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CLASSICAL MECHANICS

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Chapter 1

Newtonian Mechanics

Introduction

Mechanics is a branch of physics which deals with physical objects in motion and at rest under the influence of external and internal interactions.

When we deal with Newton's laws and their consequences, it may be called Newtonian Mechanics or vectorial mechanics, because in this scheme, the quantities such as force, acceleration, momentum etc. are used which is essentially vector.

In this chapter we shall summarize basic concepts from introductory mechanics.

Newton's Law of Motion (NLM)

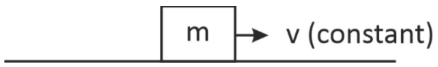
First law of motion:-

"A body moves with constant velocity (which may be zero) unless acted on by an external force." Newton's first law is often referred to as **principle of inertia**.

$\vec{F}_{net} = 0 \Rightarrow \vec{a} = 0$

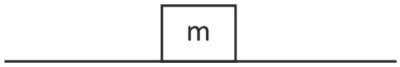
Or vice versa

$\vec{a} = 0 \Rightarrow \vec{F}_{net} = 0$



$\vec{a} = 0$

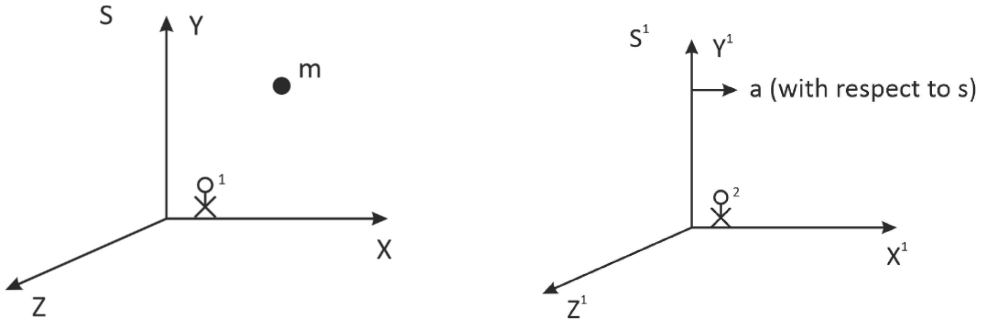
$\Rightarrow \vec{F}_{net} = 0$



$\vec{F}_{net} = 0$

$\Rightarrow \vec{a} = 0$

It fails when we use non inertial frame



Mass m is at rest with respect to S frame.

For Observer-1

Acceleration of the particle is zero and force acting on the particle is also zero

$a = 0$

$F = 0$

For Observer-2

Net force acting on the particle is zero but acceleration of particle is non zero

$F = 0$

$\vec{a} \neq 0$

So N.L.M is not valid in this case (because observer -2 is non-inertial)

If we want to apply N.L.M in non -inertial frame we have to consider an extra force known as **pseudo force**

Newton's Second Law

The second law states that the rate of change of momentum of a body over time is equal to force applied $F_{ext} = \frac{d\boldsymbol{P}}{dt}$, where P is the momentum, $\boldsymbol{P} = m\boldsymbol{v}$.

For objects and systems with constant mass, the second law can be re-stated in terms of an object acceleration.

$\vec{F} = m\vec{a}$ (acceleration measures from inertial frame)

$$\boldsymbol{F_{ext}} = \frac{d(m\boldsymbol{v})}{dt} = m\frac{d\boldsymbol{v}}{dt} = m\boldsymbol{a}$$

Newton's Third Law:-

For every action there is always equal and opposite reaction.

→ Action and Reaction always act on different bodies.

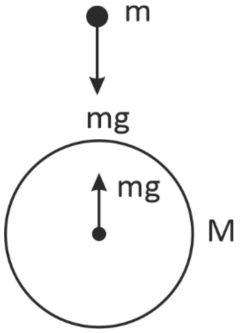
They act in pair.

Example 1: A body of mass m is falling from height h under gravity, discuss about action-reaction pair (neglect air resistance)

Explanation-

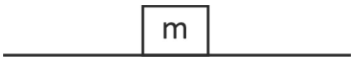
Since body is moving under gravity, action force is the gravitational force acing on body due to earth .So now where is reaction force?

Since earth is applying mg force on the body, body will also apply the same force (mg) on the earth in opposite direction.



