P R E F A C E

The principal purpose of writing this book "**GATE Mathematics Practice Book**" is to cater to the needs of the candidates aspiring for "**GATE Mathematics**" exam. To prepare for such competitive entrance exam, it is very important to solve quality practice questions and this book serves this purpose. **Book includes 10 chapters** according to the exam syllabus. **Each chapter contains 60 practice questions, of which 40 are MCQ, 10 are MSQ and 10 are NAT**. To help students, we have compiled this book in a user-friendly and well-structured manner.

Regular practice is the key to success in any competitive exam, and this book allows you to hone your problem-solving skills. We have tried to keep the material as simple as possible with easy explanations. The practice of these questions will help the students not only for GATE exam but also for other competitive exams like IIT-JAM, TIFR and other PhD entrances.

A word or suggestion from your side may add another feather to the cap in the compilation of questions in this book. The author looks forward to the comments, suggestions, and criticism from the students. Constructive suggestions and user feedback would be highly appreciated, gratefully acknowledged, and suitably incorporated. We wish all the best to all aspirants who are preparing for these exams.

A C K N O W L E D G E M E N T

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EXAM DETAILS

Mode of Examination	Computer Based Test
Duration of Exam	180 minutes
Total number of Questions	65
Total Marks	100
Language of Examination	English

EXAM PATTERN

Sections	Section I (General Aptitude)	Section II (Mathematics)
Number of Questions	10	55
Marks	5 questions of 1 mark each and 5 questions of 2 marks each	25 questions of 1 mark each and 30 questions of 2 marks each
Negative Marking	For 1-mark MCQ, 1/3 mark will be deducted for a wrong answer. For 2-mark MCQ, 2/3 mark will be deducted for a wrong answer. There will be no negative marking for MSQs & NATs.	
Types of Question	MCQ	MCQ, MSQ, NAT

GATE MATHEMATICS CUT - OFF

Year	General	EWS/OBS-NCL	SC / ST / PwD	Organizing Institute
2024	25	22.5	16.6	IISc Bengaluru
2023	25	22.5	16.6	IIT Kanpur
2022	27.3	24.5	18.2	IIT Kharagpur
2021	29	26.1	19.3	IIT Bombay
2020	25	22.5	16.6	IIT Delhi
2019	25	22.5	16.7	IIT Madras
2018	25	22.5	16.6	IIT Guwahati
2017	25	22.5	16.6	IIT Roorkee
2016	25	22	16	IISc Bengaluru

Syllabus

Calculus: Functions of two or more variables, continuity, directional derivatives, partial derivatives, total derivative, maxima and minima, saddle point, method of Lagrange's multipliers; Double and Triple integrals and their applications to area, volume and surface area; Vector Calculus: gradient, divergence and curl, Line integrals and Surface integrals, Green's theorem, Stokes' theorem, and Gauss divergence theorem.

Linear Algebra: Finite dimensional vector spaces over real or complex fields; Linear transformations and their matrix representations, rank and nullity; systems of linear equations, characteristic polynomial, eigenvalues and eigenvectors, diagonalization, minimal polynomial, Cayley-Hamilton Theorem, Finite dimensional inner product spaces, Gram-Schmidt orthonormalization process, symmetric, skew-symmetric, Hermitian, skew-Hermitian, normal, orthogonal and unitary matrices; diagonalization by a unitary matrix, Jordan canonical form; bilinear and quadratic forms.

Real Analysis: Metric spaces, connectedness, compactness, completeness; Sequences and series of functions, uniform convergence, Ascoli-Arzela theorem; Weierstrass approximation theorem; contraction mapping principle, Power series; Differentiation of functions of several variables, Inverse and Implicit function theorems; Lebesgue measure on the real line, measurable functions; Lebesgue integral, Fatou's lemma, monotone convergence theorem, dominated convergence theorem.

Complex Analysis: Functions of a complex variable: continuity, differentiability, analytic functions, harmonic functions; Complex integration: Cauchy's integral theorem and formula; Liouville's theorem, maximum modulus principle, Morera's theorem; zeros and singularities; Power series, radius of convergence, Taylor's series and Laurent's series; Residue theorem and applications for evaluating real integrals; Rouche's theorem, Argument principle, Schwarz lemma; Conformal mappings, Mobius transformations.

Ordinary Differential Equations: First order ordinary differential equations, existence and uniqueness theorems for initial value problems, linear ordinary differential equations of higher order with constant coefficients; Second order linear ordinary differential equations with variable coefficients; Cauchy-Euler equation, method of Laplace transforms for solving ordinary differential equations, series solutions (power series, Frobenius method); Legendre and Bessel functions and their orthogonal properties; Systems of linear first order ordinary differential equations, Sturm's oscillation and separation theorems, Sturm-Liouville eigenvalue problems, Planar autonomous systems of ordinary differential equations: Stability of stationary points for linear systems with constant coefficients, Linearized stability, Lyapunov functions.

Algebra: Groups, subgroups, normal subgroups, quotient groups, homomorphisms, automorphisms; cyclic groups, permutation groups, Group action, Sylow's theorems and their applications; Rings, ideals, prime and maximal ideals, quotient rings, unique factorization domains, Principle ideal domains, Euclidean domains, polynomial rings, Eisenstein's irreducibility criterion; Fields, finite fields, field extensions, algebraic extensions, algebraically closed fields

Functional Analysis: Normed linear spaces, Banach spaces, Hahn-Banach theorem, open mapping and closed graph theorems, principle of uniform boundedness; Inner-product spaces, Hilbert spaces, orthonormal bases, projection theorem, Riesz representation theorem, spectral theorem for compact self-adjoint operators.

