac{5q_0}{4\pi\epsilon_0 e^2 a^2}\hat{r}$ (d) $\frac{3q_0}{4\pi\epsilon_0 e^2 a^2}\hat{r}$

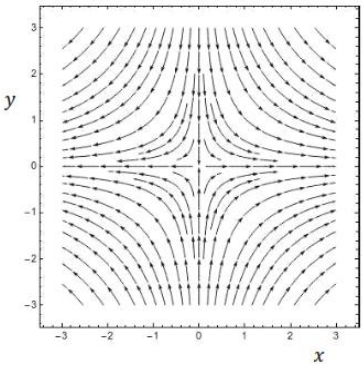
[TIFR-2020]

8. A two-dimensional electrostatic field is defined as $\vec{E}(x, y) = -x\hat{\imath} + y\hat{\jmath}$. A correct diagram for the lines of force is

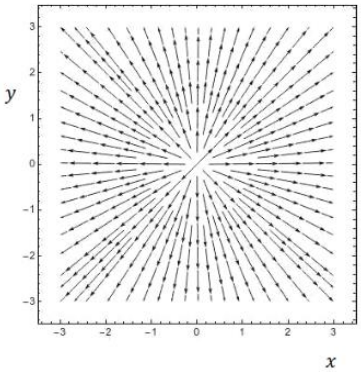
(a)

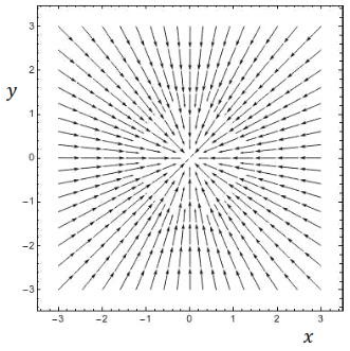


(b)



(c)

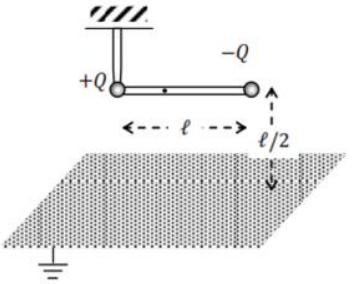




(d)

[TIFR-2020]

9. A light rigid insulating rod of length ℓ is suspended horizontally from a rigid frictionless pivot at one of the ends (see figure). At a vertical distance h below the rod there is an infinite plane conducting plane, which is grounded. If two small, light spherical conductors are attached at the ends of the rod and given charges +Q and - as indicated in the figure, the torque on the rod will be



(a) $\frac{Q^2}{4\pi\epsilon_0\ell}\hat{k}$ (b) $\frac{Q^2}{-4\pi\epsilon_0\ell}\hat{k}$

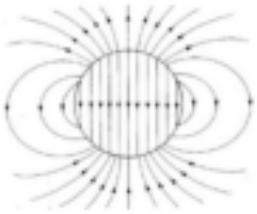
(c) $\frac{(4-\sqrt{2})}{16\pi\epsilon_0}\frac{Q^2}{\ell}\hat{k}$ (d) $-\frac{(4-\sqrt{2})}{16\pi\epsilon_0}\frac{Q^2}{\ell}\hat{k}$

[TIFR-2023]

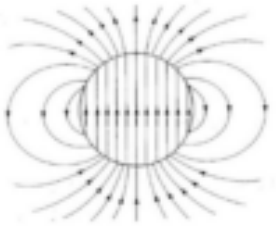
10. The electric field lines due to a uniformly polarized dielectric sphere (polarization along the positive Z-axis as shown in the figure)



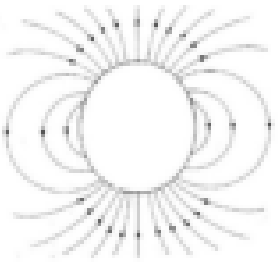
Will look like



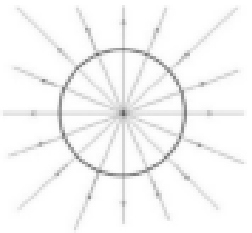
(a)



(b)



(c)



(d)

[TIFR-2024]

11. A thin spherical shell of radius R has a constant surface charge density σ. This shell is cut symmetrically into two pieces. What is the electrostatic force between the two halves?

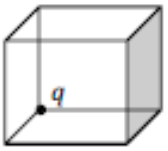
(a) $\frac{\pi}{2}\frac{\sigma^2R^2}{\epsilon_0}$ (b) $\frac{\pi}{2}\frac{\sigma^2R^2}{\epsilon_0}$

(c) $2\pi\frac{\sigma^2R^2}{\epsilon_0}$ (d) $\pi\frac{\sigma^2R^2}{\epsilon_0}$

GAUSS LAW

[TIFR-2013]

12. Point charge q sits at a corner of a cube of side a, as shown in the figure on the right. The flux of the electric field vector through the shaded side is



(a) $\frac{q}{8\varepsilon_0}$ (b) $\frac{q}{16\varepsilon_0}$

(c) $\frac{q}{24\varepsilon_0}$ (d) $\frac{q}{6\varepsilon_0}$

(c) A left circular-polarised travelling wave along the positive z-axis.

(d) A right circular-polarised travelling wave along the positive z-axis.

INTERFERENCE OF ELECTROMAGNETIC WAVES

[TIFR-2013]

48. A plane electromagnetic wave travelling in a vacuum is characterized by the electric and magnetic fields $\vec{E} = \hat{\imath}(30\pi\ Vm^{-1}) \exp i\,(\omega t + kz)$ $\vec{H} = \hat{\jmath}(H_0\ Am^{-1}) exp i(\omega t + kz)$. If $\omega, k > 0$ the value of H_0 must be

(a) 2π (b) 0.67

(c) 0.25 (d) 0.94

[TIFR-2020]

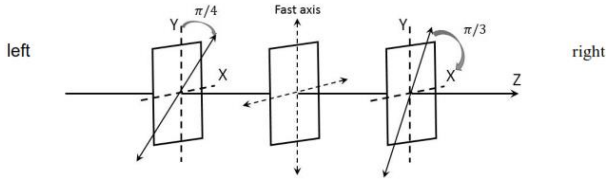
49. The component of the electric and magnetic field corresponding to a plane electromagnetic field propagating in vacuum satisfy $E_x = E_y = -E_z = \frac{|\vec{E}|}{\sqrt{3}}$, $B_x = -B_y = \frac{|\vec{B}|}{\sqrt{2}}$, $B_z = 0$. A unit vector along the direction of propagation of the plane wave is

(a) $\frac{\hat{\imath}+\hat{\jmath}+2\hat{k}}{\sqrt{6}}$ (b) $-\frac{\hat{\imath}+\hat{\jmath}+2\hat{k}}{\sqrt{6}}$

(c) $\frac{2\hat{\imath}+2\hat{\jmath}+\hat{k}}{\sqrt{3}}$ (d) $-\frac{2\hat{\imath}+2\hat{\jmath}+\hat{k}}{\sqrt{3}}$

[TIFR-2020]

50. A plane polarised light wave with electric field expressed as $\vec{E}(z,t) = E_0\hat{\jmath}\cos(kz - \omega t)$, is incident from the left on the apparatus as sketched below.



The apparatus consists of (from left to right) a polariser with transmission axis at $\pi/4$w.r.t. the y-axis, followed by a quarter-wave plate with fast axis along the y-axis, andfinally, a polariser with transmission axis at $\pi/3$ about the x-axis. If the incident intensity of the wave isI_0, what will be the intensity of the light emerging out of the apparatus (on the right)?

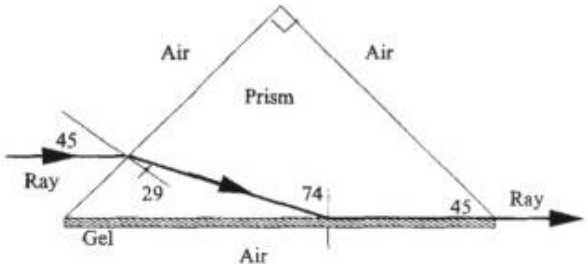
(a) $I_0/4$ (b) $I_0/8$

(c) $3I_0/8$ (d) $I_0/16$

REFLECTION AND REFRACTION

[TIFR-2010]

51. A ray of light is incident on a right-angled prism as shown in the figure below. The lower surface of this prism is coated with a gel. If the incident ray makes angles (marked in degrees) as shown in the figure, the refractive index of the gel must be

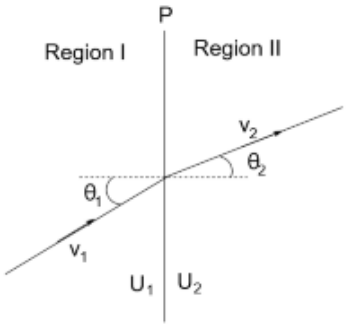


(a) 1.40 (b) 1.46

(c) 1.50 (d) 1.52

[TIFR-2011]

52. A region of space is divided into two parts by a plane P, as shown in the figure below. A particle of mass m passes from region I to region II. Where it has speed v_1 and v_2 respectively. There is a constant potential U_1 in region I and U_2 in region II.