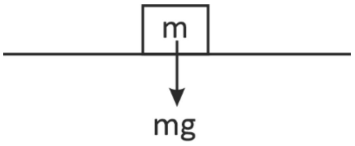
So now you can see both force are same in magnitude but opposite in direction. They are also acting on different bodies

Example 2: Draw action reaction pair



Basics about Some Forces

Weight:- Net gravitational force exerted by the earth on body. Direction of this force is always towards the centre of the earth.



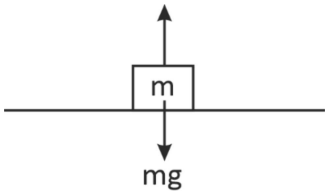
Normal Force:-

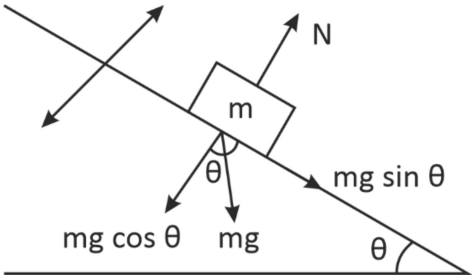
Normal force is a contact force. If two surface are not in contact, they can't exert a normal force on each other.

Normal force "constrained to move in any particular direction"

$N = mg$

Direction → always perpendicular to contact surface



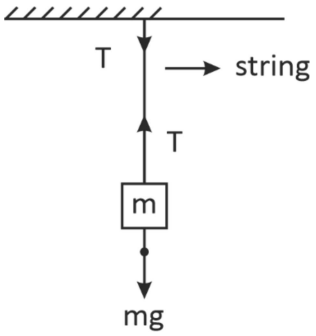


$N = mg\cos\theta$

Tension:-

The tension force is a force experienced by a rope, string or cable when it is pulled from the opposite end. It acts along the length of the wire and is equal and opposite to the applied force.

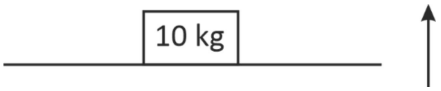
Direction:- Along the string



String → inextensible

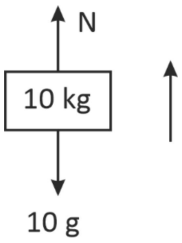
Example:

1. System is moving upward with acceleration $2\,m/sec^2$ what will be the normal force acting on the body due to earth ($g = 10m/sec^2$)



Given

$a = 2m/sec^2$



$\Rightarrow \vec{F}_{net} = m\vec{a}$

Since acceleration is upward, value of Normal force will be more than gravitational force

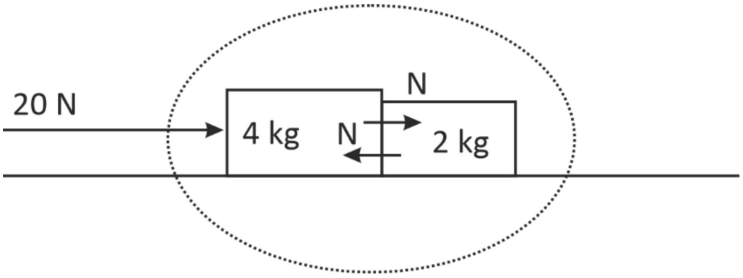
Upward force > downward force

$\Rightarrow N - 10g = 10 \times 2$

$N = 10g + 20$

$N = 120$ Newton

2. Find acceleration of each block and normal reaction acting between block



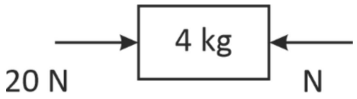
Solution:-

We can consider both the particle as a single system, the acceleration of both the particle will be same