Let T_1 be the kinetic energy of the particle in region. I if the trajectory of the particle is inclined to the normal to the plane P by angles θ_1 and θ_2, as shown in the figure then the ratio $\sin\theta_1 / \sin\theta_2$ is given by

(a) $\sqrt{1 - T_1/(U_1 - U_2)}$

(b) $\sqrt{1 + T_1/(U_1 - U_2)}$

(c) $\sqrt{1 - (U_1 + U_2)/T_1}$

(d) $\sqrt{1 + (U_1 + U_2)/T_1}$

(e) $\sqrt{1 - T_1/(U_1 - U_2)}$

(f) $\sqrt{1 + (U_1 - U_2)/T_1}$

[TIFR-2013]

53. A ray of light is incident on the surface of a thin prism at a small angle $\theta_1$with the normal, as shown in the figure on the right. The material of the prism has refractive index n and you may assume the

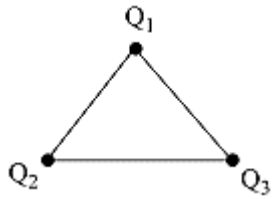
Answer Key									
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
(d)	(c)	(d)	(a)	(d)	(d)	(c)	(a)	(d)	(a)
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
(a)	(c)	(b)	(a)	(d)	(4t)	(a)	(a)	(0.9142)	(c)
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
(529)	(c)	(b)	(150)	(a)	(a)	(b)	(c)	(a)	(a)
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
(b)	(a)	(d)	(c)	(a)	(b)	(a)	(a)	(a)	(f)
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
(d)	(a)	(a)	(a)	(a)	(a)	(a)	(c)	(b)	(a)
51	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
(a)	(f)	(d)	(a)	(d)	(c)	(b)	(090)	(c)	(b)
61	**62**	**63**	**64**	**65**	**66**				
(a)	(a)	(a)	(a)	(a)	(c)				

SOLUTIONS

1. Solution: (d)

$Q_1 = Q_2 = Q_3 = Q$

When Q_1 is brought, there is no work done



So, $\Delta_1 = 0$

When Q_2 is brought, the increase in electrostatic energy

$\Delta_2 = \frac{Q^2}{4\pi\epsilon_0 a}$

When Q_3 is brought, the increase in electrostatic energy

$\Delta_3 = \frac{2Q^2}{4\pi\epsilon_0 a}$

2. Solution: (c)

Let first calculate the force in wire B due to magnetic field produced by wire A at the position of wire B

$B = \frac{\mu_0 I}{2\pi d}$

Force per unit length on wire B due to wire A

$F = \frac{\mu I^2}{2\pi d}$ towards right

Similarly, force per unit length on wire B due magnetic field produced by wire C

$F = \frac{\mu_0 I^2}{2\pi d}$ towards right

If wire B is displaced towards right, then net restoring force will be

$= \frac{\mu I^2}{2\pi(d-x)} - \frac{\mu_0 I^2}{2\pi(d+x)}$ towards left

$= \frac{\mu_0 I^2 x}{2\pi(d^2-x^2)}$

$\approx \frac{\mu_0 I^2}{2d^2} x$

So, $F = \frac{\mu_0 I^2}{\pi d^2} x$

$\Rightarrow m\frac{d^2x}{dx^2} = \frac{\mu_0 I^2}{\pi d^2} x$

$\Rightarrow \frac{d^2x}{dt^2} + \frac{\mu_0 I^2}{\pi m d^2} x = 0$

So, $\omega = \sqrt{\frac{\mu_0 I^2}{\pi m d^2}} = \sqrt{\frac{\mu_0}{\pi m}}\frac{I}{d}$

Time period $T = 2\pi\sqrt{\frac{\pi m}{\mu_0}}\frac{d}{I}$

3. Solution: (d)

By induction of charge option (d) is correct.

4. Solution: (a)

$\phi(r) \propto r^{-3}$ For quadrupole. The charge distribution given in (a) is a quadrupole.

5. Solution: (d)

For charge $2Q$ to be in equilibrium

Net force acting on $2Q$ should be zero or electric field at the position of $2Q$ due to Q and $4Q$ should be zero.

i.e. $\frac{Q}{4\pi\epsilon_0 x^2} = \frac{4Q}{4\pi\epsilon_0(10-x)^2}$

$(10-x)^2 = 4x^2$

$10 - x = 2x$

$\Rightarrow x = \frac{10}{3}$

6. Solution. (d)

In absence of grounding the charge distribution will be as follows.

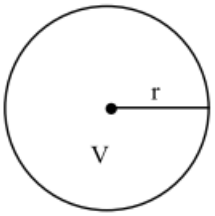
Net charge on inner surface$1 = 0$, because if we take Gaussian surface in the bulk of inner shell, it must enclose zero charge. Now, sum of charge as surface 2 and 3 plus charge q must add to zero as the Gaussian surface in the bulk to outer shell must enclose zero charge. Net charge on outer surface 4 must be equal to q, which will go to ground as it is grounded.



You must also keep in mind that electric field lines are perpendicular to conductions surface.

7. Solution: (c)

Let us take spherical surface of radius r and centred at q_0. The charge distribution has spherical symmetry hence, electric field also has spherical symmetry. Hence, electric field is either constant or a function of r only and field lines will be radial.



So, $\oint \vec{E}.ndS = \oint EdS = E\oint dS$

$= E.4\pi r^2$

Charge enclosed

$q_0 = q_0 + \int_V \rho d\tau$

$= q_0 + \int_0^r -\frac{q_0 e^{-2r/a}}{\pi a^3} dr$

$= q_0 - \frac{4q_0}{a^3}\left[-\frac{a}{2}e^{-\frac{2r}{a}}r^2 + a\int_0^r re^{-\frac{2r}{a}}dr\right]_0^r$

$= q_0 - \frac{4q_0}{a^3}\left[-\frac{a}{2}e^{-\frac{2r}{a}}r^2 + a\int_0^r re^{-\frac{2r}{a}}r + \frac{a}{2}\int_0^r e^{-\frac{2r}{a}}dr\right]_0^r$

$= q_0 - \frac{4q_0}{a^3}\left[-\frac{a}{2}e^{-\frac{2r}{a}}r^2 + a\left\{-\frac{a}{2}e^{-\frac{2r}{a}} + \frac{a}{2}e^{-\frac{2r}{a}}\right\}\right]_0^r$

$= q_0 - \frac{4q_0}{a^3}\left[-\frac{a}{2}e^{-\frac{2r}{a}}r^2 + a - \frac{a^2}{2}e^{-\frac{2r}{a}}r - \frac{a^3}{2}e^{-\frac{2r}{a}} + \frac{a^3}{4}\right]$

$= q_0 - \frac{4q_0}{a^3}(2r^2 + 2ar + a^2)e^{-2r/a}$

From Gauss law of electrostatics

$\oint \vec{E}.ndS = \frac{q_0}{\varepsilon_0}$

$\Rightarrow E.4\pi r^2 = \frac{1}{\varepsilon_0}\frac{q_0}{a^2}(2r^2 + 2ar + a^2)e^{-2r/a}$

$\Rightarrow E = \frac{q_0}{4\pi\varepsilon_0 a^2}(2a^2 + 2a^2 + a^2)e^{-2}$

$= \frac{5q_0}{4\pi\varepsilon_0 e^2 a^2}$

$\vec{E} = \frac{5q_0}{4\pi\varepsilon_0 e^2 a^2}\hat{e}_r$

8. Solution: (a)

$\nabla.E = 0$

So E is divergence less

(iii) & (iv) are incorrect. For negative side of x-axis, E is positive and the test charge will move towards right. So (a) is correct.

9. Solution: (d)

Torque on rod $\tau = \frac{Q^2}{4\pi\epsilon_0 l^2}.l - \frac{Q^2}{4\pi\epsilon_0(\sqrt{2}\,l)^2}.\frac{l}{\sqrt{2}}$

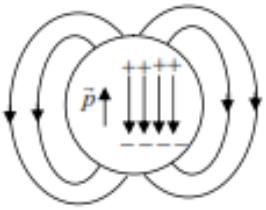
$= \frac{Q^2}{4\pi\,\epsilon_0\,l^2}\left(1 - \frac{1}{2\sqrt{2}}\right)$

$= \frac{Q^2}{16\pi\,\epsilon_0\,l}(4 - \sqrt{2})clockwise$

In vector form, $\bar{\tau} = = \frac{Q^2}{16\pi\epsilon_0 l}(4 - \sqrt{2})(-\hat{k})$

10. Solution: (a)

The direction of polarization is from negative to positive charge. However, the direction of electric field is from positive to negative charge.



11. Solution: (a)

Electrostatic pressure is $P = \frac{\sigma^2}{2\varepsilon_0}$

Force =pressure × area

$F = \frac{\sigma^2}{2\varepsilon_0}\,\pi R^2$

Option (a) is correct.

12. Solution: (c)

Since charge is placed at one corner to apply Gauss law, consider 7 imaginary cube surrounding to it such the this charge comes to center of cube of side '2a'.Hence the charge is equally shared by 8 cubes. To find flux we take surface area perpendicular to electric fields lines of a charge. Out of 6 only 3 faces will contribute to flux.