$\vec{F}_{net} = m_T\vec{a}$

$20 = 6a$

$\Rightarrow \boldsymbol{a = \frac{10}{3} m/sec^2}$



$\vec{F}_{net} = m\vec{a}$

$20 - N = ma$

$20 - 4 \times \frac{10}{3} = N$

$\Rightarrow N = \frac{20}{3} Newton$

$\rightarrow a = \frac{10}{3} m/sec$

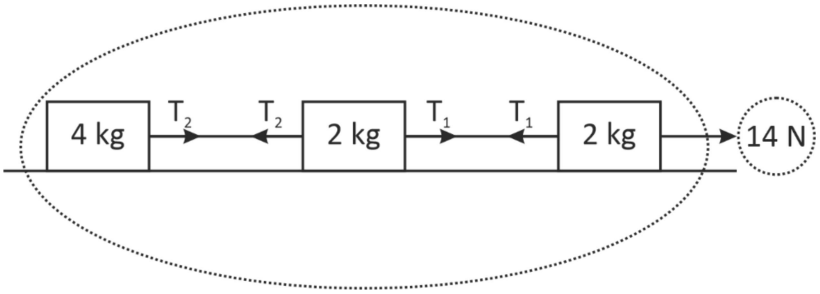
$\vec{N}$ (2kg)

$F_{net} = ma$

$N = 2 \times \frac{10}{3}$

$N = \frac{20}{3} newton$

3. Find acceleration of the system and tension in the string.



Solution: →

Since we have two strings, so there will be different tension force (in general) in both the string.

For acceleration of each block we can take all blocks as a system.

$F_{net} = m_T a$

$14 = 7a$

$\Rightarrow a = 2m/sec^2$

Now free body diagram of 4 kg block

$\rightarrow 2m/sec^2$

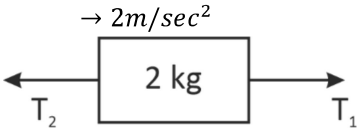


$\vec{F}_{net} = m\vec{a}$

$T_2 = 4 \times 2$

$\boldsymbol{T_2 = 8N}$

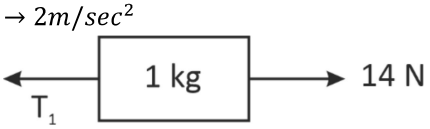
Free body diagram of 2 kg block



$T_1 - T_2 = 2 \times 2$

$T_1 - T_2 = 4$

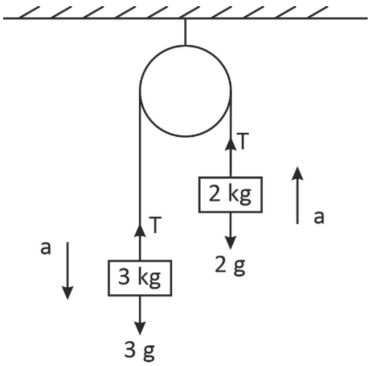
Free body diagram of 1 kg block



$14 - T_1 = 1 \times 2$

$\boldsymbol{T_1 = 12N}$

4. Atwood machine: Find acceleration of each block and tension in the string pully is massless & frictionless (consider $g = 10\ m/sec^2$) string is inextensible



Solution: we can observe the problem that acceleration of 3 kg block will be downward and acceleration of 2 kg block will be upward

Newton's equation for 3 kg block

$3g - T = 3a$ ---------- (1)

Newton's equation for 2 kg block

$T - 2g = 2a$ ---------- (2)

By adding both the equation,

$3g - 2g = 3a + 2a$

$g = 5a$

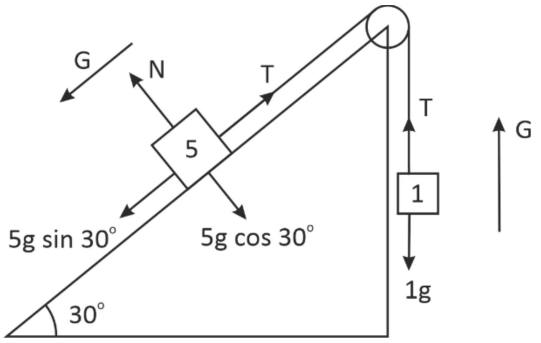
$\Rightarrow a = \frac{g}{5}$

$a = 2m/sec^2$

Put the value of a in equation 1

$T = 24N$

5. Find acceleration of the system (each block) and tension in the string.



Given, mass $m_1 = 5\ kg$, $m_2 = 1\ kg$ and $g = 10\ m/second^2$

Solution:

Since both the block is connected by a single string and string is inextensible, so acceleration of both the block will be same.

Newton's equation for 5 kg block

$5g \sin 30° - T = 5a$ ---- (1)

Newton's equation for 1 kg block

$T - 1g = 1a$ ---------- (2)

By adding both the subtracting both the equation, we will get

$a = \frac{g}{4}$, $T = \frac{5g}{4}$

Newton's Law for Variable Acceleration

When a particle is moving in constant acceleration, we can simply use equation of motion like $v = u + at$, $s = ut + \frac{1}{2}\ at^2\ and\ v^2 = u^2 + 2\ a\ s$

But when particle is moving in variable acceleration, we can't use these equation's.

To solve the problem of variable acceleration we have to use integration from initial to final.

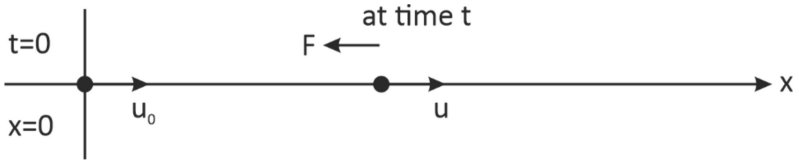
We face these type of problem when we deal with motion in viscous medium

$\vec{F} = ma = m\frac{dv}{dt} = mv\frac{dv}{dx}$

→ use integration

Example: A particle of mass m travelling along x-axis such that at $t = 0$ it is located at $x = 0$ and has speed v_0. The particle is called upon by a force which opposes the motion and proportional to square of instantaneous speed. Find acceleration of particle at any time t.

Solution: →



$F = -\beta v^2$ (since force is opposite to the direction of velocity

$ma = -\beta v^2$

Where β is constant

$a = -\frac{\beta v^2}{m}$

$\frac{dv}{dt} = -\frac{\beta v^2}{m}$

$\int_{v_0}^{v} \frac{dv}{v^2} = \int_0^t -\frac{\beta}{m} dt$

$\Rightarrow \left[-\frac{1}{v}\right]_{v_0}^{v} = -\frac{\beta}{m} t$

$\Rightarrow \frac{1}{v} - \frac{1}{v_0} = \frac{\beta t}{m}$

$\Rightarrow \frac{1}{v} = \frac{1}{v_0} + \frac{\beta t}{m}$

$\Rightarrow v = \frac{m v_0}{m + \beta v_0 t}$

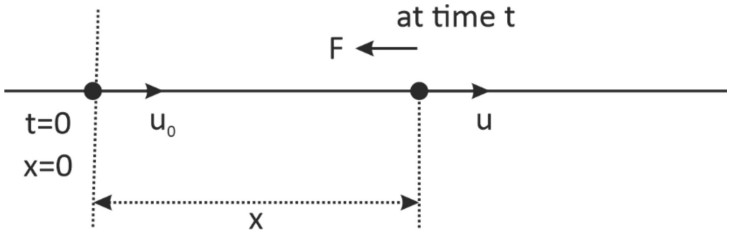
$\Rightarrow \boldsymbol{a = \frac{dv}{dt} = \frac{-m\beta v_0^2}{(m+\beta v_0 t)^2}}$

Example: A particle of mass m thrown horizontally with initial velocity v_0. A resistive force $(-kv)$ acts on the particle.

i) Distance travelled by the particle in time t

ii) Draw $v - t$ & $x - t$ graph.

Solution:



$F = -kv$

$m\frac{dv}{dt} = -kv$

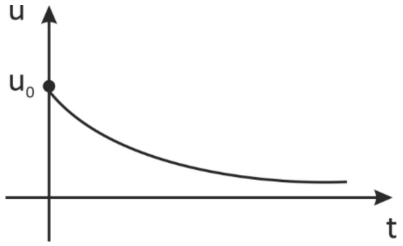
$\int_{v_0}^{v} \frac{dv}{v} = \int_0^t -\frac{k}{m} dt$

$\Rightarrow [\ln v - \ln v_0] = -\frac{k}{m} t$

$\ln \frac{v}{v_0} = -\frac{kt}{m}$

$\Rightarrow \frac{v}{v_0} = e^{-\frac{kt}{m}}$

$\boldsymbol{v = v_0 e^{-\frac{kt}{m}}}$



$t = 0, v = v_0$

$t \to \infty, v \to 0$

Now,

$\frac{dx}{dt} = v_0 e^{-\frac{kt}{m}}$

$\Rightarrow \int_0^x dx = \int_0^t v_0 e^{-\frac{kt}{m}} dt$

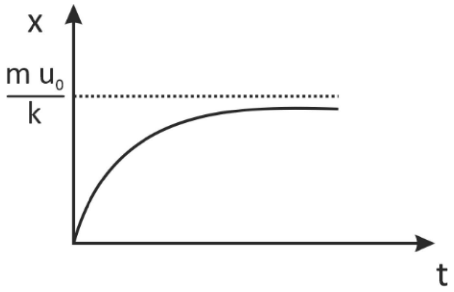
$x = -\frac{m v_0}{k}\left[e^{-\frac{kt}{m}}\right]_0^t$