Hence flux will be, $\frac{q}{24\varepsilon_0}$

13. Solution: (b)

We know that, Potential at Centre O of cube

$V = \frac{\oint \phi.dS}{\oint dS}$

$= \frac{\phi a^2}{6a^2} = \frac{\phi}{6}$

14. Solution: (a)

$\nabla^2 V = -\frac{\rho}{\varepsilon_0}$

$\frac{1}{r}\frac{d}{dr}\left(r\frac{dV}{dr}\right) = -\frac{kr}{\varepsilon_0}$

$r\frac{dV}{dr} = -\frac{kr^2}{3\varepsilon_0} + c_1$

$\frac{dV}{dr} = -\frac{kr^2}{3\varepsilon_0} + \frac{c_1}{r}$

$V = -\frac{kr^3}{9\varepsilon_0} + c_1\log r + c_2$

Outside cylinder $\nabla^2 V = 0$

$\frac{1}{r}\frac{d}{dr}\left(r\frac{dV}{dr}\right) = 0$

$r\frac{dV}{dr} = c_r$

CHAPTER-5 THERMODYNAMICS & STATISTICAL MECHANICS

TIFR - PREVIOUS YEAR EXAM QUESTIONS

KTG

[TIFR-2014]

1. Consider the CO molecule as a system of two point particles which has both translational and rotational degrees of freedom. Using classical statistical mechanics, the molar specific heat C_v of CO gas is given in terms of the Boltzmann constant k_B by

(a) $\frac{5}{2}k_B$ (b) $2k_B$

(c) $\frac{3}{2}k_B$ (d) $\frac{1}{2}k_B$

[TIFR-2016]

2. Two containers are maintained at the same temperature and are filled with ideal gases whose molecules have mass m_1, and m_2 respectively. The mean speed of molecules of the second gas is 10 times the r.m.s. speed of the molecules of the first gas. Find the ratio of m_1/m_2, to the nearest integer

[TIFR-2016]

3. A gas of non-interacting particles, each of rest mass 1 MeV, is at a temperature T = 2.0 x 10^7 K and has an average particle density n = 2.7 x 10^{34} cm. We can obtain a reasonably correct treatment of this system

(a) Only by using special relativity as well as quantum mechanics.

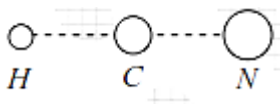
(b) By neglecting quantum mechanics but not special theory of relativity.

(c) By neglecting special relativity but not quantum mechanics.

(d) By neglecting both special relativity and quantum mechanics.

[TIFR-2019]

4. Consider the following linear model of a molecule of hydrogen cyanide (HCN) depicted below



It follows that the molar specific heat of hydrogen cyanide gas at constant pressure must be

(a) 4.5 R (b) 5 R

(c) 5.5 R (d) 6 R

LAWS OF THERMODYNAMICS

[TIFR-2014]

5. An ideal gas at a temperature T is enclosed in a rigid container whose walls are initially at temperature T_1, where $T_1 < T$. The walls are covered on the outside with perfect thermal insulation and the system is allowed to come to equilibrium. The pressure exerted by the gas on the walls of the container

(a) remains constant throughout.

(b) is lower at the initial stage than at the final stage.

(c) is higher at the initial stage than at the final stage.

(d) Is the same at the initial and final stage.

[TIFR-2015]

6. The equation of state of a gas is given by

$V = \frac{RT}{P} - \frac{b}{T}$

Where R is the gas constant and b is another constant parameter. The specific heat at constant pressure C_P and the specific heat at constant volume C_V for this gas is related by $C_P - C_V =$

(a) R (b) $R\left(1 + \frac{RT^2}{bP}\right)^2$

(c) $R\left(1 + \frac{bP}{RT^2}\right)^2$ (d) $R\left(1 - \frac{bP}{RT^2}\right)^2$

CALORIMETRY

[TIFR-2013]

7. A certain amount of fluid with heat capacity C_FJoules/^{0}C is initially at a temperature 0^o C. It is then brought into contact with a heat bath at a temperature of 100^o C, and the system is allowed to come into equilibrium. In this process, the entropy (in Joules/0C) of the Universe changes by

(a) $100C_F$ (b) 0

(c) 0.055 C_F (d) 0.044 C_F

[TIFR-2015]

8. In a cold country, in winter, a lake was freezing slowly. It was observed that it took 2 hours to form

a layer of ice 2 cm thick on the water surface. Assuming a constant thermal conductivity throughout the layer, the thickness of ice would get doubled after

(a) 8 more hours (b) 4 more hours

(c) 2 more hours (d) 6 more hours

[TIFR-2016]

9. In the temperature range 100 - 1000 C, the molar specific heat of a metal varies with temperature T (measured in degrees Celsius) according to the formula $C_p = (1 + T/5)\, J - \deg C^{-1} - mol^{-1}$If 0.2 kg of the metal at 600 C is brought in thermal contact with 0.1 kg of the same metal at 300 C, the final equilibrium temperature, in°C, will be [Assume that no heat is lost due to radiation and/or other effects.]

(a) 466 (b) 567

(c) 383 (d) 519

[TIFR-2018]

10. A many-body system undergoes a phase transition between two phases A and B at a temperature T_c. The temperature-dependent specific heat at constant volume C_V of the two phases are given by $C_V^{(A)} = aT^3 + bT$ and $C_V^{(B)} = cT^3$. Assuming negligible volume change of the system, and no latent heat generated in the phase transition, T_c is

(a) $\sqrt{\frac{4b}{c-a}}$ (b) $\sqrt{\frac{3b}{c-a}}$

(c) $\sqrt{\frac{2b}{c}}$ (d) $\sqrt{\frac{b}{c-a}}$

[TIFR-2019]

11. A thermally insulated coffee mug contains 500 g of warm coffee at $80°C$. Assuming that the heat capacity of this liquid is $1\ cal\ g^{-1}{}^0C^{-1}$ and the latent heat of fusion for ice is $80\ calg^{-1}$, the amount of ice that must be dropped into the cup to convert it into cold coffee at $5°\ C$ is about

(a) 471 g (b) 469 g

(c) 421g (d) 441g

[TIFR-2021]

12. Which of the following is the entropy generated when two identical blocks at temperatures 27^0 and T are brought into thermal contact and allowed to reach equilibrium? [Assume that the heat capacity of each block is C]

(a) $C(2\ln 3 - 3\ln 2)$ (b) Zero

(c) $2C\ln\frac{3}{2}$ (d) $C(2\ln 2 - 3\ln 3)$

THERMODYNAMIC PROCESS

[TIFR-2010]

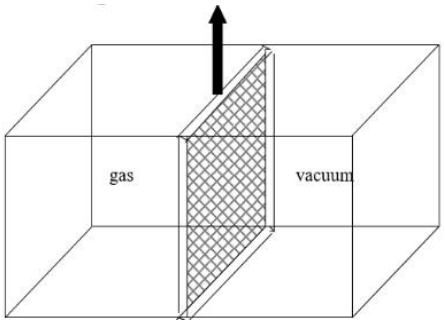
13. A car tyre is slowly pumped up to a pressure of 2 atmospheres in an environment at 15°C. At this point, it bursts. Assuming the sudden expansion of the air (a mixture of O2 and N2) that was inside the tyre to be adiabatic, its temperature after the burst is

(a) $-55°$ C (b) $-37°$ C

(c) $-26°$ C (d) $+9°$ C

[TIFR-2012]

14. Consider a sealed but thermally container of total volume V, which is in equilibrium with a thermal bath at temperature T. The container is divided into two equal chambers by a thin but impermeable partition. One of these chambers contains an ideal gas, while the other half is a vacuum (see figure)



If the partition is removed and the ideal gas is allowed to expand and fill the entire container, then the entropy per molecule of the system will increase by an amount

(a) $2k_B$ (b) $k_B\ In(1/2)$

(c) $k_B\ In2$ (d) $\frac{(k_B\ In\ 2)}{2}$

[TIFR-2013]

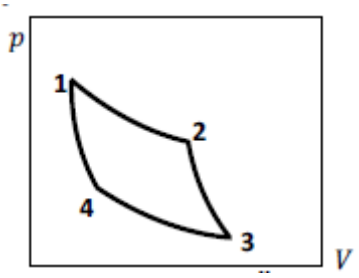
15. A monatomic gas is described by the equation of state$p(V - bn) = nRT$ where b and R are constants and other quantities have their usual meanings. The maximum density (in moles per unit volume) to which this gas can be compressed is

(a) $1/b_n$ (b) b

(c) $^1/_b$ (d) Infinity

[TIFR-2013]

16. The pV-diagram for a Carnot cycle executed by an ideal gas with $C_p/C_v = \gamma > 1$ Is shown below. Note that 1,2,3 and 4 label the change over point in the cycle If, for this cycle,



$\frac{T_2}{T_3} = \left(\frac{p_2}{p_3}\right)^x$

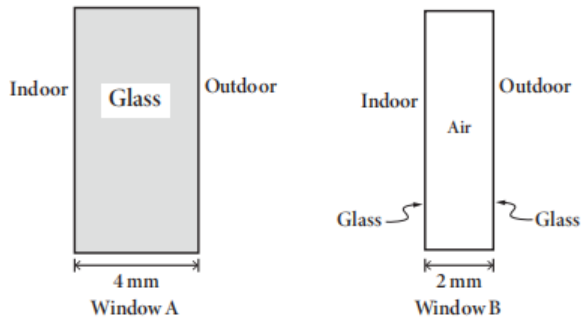
Then $X =$

(a) $1 - {}^1/_\gamma$ (b) 0

(c) 1 (d) $- {}^1/_\gamma$

[TIFR-2014]

17. A manufacture is able to offer two models of heat-conserving windows, as described below.



Window A is a simple pane of glass 4mm thick Window B, on the other hand consists of two extremely thin panes of glass, separated by an air gap of 2mm, as shown in the figure above. If the thermal conductivity of glass is known to be 0.8 $Wm^{-1}K^{-1}$ and that of air is $0.025Wm^{-1}K^{-1}$, then the ratio of heat flow Q_A throught Window A to heat flow Q_B throught Window B is given by

$\frac{Q_A}{Q_B} =$

(a) $\frac{1}{16}$ (b) $\frac{1}{4}$

(c) 4 (d) 16

[TIFR-2015]

18. An ideal diatomic gas in initially at a temperature $T = 0°C$. Then it expands reversibly and adiabatically to 5 times its volume. Its final temperature will be approximately

(a) $0°C$ (b) $-150°C$

(c) $-130°C$ (d) $-180°C$

[TIFR-2016]

19. The equation of states for a gas is given by $[p + \left(\frac{\alpha N}{V}\right)^2 (V - \beta N) = Nk_BT$ where P.V,T,N, and k_B represent pressure, volume, temperature, number of atoms and the Boltzmann constant, respectively, while a and B are constants specific to the gas.

If the critical point C corresponds to a point of inflexion of the p-V curve then the critical volume V_c and critical pressure p_c for this gas are given by

(a) $V_c = 3\beta N, p_c = \alpha^2/3\beta^2$

(b) $V_c = 3\beta N, p_c = \alpha^2/27\beta^2$

(c) $V_c = 3\beta N, p_c = 8\alpha^2/27\beta^2$

(d) $V_c = 3\beta N, p_c = \alpha^2/27\beta^2$

[TIFR-2017]

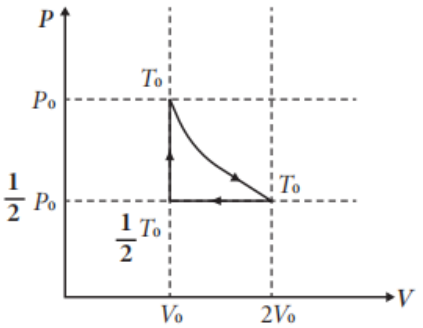
20. One mole of monoatomic ideal gas is= initially at pressure ρ_0 and volumeV_0.

The gas then undergoes a three-stage cycle consisting of the following processes:

i) An isothermal expansion till it reaches volume $2V_0$, and heat Q flows into the gas

ii) An isobaric compression back to the original volume V_0

iii) An isochoric increase in pressure till the original pressure ρ_0 is regained.



The corresponding P-V diagram is shown above.

The efficiency of this cycle can be expressed as

(a) $\epsilon = \frac{4Q+2RT_0}{4Q+RT_0}$ (b) $\epsilon = \frac{4Q+2RT_0}{4Q-3RT_0}$

(c) $\epsilon = \frac{4Q-2RT_0}{4Q+RT_0}$ (d) $\epsilon = \frac{4Q-2RT_0}{4Q+3RT_0}$

[TIFR-2017]

21. A thermally isolated container stores N_2 gas at $27.24^0\ C$ at one atmospheric pressure. Suddenly the pressure of the gas is increased to two atmospheric pressures. Assuming N_2 to behave as an ideal gas, estimate the change in temperature of the gas, in Celsius degrees (^{0}C).

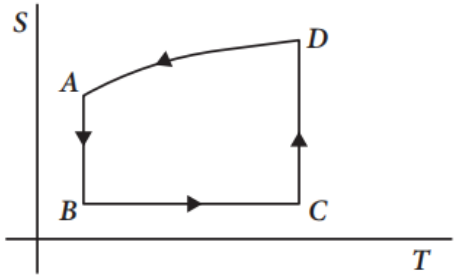
[TIFR-2018]

22. A heat engine is operated between two bodies that are kept at constant pressure. The constant-pressure heat capacity c_p of the reservoirs is

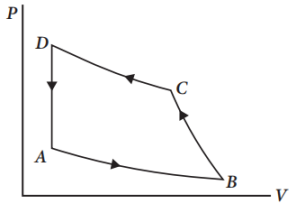
independent of temperature. Initially the reservoirs are at temperatures $300K$ and $402K$. If, after some time, they come to a common final temperature T_f, the process remaining adiabatic, what is the value of T_f (in Kelvin)?

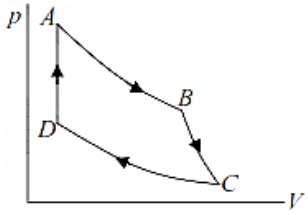
[TIFR-2019]

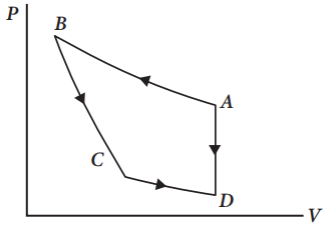
23. An ideal gas engine is run according to the cycle shown in the $s-T$ diagram below, where the process from D to A is known to be isochoric (i.e. maintaining V = constant)

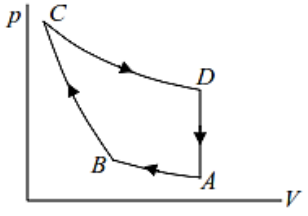


The corresponding cycle in the $p-V$ diagram will most closely resemble

(a)


(b)


(c)


(d)


[TIFR-2019]

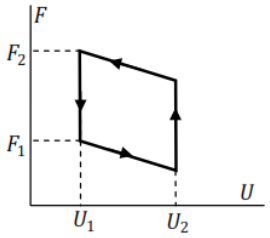
24. The molar equation of state of a gas at temperature T, pressure P and volume V is given by $p = \frac{RT}{V-b} - \frac{a}{TV^2}$. Where a and b are two constants and R is the gas constant. The critical temperature and pressure for the gas will be

(a) $T_c = \sqrt{\frac{4a}{27\,Rb}}, P_c = \frac{RT_c}{4b}$ (b) $T_c = \sqrt{\frac{8a}{3\,Rb}}, P_c = \frac{RT_c}{8b}$

(c) $T_c = \sqrt{\frac{8a}{27\,Rb}}, P_c = \frac{RT_c}{8b}$ (d) $T_c = \sqrt{\frac{a}{27\,Rb}}, P_c = \frac{RT_c}{b}$

[TIFR-2020]

25. An ideal gas is passed through a cyclic process where the corresponding changes in the thermodynamic potentials are plotted on the adjoining graph. Here U is the internal energy and F is the Helmholtz free energy.



The efficiency of this cycle is given by

(a) $1 - \frac{U_1}{U_2}$

(b) $1 - exp\left(-\frac{F_2}{F_1}\right)$

(c) $1 - \frac{U_1}{U_2} exp\left(-\frac{F_2}{F_1}\right)$

(d) $exp\left(\frac{U_1}{U_2}\right) - exp\left(-\frac{F_2}{F_1}\right)$

[TIFR-2021]

26. A spherical balloon of radius R is made of a material with surface tension γ and filled with N particles of an ideal gas. If the outside air pressure is P, the pressure P_b inside the balloon is given by

(a) $P_b = P + 2\gamma/R$ (b) $P_b = P$

(c) $P_b = P - 2\gamma/R$ (d) $P_b = P + 3\gamma/R$

[TIFR-2022]

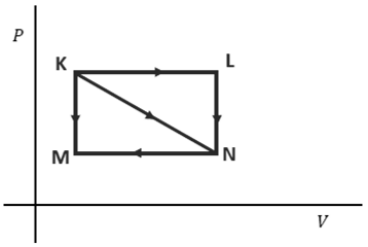
27. A bicycle tyre is pumped with air to an internal pressure of 6 atm at 20°C, at which point it suddenly bursts. Assuming the external pressure to be 1 atmosphere and the subsequent sudden expansion to be adiabatic, the temperature immediately after the burst is approximately

(a) -108.5°C (b) 216.0°C

(c) 45.5°C (d) -97.5°C

[TIFR-2024]

28. An ideal gas on the pressure (P) volume (V) diagram can be taken from point K to point N along three different paths, as shown below. $K \to L \to N$, $K \to N$ and $K \to M \to N$. Which of the following potions is a true statement. ?



(a) The change in internal energy is the same along each path

(b) The same work is done along each path

(c) The same amount of heat is added to each the gas along each path

(d) There is no work done along the path $K \to N$

[TIFR-2024]

29. An ideal heat pump delivers heat at a rate $\beta(T_H - T_S)$ from the colder surroundings (S) at a temperature T_s to a heater (H) kept at a constant temperature T_H. Here β is a constant. What is the power consumption of the heat pump?