$x(t) = -\frac{mv_0}{k}\left[e^{-\frac{kt}{m}} - 1\right]$

$x(t) = \frac{mv_0}{k}\left(1 - e^{-\frac{kt}{m}}\right)$

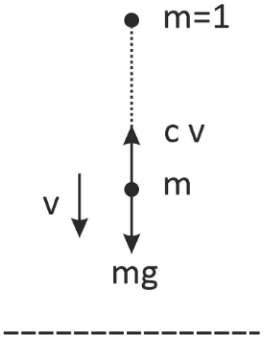
$t = 0, x = 0$

$t \to \infty, x = \frac{mv_0}{k}$



Example: Consider a body of unit mass falling from rest under gravity. Air resistance retards the acceleration by cv, where c is constant then $v(t) =$?

Solution: →



At $t = 0, v = 0$

Net force acting on the particle,

$F = (mg - cv)$

$m\frac{dv}{dt} = mg - cv$

$\Rightarrow \frac{dv}{dt} = g - cv$

$\Rightarrow \int_0^v \frac{dv}{g-cv} = \int_0^t dt$

$\Rightarrow \left[-\frac{1}{c}\ln(g - cv)\right]_0^v = t$

$\Rightarrow [\ln(g - cv) - \ln g] = -ct$

$\Rightarrow \ln\left(\frac{g-cv}{g}\right) = -ct$

$\frac{g-cv}{g} = e^{-ct}$

$\Rightarrow 1 - \frac{cv}{g} = e^{-ct}$

$\Rightarrow \frac{cv}{g} = (1 - e^{-ct})$

$\Rightarrow \boldsymbol{v = \frac{g}{c}(1 - e^{-ct})}$

Work Power Energy

Kinetic Energy



Suppose a particle of mass m is moving with velocity v, kinetic energy of particle is

$K = \frac{1}{2}mv^2$

For system particle, we can directly add kinetic energy of all particles because it is a scalar quantity.

$K = \sum_i ki = \sum_i \frac{1}{2} m_i v_i^2$



We can change the kinetic energy of particle by applying some force on the particle.

(i) If force on the particle is parallel to velocity, kinetic energy of the particle increases.

(ii) If force on the particle antiparallel to velocity, kinetic energy of the particle decreases.

(iii) If force acting on the particle is zero or force is always perpendicular to the velocity, then kinetic energy remains same (in this case $\vec{F}$ will only change the direction of velocity)

Kinetic energy of a particle of mass m moving with velocity v

$K = \frac{1}{2}mv^2$

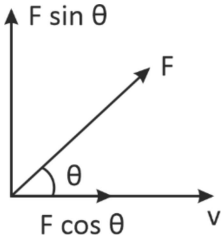
$\frac{dK}{dt} = \frac{d}{dt}\left[\frac{1}{2}mv^2\right]$

$\frac{dK}{dt} = \frac{1}{2}m.2v\frac{dv}{dt}$

$\frac{dK}{dt} = \left(m\frac{dv}{dt}\right)v$

$\frac{dK}{dt} = F_{\parallel}\vec{v}$

$\Rightarrow \frac{dK}{dt} = (F\cos\theta)v$



$\frac{dK}{dt} = \vec{F}.\vec{v}$

$\frac{dK}{dt} = \vec{F}.\frac{d\vec{r}}{dt}$

$\Rightarrow \boldsymbol{dK = \vec{F}.d\vec{r}}$

$\int_{K_i}^{K_f} dK = \int \vec{F}.d\vec{r}$

$\Rightarrow \boxed{\boldsymbol{K_f - K_i = \int \vec{F}.d\vec{r} = W_{all}}}$

It is known as Work energy theorem, it is valid for all type of forces