(a) $\frac{\beta(T_H - T_S)^2}{T_H}$ (b) $\beta(T_H - T_s)$

(c) $\frac{\beta(T_H - T_S)^2}{T_S}$ (d) $\frac{\beta(T_H - T_S)T_H}{T_S}$

THERMODYNAMIC POTENRTIALS

[TIFR-2011]

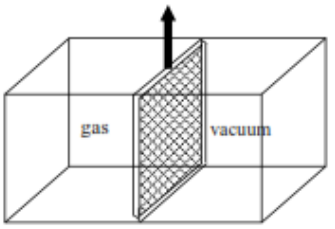
30. A closed thermally insulated box contains one mole of an ideal monatomic gas G in thermodynamics equilibrium with blackbody radiation B. The total internal energy of the system is $U = U_G + U_B$ where U_G and $U_B(\propto T^4)$ are the energies of the ideal gas and the radiation respectively. If $U_G = U_B$ at a certain temperature$T_0 K$, then energy required to raise the temperature from $T_0 K$ to$(T_0 + 1)K$, in terms of the gas constant R, is

(a) $7.5\,R$ (b) $6R$

(c) $1.5R$ (d) $0.33R$

[TIFR-2023]

31. Consider a sealed but thermally conducting container of total volume V, which is in equilibrium with a thermal bath at temperature 7. The container is divided into two equal chambers by a thin partition, which is thermally conducting but impermeable to particles. One of the chambers contains an ideal gas, while the other is a vacuum.



If the partition is removed suddenly and the ideal gas is allowed to expand and fill the entire container, then, once equilibrium has been reached, the entropy per molecule will increase by an amount

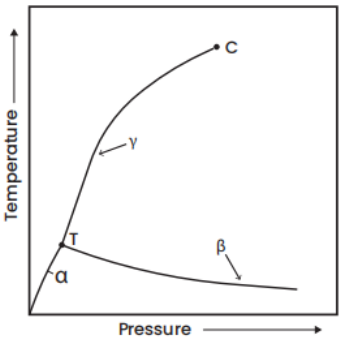
(a) $k_B\, In\, 2$ (b) $-k_B\, In\, 2$

(c) $2k_B$ (d) $\frac{k_B}{2} In2$

PHASE TRANSITION

[TIFR-2011]

32. The phase diagram of a pure substance is given in the figure below, where $'T'$ denotes the triple point and $'C'$ denotes the critical point.



The phase transitions occurring along the lines marked α, β and γ are

(a) $\alpha =$ Melting; $\beta =$ condensation; $\gamma =$ sublimation

(b) $\alpha =$ Sublimation; $\beta =$ vaporization; $\gamma =$ melting

(c) $\alpha =$ Melting; $\beta =$ vaporization; $\gamma =$ condensation

(d) $\alpha =$ Sublimation; $\beta =$ melting; $\gamma =$ vaporization

ENSEMBLES AND ITS TYPES

[TIFR-2011]

33. A quantum system has three energy levels $-0.12eV, -0.2eV$ and $-0.44eV$ respectively. Three electrons and distributed among these levels. At a temperature of $1727°C$ the system has total

rotational degrees of freedom and 4 vibrational degrees of freedom.

So, $DOf, f = 3 + 2 + 4 = 9$

$C_V = \frac{fR}{2} = \frac{9R}{2}$

$C_p = C_V + R = \frac{9R}{2} + R = \frac{11R}{2} = 5.5R$

5. Solution: (c)

When the gas has temperature $T > T_1$ and pressure exerted by gas on walls is proposal to V_{rms} where

$V_{rms} = \sqrt{\frac{3k_BT}{m}}$

Thus pressure $\propto T$

Since initial gas has temperature $T > T_1$ after thermal equilibrium it reduces from T.

∴ The pressure is higher at initial stage that the final stage.

6. Solution: (c)

$C_p = C_r = T\left(\frac{\partial p}{\partial T}\right)_V \left(\frac{\partial V}{\partial T}\right)_P$

Equation of state given by

$V = \frac{RT}{p} - \frac{b}{T}$

$\left(\frac{\partial V}{T\partial}\right)_p = \frac{R}{P} + \frac{b}{T^2}$

Differentiating w.r.t T keeping V constant

$0 = \frac{R}{P} + \frac{RT}{p^2}\left(\frac{\partial p}{\partial T}\right)_V + \frac{b}{T^2}$

$\left(\frac{\partial p}{\partial T}\right)_V = \frac{\left(\frac{R}{p} + \frac{b}{T^2}\right)}{\frac{RT}{p^2}} + \frac{p^2}{RT}\left(\frac{R}{p} + \frac{b}{T^2}\right)$

$C_p = C_r = T\left(\frac{\partial p}{\partial T}\right)_V \left(\frac{\partial V}{\partial T}\right)_P$

$= T.\frac{p^2}{RT}\left(\frac{R}{p} + \frac{b}{T^2}\right).\left(\frac{R}{p} + \frac{b}{T^2}\right)$

$= T.\frac{p^2}{RT}\frac{R^2}{p^2}\left(1 + \frac{bp}{RT^2}\right)^2$

7. Solution: (d)

Entropy change of fluid $\Delta S_1 = \int \frac{dQ}{T} = \int_{273}^{373} \frac{C_F dT}{T}$

$C_F \log\frac{373}{273} = 0.313C_F$

Entropy change in heat bath $\Delta S_2 = -\frac{\Delta Q}{T}$

$= \frac{-C_F \Delta T}{T}$

$= -\frac{C_F \times 100}{373} = -0.268C_F$

Total entropy change in universe

$\Delta S = \Delta S_1 + \Delta S_2$

$= 0.312C_F - 0.268C_F$

$= 0.044C_F$

8. Solution: (d)

Rate of heat flow $\frac{dQ}{dt} = \frac{\Delta T}{(x/kA)}$

Where $\Delta T =$ temperature difference, x is thickness, A is area of cross – section of heat flow and k is thermal conductivity

Let dx thickness of water freezes in dt time

So, heat flow in dt time, $dQ = (\rho A dx)L$

Where L is latent heat

So, $\frac{dQ}{dt} = \rho AL\frac{dx}{dt} = \frac{(\Delta T)kA}{x}$

$\Rightarrow xdx = \left(\frac{k\Delta T}{\rho L}\right)dt$

$\Rightarrow \int_0^x xdx = \left(\frac{k\Delta T}{\rho L}\right)\int_0^t dt$

$x^2 = \left(\frac{2k\Delta T}{\rho L}\right)t$

$= kt$

So, time taken for 2 cm of water of freeze = 2 hrs.

Let time taken for 4 cm of water of freeze is t

So, $(2cm)^2 = k(2\text{hrs})$

$(4cm)^2 = k(t\text{ hrs})$

Dividing $\frac{16}{4} = \frac{t}{2} \Rightarrow t = 8\,hrs$

9. Solution: (d)

Let T be the equilibrium temperature.

Heat lost by 0.2 kg = Heat gained by $0.1kg$

$\int_{600}^{T} m_1 C_p dT + \int_{300}^{T} m_2 C_p dT = 0$

$\Rightarrow 0.2\int_{600}^{T}\left(1 + \frac{T}{5}\right)dT + 0.1\int_{300}^{T}\left(T + \frac{T}{5}\right)dT = 0$

$\Rightarrow 2\left(T + \frac{T^2}{10}\right)\Big|_{600}^{T} + T + \frac{T^2}{10}\Big|_{300}^{T} = 0$

$\Rightarrow 2\left(T + \frac{T^2}{10} - 600 - 3600\right) + \left(T + \frac{T^2}{10} - 300 - 9000\right) = 0$

$\Rightarrow 3\frac{T^2}{10} + 3T - 82500 = 0$

$T^2 + 10T - 275000 = 0$

$T = \frac{-10 + \sqrt{10^2 + 4 \times 275000}}{2} = 519.428$

$\approx 519°C$

10. Solution: (b)

Since from given problem, volume change in negligible and C_V is discontinuous in transition from phase A to phase B.

∴ It is 2nd order phase transition.

Hence entropy should be continuous

$S_A = S_B$

$\left(\int \frac{\mathrm{dQ}}{\mathrm{T}}\right)_{\mathrm{A}} = \left(\int \frac{\mathrm{dQ}}{\mathrm{T}}\right)_{\mathrm{B}}$

$\int \frac{\mathrm{CvdT}}{\mathrm{T}} = \int \frac{\mathrm{CvdT}}{\mathrm{T}}$

$\int \frac{(aT^3 + bT)}{T} dT = \int \frac{cT^3}{T} dT$

$\int (aT^2 + b) dT = \int cT^2 dT$

$\frac{aT^3}{3} + bT = \frac{cT^3}{3} \Rightarrow T = \sqrt{\frac{3b}{C-a}}$

11. Solution: (d)

Heat lost by coffee

$H_1 = mc\Delta T$

$= 500 \times 1 \times 75$

$= 37500\ cal$

Let amount of ice required is m (in grams)

Heat gained by ice,

$H_2 = mL + mc\Delta T$

$80cm + m \times 5 = 85m$

Now, $H_2 = H_1$

$85m = 37500$

$m = \frac{37500}{85} = 441g$

12. Solution: (a)

Let two black with hear capacity c are in thermal contact with initial temperatures 2T and T.

By calorimetry the equilibrium temp. is

$T_f = \frac{C_1T_1 + C_2T_2}{C_1 + C_2}$

$= \frac{T_1 + T_2}{2}$

$= \frac{2T + T}{2}$

$= T_F = \frac{3T}{2}$

Total entropy change,

$\Delta S = \Delta S_1 + \Delta S_2$

$\int_{T_1}^{T_F} \frac{CvdT}{T} + \int_{T_2}^{T_f} \frac{CvdT}{T}$

$= C \int_{2T}^{T_f} \frac{dT}{T} + C \int_{T}^{T_f} \frac{dT}{T}$

$= C\ \ell n \left(\frac{T_f}{2T}\right) + c\ \ell n \left(\frac{T_F}{T}\right)$

$= c\ \ell n \left(\frac{T_f^2}{2T^2}\right)$

$= c\ \ell n \left(\frac{9T^2}{4(2T^2)}\right)$

$= c\ \ell n \left(\frac{9}{8}\right)$

$\Delta S = c(2\ell n 3 - 3\ell n^2)$

∴ Option (a) is correct

13. Solution: (b)

Here, $\gamma = \frac{C_p}{C_v} = \frac{7}{5} = 1.4$

$Tp^{\frac{1-\gamma}{\gamma}} = constant$

$T_i p_i^{\frac{1-\gamma}{\gamma}} = T_f p_f^{\frac{1-\gamma}{\gamma}}$

$T_f = \left(\frac{P_i}{P_f}\right)^{\frac{1-\gamma}{\gamma}} T_i$

$= T_i 2^{-2/7}$

$\frac{288}{2^{2/7}} = 236.3K$

$= -36.7°\mathrm{C}$

So $-37°$ C is correct answer.

14. Solution: (c)

Here the expansion of gas is isothermal,

Hence, Change in entropy

$\Delta S = \int \frac{dQ}{T} = \int \frac{dW}{T}$

$= \int \frac{PdV}{T}$

$= nR \int_{V/2}^{V} \frac{dV}{V}$

$= \frac{N}{N_0} R\ In\ 2$

$= Nk_B\ In\ 2$

Increase in entropy per molecule $= \frac{\Delta S}{N} = k_B\ In\ 2$

15. Solution: (c)

$P(V - bn) = nRT$

$P = \frac{nRT}{(V-bn)}$

$PV = n(RT + pB)$

$\frac{n}{V} = \frac{P}{RT+pb}$

CHAPTER-7 ATOMIC & MOLECULAR PHYSICS

TIFR - PREVIOUS YEAR EXAM QUESTIONS

HYDROGEN ATOM

[TIFR-2011]

1. Given that the ionization energies of hydrogen (^1H) and Lithium (^3Li) are $13.6eV$ and $5.39\ eV$, Respectively, the effective nuclear charge experienced by the valence electron of a ^3Li atom may be estimated in terms of the proton charged e as

(a) $0.63\ e$ (b) $1.26\ e$

(c) $1.59\ e$ (d) $3.00\ e$

[TIFR-2012]

2. If we model the electrons as a uniform sphere of radius r_e, spinning uniformly about an axis passing through its Centre with angular momentum $L_e = \frac{\hbar}{2}$, and demand that the velocity of ration at the equator cannot exceed the velocity c of light in vacuum, then the minimum value of r_e is

(a) $19.2\ fm$ (b) $0.192\ fm$

(c) $4.8\ fm$ (d) $1960\ fm$

(e) $480\ fm$

[TIFR-2013]

3. The velocity of an electron in the ground state of a hydrogen atom isv_H. If v_p be the velocity of an electron in the ground state of positronium, then

(a) $v_p = v_H$ (b) $v_p = 2v_H$

(c) $v_p = {}^{v_H}/_2$ (d) $V_p = \sqrt{2}v_H$

[TIFR-2015]

4. A sample of ordinary hydrogen (^1_1H) gas in a discharge tube was seen to emit the usual Balmer spectrum. On careful examination, however, it was found that the H_α line in the spectrum was split into two fine lines, one an intense line at 656.28 nm, and the other a faint line at 656.04 nm. From this one can conclude that the gas sample had a small impurity of

(a) ^3_1H (b) ^2_1H

(c) ^4_2H (d) H_2O

[TIFR-2017]

5. Hydrogen atoms in the atmosphere of a star are in thermal equilibrium, with an average kinetic energy of 1 eV. The ratio of the number of hydrogen atoms in the 2nd excited state $(n = 3)$ to the number in the ground state $(n = 1)$ is

(a) 3.16×10^{-11} (b) 1.33×10^{-8}

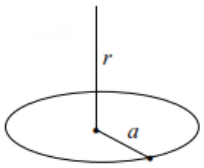
(c) 3.16×10^{-8} (d) 5.62×10^{-6}

[TIFR-2017]

6. The energy of an electron in the ground state of the He atom is $-79eV$. Considering the Bohr model of the atom, what would be 10 times the first ionization potential for a He^+ ion, in units of eV?

[TIFR-2019]

7. Consider a hydrogenic atom in its ground state ass conceived in Bohr's theory, where an electron of charge $-e$ is rotating about a central nucleus of charge $+Ze$ in a circular orbit of radius $a = 4\pi \in_0 \hbar^2/Ze^2 in$. In this model, the magnetic field at a distance r from the nucleus, perpendicular to the orbit will be



(a) $\frac{Ze^2\mu_0}{4\pi^2\hbar\epsilon_0}\frac{1}{a}\left(1+\frac{r^2}{a^2}\right)^{-1/2}$

(b) $\frac{Ze^3\mu_0}{8\pi\hbar\epsilon_0}\left(1+\frac{r^2}{a^2}\right)^{-3/2}$

(c) $\frac{Ze\mu_0}{\hbar\epsilon_0}\left(1+\frac{r^2}{a^2}\right)^{-3/2}$

(d) $\frac{Ze^2\mu_0}{16\pi^2\hbar\epsilon_0}\frac{1}{a}\left(1+\frac{r^2}{a^2}\right)^{-1/2}$

[TIFR-2020]

8. A sample of hydrogen gas was placed in a discharge tube and its spectrum was measured using a high-resolution spectrometer. The H_α line in the spectrum was found to be split into two lines, a high intensity line at 656.28 nm, and a low intensity line at 656.01 nm. This indicates that the hydrogen sample was contaminated with

(a) Deuterium (b) Tritium
(c) Helium (d) Water vapour

[TIFR-2021]

9. A hydrogen atom in its ground state collides with an electron of energy 13.377eV, absorbs most of the energy of the electron, and goes into an excited state. The maximum possible fraction $f = \frac{R_{final}-R_{initial}}{R_{initial}}$. By which its radius R would increase will be
(a) $f = 0.63$ (b) $f = 0.48$
(c) $f = 0.60$ (d) $f = 0.07$

[TIFR-2023]

10. The energy gap between the $n = 1$ and the $n = 2$ energy levels of a hydrogen atom is denoted E_0 Now, considera muonic carbon ion C^{5+} i.e., a carbon nucleus $({}_6C^{12})$ orbited by a muon $\mu(q = -e, M_\mu = 210m_e)$.The energy of the photon emitted in the transition of the muon from the n=3 level to the n=2 level of this ion will be approximately
(a) $1400E_0$, (b) $235E_0$
(c) $1050E_0$ (d) $7560E_0$

[TIFR-2024]

11. The ionization potential of the H atom is 13.598 eV. If the mass of a proton is 1.673 x 10-27 kg, the mass of an Electron is 9.109 x 10-31 kg and the mass of the D nucleus is 3.344 × 10−27 kg, the ionization potential of the D atom is given by:
(a) 13.602 eV (b) 13.594 eV
(c) 13.598 eV (d) 27.188 eV

VECTOR ATOM MODEL AND STERN GERLACH EXPERIMENT

[TIFR-2014]

12. The ground state electronic configuration for a carbon atom is$(1s)^2(2s)^2(2p)^2$. The first excited state of this atom would be achieved by
(a) Re-alignment of the electron spins within the $2p$ orbital.
(b) Transition of an electron from the $2s$orbital to the $2p$ orbital.
(c) Transition of an electron from the $2p$ orbital to the $3s$orbital.
(d) Transition of an electron from the $2s$ orbital to the $3s$orbital.

LS & JJ COUPLING

[TIFR-2023]

13. The number of hyperfine states found in the He^3 atom for the electronic configuration $1s^1\ 2s^0\ 2p^3$ would be
(a) 7 (b) 2
(c) 4 (d) 1

ELECTRINIC TRANSITIONS

[TIFR-2010]

14. When white light is scattered from a liquid, a strong absorption line is seen at 400 nm, and two emission lines are observed, one of which is at 500 nm, and another in the infra-red portion of the spectrum. The wavelength of this second emission line is
(a) 900 nm (b) 2000 nm
(c) 100 nm (d) 222 nm

[TIFR-2010]

15. An atom is capable of existing in two states: a ground state of mass M and an excited state of mass $M + \Delta$. If the transition from the ground state to the excited state proceeds by the absorption of a photon, the photon frequency in the laboratory frame (where the atom is initially at rest) is
(a) $\frac{\Delta c^2}{h}$ (b) $\frac{\Delta c^2}{h}\left(1+\frac{\Delta}{2M}\right)$
(c) $\frac{Mc^2}{h}$ (d) $\frac{\Delta c^2}{h}\left(1-\frac{\Delta}{2M}\right)$
(e) $\frac{Mc^2}{h}\left(1+\frac{\Delta}{2M}\right)$ (f) $\frac{Mc^2}{h}\left(1-\frac{\Delta}{2M}\right)$

[TIFR-2011]

16. An excited atomic electron undergoes a spontaneous transition
$$3d_{3/2} \rightarrow 2p_{1/2}$$
The interaction responsible for this transition must be of the type
(a) Electric dipole (E1) or magnetic quadrupole (M2)
(b) Electric dipole (E1) or magnetic dipole (M1)
(c) Electric quadrupole (E2) or magnetic quadrupole (M2)
(d) Electric quadrupole (E2) or magnetic dipole (M1)

[TIFR-2013]

17. When a pure element is vaporised and placed in a uniform magnetic field B_0, it is seen that a particular spectral line of wavelength λ , corresponding to a $J = 1 \rightarrow J = 0$ transition, gets split into three components $\lambda, \lambda \pm \Delta\lambda$. It follows that the Landé g-factor for the transition $J = 1 \rightarrow J = 0$ is given by

(a) $g = \frac{hc}{\mu_B B_0}\frac{\Delta\lambda^2}{\lambda}$ (b) $g = \frac{hc}{\mu_B B_0}\frac{\lambda^2}{\Delta\lambda}$

(c) $g = \frac{hc}{\mu_B B_0}\frac{\lambda}{\Delta\lambda^2}$ (d) $g = \frac{hc}{\mu_B B_0}\frac{\Delta\lambda}{\lambda^2}$

ROTATIONAL SPECTROSCOPY

[TIFR-2012]

18. The strongest three lines in the emission spectrum of an interstellar gas cloud are found to have wavelength $\lambda_0, 2\lambda_0$ and $6\lambda_0$ respectively, where λ_0 is a known wavelength. From this we can deduce that the radiating particles in the cloud behave like
    (a) Free particles
    (b) Particles in a box
    (c) Harmonic oscillators
    (d) Rigid rotators
    (e) Hydrogeninc atoms

[TIFR-2016]

19. Two homo-nuclear diatomic molecules produce different rotational spectra, even though the atoms are known to have identical chemical properties. This leads to the conclusion that the atoms must be
    (a) isotopes, i.e. with the same atomic number
    (b) isobars, i.e. with the same atomic weight.
    (c) isotones, i.e. with the same neutron number.
    (d) isomers, i.e. with the same atomic number and weight.

[TIFR-2017]

20. The separation between neighbouring absorption lines in a pure rotational spectrum of the hydrogen bromide (HBr) molecule is 2.23meV. If this molecule is considered as a rigid rotor and the atomic mass number of Br is 80, the corresponding absorption line separation in deuterium bromide (DBr) molecule, in units of meV, would be
    (a) 2.234 (b) 1.115
    (c) 1.128 (d) 4.461

[TIFR-2022]

21. Treat the hydrogen molecule H_2 as a rigid rotator. The next-to-largest wavelength in its rotational spectrum is about 111 μm. From this is can be estimated that the separation between the pair of hydrogen atoms is about
    (a) 24.4 nm (b) 61.4 nm
    (c) 3.07 μm (d) 0.12 nm

RAMAN SPECTROSCOPY

[TIFR-2010]

22. Light transmitted along an optical fibre incurs losses due to Rayleigh scattering from in homogeneities. If a fibre of given length transmits 50% of the monochromatic light coupled into it at a wavelength of 1350 nm, the transmitted fraction for the same fibre at 1550 nm will be
    (a) 55% (b) 57%
    (c) 62% (d) 67%
    (e) 74% (f) 87%

LASER

[TIFR-2012]

23. When light is emitted from a gas of excited atoms, the lines in the spectrum are Doppler-broadened due to the thermal motion of the emitting atoms.
    The Doppler width of an emission line of wavelength 500 nanometer (nm) emitted by an excited atom of Argon $(^{40}_{20}A)$ at room temperature $(27°C)$ can be estimated as
    (a) $5.8 \times 10^{-4} nm$ (b) $3.2 \times 10^{-4} nm$
    (c) $3.2 \times 10^{-3} nm$ (d) $2.5 \times 10^{-3} nm$

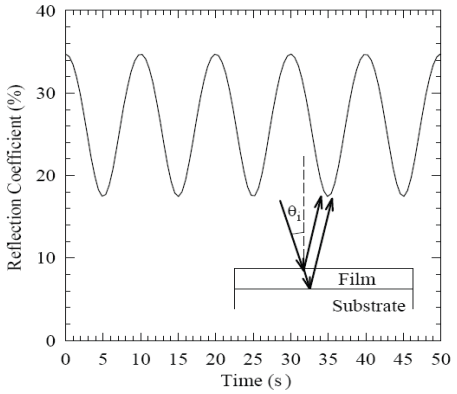
[TIFR-2014]

24. A beam of atoms moving in a certain direction can be slowed down if they absorb photons from a laser beam moving in the opposite direction and subsequently spontaneously emit photons isotopically. For a beam of Sodium atoms (mass number$A = 23$) with speed 600 ms^{-1}, if a laser beam of wavelength 589 nm is used, the number of such absorption and emission cycles needed to bring a Sodium atom to rest would be approximately
    (a) 1.3×10^5 (b) 1.3×10^4
    (c) 2.1×10^3 (d) 2.1×10^4

[TIFR-2013]

25. The rate of deposition of a dielectric thin film on a thick dielectric substrate was monitored by the following experiment: a laser beam of wavelengthλ= 633 nm, at near normal incidence θ_i, was reflected from the thin film (see inset figure on the right), and the reflection coefficient R was measured. As the film thickness increased R varied with time as shown on the right. The refractive index of the film is 3.07 and is less than that of the substrate. Using

the graph, the approximate thickness of the film at the end of 25 seconds can be estimated to be



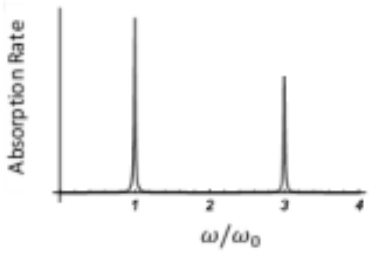
(a) 0.017 μm (b) 0.26 μm

(c) 0.51 μm (d) 2.2 μm

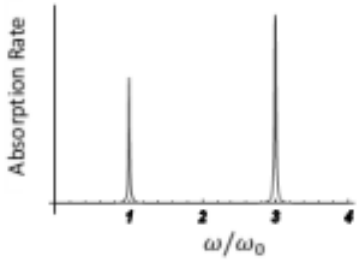
[TIFR-2021]

26. Consider a one-dimensional simple harmonic oscillator with frequency ω_0 in its ground state. An external wave passes through this system, creating a small time-dependent potential of the form $V(x,t) = Ax^3 \cos \omega t$, where À and ω are constants. If the absorption rate of the wave is measured as a function of ω which of the following graphs is the likely result of such a measurement?

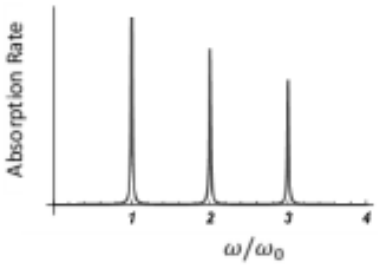
(a)



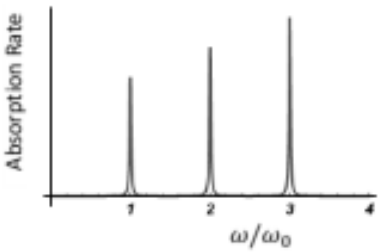
(b)



(c)



(d)



[TIFR-2016]

27. A continuous monochromatic (λ = 600 nm) laser beam is chopped into 0.1 ns pulses using some sort of shutter. Find the resultant linewidth $\Delta\lambda$ of the beam in units of 10^{-3} nm.

[TIFR-2022]

28. A gas of atoms, each of mass m, in thermal equilibrium at a temperature T, is radiating with a frequency v_0. The Doppler broadening (full width at half maximum, or FWHM) of the observed spectral line would be given by

(a) $\frac{2v_0}{C}\sqrt{\frac{2k_BT}{m}}$ (b) $\frac{2v_0}{C}\sqrt{\frac{2In2k_BT}{m}}$

(c) $\frac{2v_0}{C}\sqrt{\frac{In2k_BT}{m}}$ (d) $\frac{v_0}{C}\sqrt{\frac{2k_BT}{m}}$

Answer Key									
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
(b)	(e)	(a)	(a)	(b)	246	(b)	(c)	(a)	(a)
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
(a)	(b)	(a)	(b)	(a)	(a)	(d)	(c)	(b)	(c)
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**		
(d)	(e)	(a)	(d)	(b)	(a)	$0.012nm$	(b)		

SOLUTION

1. Solution: (b)

Energy of one electron species are given by

$$E_n = -\frac{13.6Z^2}{n^2}eV$$

In Li, there are three electrons of which two electrons occupy first shell and third electron occupies second shell. Here we have to find out that what is effective nuclear charge experienced by the valence electron.

$$E_2 = \frac{13.6Z^2}{4} = -5.3$$

$= z = 1.26e$

2. Solution: (e)

Angular momentum

$L = \frac{\hbar}{2}$

$\Rightarrow \frac{2}{3}mr^2\omega = \frac{\hbar}{2}$

$\Rightarrow \frac{2}{5}mrv = \frac{\hbar}{2}$

$r_\min = \frac{5h}{4mv_{max}}$

$= \frac{5\times6.63\times10^{-34}}{2\times3.14\times4\times9.1\times10^{-31}\times3\times10^{8}}$

$= 483.4\ fm$

3. Solution: (a)

Velocity $v = \frac{e^2}{2nh\varepsilon_0}$

Velocity is independent of mass, so,

$v_p = v_H$

4. Solution: (a)

$\frac{hc}{\lambda} = \frac{\mu e^4}{8h^2\varepsilon_0^2}\left(1-\frac{1}{4}\right)$

$= \frac{hc}{656.28\times10^{-9}} = \frac{\mu e^4}{8h^2\varepsilon_0^2}\left(1-\frac{1}{4}\right)$

$\frac{hc}{656.04\times10^{-9}} = \frac{u'e^4}{8h^2\varepsilon_0^2}\left(1-\frac{1}{4}\right)$

Dividing $(2) \div (1)$

$\frac{\mu'}{\mu} = \frac{656.28}{656.04}$

$\mu' = \frac{(Am_n).m_c}{(Am_n+m_c)}$

$\mu = \frac{m_n.m_c}{m_n+m_c}$

$\Rightarrow \frac{A(m_n+m_c)}{(Am_n+m_c)} = \frac{656.28}{656.04}$

$\Rightarrow \frac{1841A}{(1840A+1)} = 1.000365$

$A = 3.06$

So, the impurity in ^3_1H

5. Solution: (b)

The ratio of the number of particles in different energy states in thermal equilibrium can be determined using the Boltzmann distribution

$$\frac{N_2}{N_1} = \frac{g_2}{g_1}e^{\frac{-E_2-E_1}{k_BT}}$$

Where, N_2 and N_1 are the number of particles in states N_2 and N_1

g_2 & g_1 : Are the pf states n_2 & n_1

E_2 & E_1 : Are the energies of states.

k_B is the Boltzmann constant

The degeneracy (g) of a state = g = $2n^2$

$$E_n = -\frac{13.6}{n^2}\ eV$$

For ground state $(n=1)$: $g_1 = 2, E_1 = -13.6eV$

For 2nd ground state $(n=3)$; $g_3 = 18, E_2 = -\frac{13.6}{g}eV$

$\therefore \frac{N_3}{N_1} = \frac{g_3}{g_1}e^{\frac{-E_3-E_1}{kT}} = \frac{18}{2}\ e^{-\frac{\left(-\frac{13.6}{9}+13.6\right)}{kBT}}$

$\frac{N_3}{N_1} = g\ e^{\frac{-13.6\left(-\frac{1}{9}+1\right)}{kBT}} = g\ e^{\frac{\left(-\frac{13.6}{9}+13.6\right)}{kBT}}$

It given, $\frac{3}{2}kBT = 1\ eV,\ kBT = \frac{2}{3}\ eV$

$\therefore \frac{N_3}{N_1} = 9\ e^{\frac{-13.6\times8\times3}{9\times2}} = 9e^{-18.13}$

$\frac{N_3}{N_1} = 1.337\times10^{-8}$

6. Solution: (246)

Ground state energy of He atoms

$E_{He} = -79eV$

After ionizing the He atom it become He^+

Ground state energy of He^+ is

$E_{He^+} = -\frac{13.6\,Z^2}{n^2}$

$= -\frac{13.6\times4}{1} = -54.4\ eV$

So, first ionization potential for a He

$E_{He^+} - E_{He} = (-54.4) - (-79)$

$= 24.6\ eV$

So, 10 times of first ionization potential

$= 10\times24.6 = 246\ eV$

7. Solution: (b)

Magnetic field at distance r is given by

$$B = \frac{\mu_0 I}{2a}\left(1+\frac{r^2}{a^2}\right)^{-\frac{3}{2}}$$

For hydro genic atom $I = \frac{eV}{2\pi a}$

So,

$$B = \frac{\mu_0 eV}{4\pi a^2}\left(1+\frac{r^2}{a^2}\right)^{-\frac{3}{2}} = \frac{Ze^3\mu_0}{8\pi\varepsilon_0 ha^2}\left(1+\frac{r^2}{a^2}\right)^{-\frac{3}{2}}$$

8. Solution: (c)

$E_n = -\frac{\mu e^4}{8n^2h^2\epsilon_0^2}$

For H_α Line

$\frac{\lambda c}{\lambda} = E_3 - E_2 = \frac{\mu e^4}{8h^2\epsilon_0^2}\left(\frac{1}{4}-\frac{1}{9}\right)$

So, $\frac{\lambda}{\lambda'} = \frac{\mu'}{\mu}$

$$\Rightarrow \frac{656.28}{656.01} = \frac{\mu'}{\mu} = \frac{\frac{xm_p.m_e}{xm_p+m_e}}{\frac{m_p.m_e}{m_p+m_e}} = x\frac{(m_p+m_e)}{xm_p+m_e}$$

$$= \frac{x\left(\frac{m_p}{m_c}+1\right)}{x\frac{m_p}{m_c}+1} = \frac{1841x}{1840x+1}$$

$\Rightarrow 656.28 \times 1840x + 656.28 = 656.01 \times 1841x$

$\Rightarrow 1207552.2x + 656.28 = 1207714.41x$

$\Rightarrow x = \frac{656.28}{162.21} = 4.046$

So, nucleus of unknown species will contain 4 nucleons. It must be He.

9. Solution: (a)

Here $E = 13.377eV$,

Hence 13.6-13.377=0.223eV

$$n^2 = \frac{13.6}{0.223} = 60.9865 \Rightarrow n = 7.8 \approx 8$$

Radius $\propto n^2$

$$\therefore f = \frac{R_{Final} - R_{Initial}}{R_{Initial}} = \frac{64^2-1}{1} = 63$$

10. Solution: (a)

$E_n = \frac{\mu}{m_e}E_n \Rightarrow E_0 = 13.6 \times \frac{3}{4} \times \frac{1836}{1837}eV \Rightarrow leV = 4 \times \frac{1837 \times E_0}{1836 \times 13.6 \times 3}$

$E_n'(C^{12}) = \frac{\mu}{m_e}E_n \Rightarrow E = \frac{12\times1836\times208}{12\times1836+208} \times 13.6 \times \frac{36\times5}{36}eV$

$\frac{12\times1836\times208}{12\times1836+208} \times 13.6 \times \frac{36\times5}{36} 4 \frac{\times1837\times E_0}{1836\times13.6\times3} = 1373E_0$

11. Solution: (a)

$$\mu_{H-atom} = \frac{m_p m_e}{m_r+m_e} = 9^{-31}$$

$$\mu_{H-atom} = 9.1044 \times 10^{-31}kg$$

$$\mu_{H-atom} = \frac{3.344 \times 10^{-27} \times 9.109 \times 10^{-31}}{(3.344 \times 10^{-27}) + (9.109 \times 10^{-31})}$$

$$E \propto \mu$$

$$E_{D-atom} = \left(\frac{\mu_{D-atom}}{\mu_{H-atom}}\right)E_{H-atom}$$

$$= \left(\frac{9.1065 \times 10^{-3}}{9.1044 \times 10^{-31}}\right) \times 13.598eV$$

$$E_{D-atom} = 13.6011eV$$

Hence option (a) is correct.

12. Solution: (b)

Since the half-filled configuration will be more stable hence option (b) is correct.

13. Solution: (a)

$$l_1 = 0, l_2 = 1, L = 1$$

$s_1 = \frac{1}{2}, s_2 = \frac{1}{2} \Rightarrow S = 1,0 \& S = 0, L = 1 \Rightarrow J = 1,$

$I = \frac{1}{2}$ $For\ He^3 \Rightarrow F = \frac{3}{2}, \frac{1}{2}$

$S = 1, L = 1, J = 2,1,0, S = 1, L = 1, J = 2, I = \frac{1}{2}, F = \frac{5}{2}, \frac{3}{2}$

$S = 1, L = 1, J = 1, I = \frac{1}{2}, F = \frac{3}{2}, \frac{1}{2}, S = 1, L = 1, J = 0, I = \frac{1}{2}, F = \frac{1}{2}$

To total seven F values are obtained for different states. So, number of hyperfine splitting is seven.

14. Solution: (b)

Energy of photon absorbed $E = \frac{1240}{400} = 3.1eV$

Energy of photon emitted $E_1 = \frac{1240}{500} = 2.48eV$

Energy of other photon emitted $E_2 = 3.1 - 2.48 = 0.62$

The corresponding wavelength $\lambda = \frac{1240}{0.62} = 2000nm$

15. Solution: (a)

$$A + \gamma = B$$

$h\nu$ =Energy of B −Energy of A

$= (M+\Delta)c^2 - Mc^2$

$= \Delta c^2$

16. Solution. (a)

The transition $3d_{3/2} \rightarrow 2P_{1/2}$ involves a change in both n and ℓ. In this case, the electron is transitioning from a 3d orbital with $\ell = 1$

The selection rules for electric dipole (E_1) transitions are $\Delta\ell = \pm1$,. and $\Delta m_1 = 0, \pm1$

For magnetic dipole (M_1) transition $\Delta\ell = 0 \pm 1$ $\Delta m_\ell = 0 \pm 1$

In this transition

$\Delta n = -1$ (Since the electron is morig from $n = 3$ to $n = 2$)