Numerical Analysis: Systems of linear equations: Direct methods (Gaussian elimination, LU decomposition, Cholesky factorization), Iterative methods (Gauss-Seidel and Jacobi) and their convergence for diagonally dominant coefficient matrices; Numerical solutions of nonlinear equations: bisection method, secant method, Newton-Raphson method, fixed point iteration; Interpolation: Lagrange and Newton forms of interpolating polynomial, Error in polynomial interpolation of a function; Numerical differentiation and error, Numerical integration: Trapezoidal and Simpson rules, Newton-Cotes integration formulas, composite rules, mathematical errors involved in numerical integration formulae; Numerical solution of initial value problems for ordinary differential equations: Methods of Euler, Runge-Kutta method of order 2.

Partial Differential Equations: Method of characteristics for first order linear and quasilinear partial differential equations; Second order partial differential equations in two independent variables: classification and canonical forms, method of separation of variables for Laplace equation in Cartesian and polar coordinates, heat and wave equations in one space variable; Wave equation: Cauchy problem and d'Alembert formula, domains of dependence and influence, nonhomogeneous wave equation; Heat equation: Cauchy problem; Laplace and Fourier transform methods.

Topology: Basic concepts of topology, bases, subbases, subspace topology, order topology, product topology, quotient topology, metric topology, connectedness, compactness, countability and separation axioms, Urysohn's Lemma.

Linear Programming: Linear programming models, convex sets, extreme points; Basic feasible solution, graphical method, simplex method, two phase methods, revised simplex method ; Infeasible and unbounded linear programming models, alternate optima; Duality theory, weak duality and strong duality; Balanced and unbalanced transportation problems, Initial basic feasible solution of balanced transportation problems (least cost method, north-west corner rule, Vogel's approximation method); Optimal solution, modified distribution method; Solving assignment problems, Hungarian method.

I N D E X

GATE MATHEMATICS

CHAPTER NO.	CHAPTER NAME	PAGE NO.
1	*CALCULUS*	*1*
2	*LINEAR ALGEBRA*	*18*
3	*REAL ANALYSIS*	*34*
4	*COMPLEX ANALYSIS*	*48*
5	*ORDINARY DIFFERENTIAL EQUATIONS*	*63*
6	*MODERN ALGEBRA*	*85*
7	*NUMERICAL ANALYSIS*	*99*
8	*PARTIAL DIFFERENTIAL EQUATIONS*	*118*
9	*TOPOLOGY*	*139*
10	*LINEAR PROGRAMMING PROBLEMS*	*157*

CHAPTER 1

CALCULUS

PRACTICE QUESTIONS

MCQ (Multiple Choice Questions)

1. Let $f:\mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x,y)=\begin{cases}\frac{x^2y}{x^4+y^2}, & (x,y)\neq 0\\ 0, & (x,y)=0\end{cases}$$

Then, the directional derivative of f at $(0,0)$ in the direction of the vector $\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ is

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$

(c) $\frac{1}{2\sqrt{2}}$ (d) $\frac{1}{4\sqrt{2}}$

2. Define $f:\mathbb{R}^2 \to \mathbb{R}$ by

$$f(x,y)=\begin{cases}1, & \text{if } xy=0\\ 2, & \text{otherwise}\end{cases}$$

If $S=\{(x,y): f \text{ is continuos at the point } (xy)\}$, then

(a) S is open (b) S is connected

(c) $S=\emptyset$ (c) S is closed

3. Consider the hemisphere $x^2+y^2+(z-2)^2=9, 2\leq x\leq 5$ and the vector field $\vec{F}(x,y,z)=x\,\hat{\imath}+y\,\hat{\jmath}+(z-2)\hat{k}$. The surface integral $\int\int(\vec{F}.\vec{n})\,d\sigma$, evaluated over the hemisphere with $\vec{n}$ denoting the unit outward normal is

(a) 9π (b) 27π

(c) 54π (c) 162π

4. Let $f,g:R^2 \to R$ be defined by $f(x,y)=x^2-2xy^2$ and $g(x,y)=4x^4-5x^2y+y^2$ for all $(x,y)\in R^2$

Consider the following statements:

P: f has a saddle point at (0,0)

Q: g has a saddle point at (0,0). Then

(a) both P and Q are True

(b) P is False but Q is True

(c) P is True but Q is False

(d) both P and Q are False

5. Let $f:R^2 \to R$ be given by $f(x,y)=4xy-2x^2-y^4$. Then f has

(a) a point of local maximum and a saddle point

(b) a point of local minimum and a saddle point

(c) a point of local maximum and a point of local minimum

(d) two saddle points

6. Let $S=\{(x,y,z)\in\mathbb{R}^3: -1<x<1,-1<y<1,-1<z<1$ and $\phi:\Gamma\longrightarrow\mathbb{R}$ be a function whose all second order partial derivatives exists and are continuous. If ϕ satisfies the Laplace equation $\nabla^2\phi=0$ for all $(x,y,z)\in\gamma$, then which one of the following statements is true Γ?

($\mathbb{R}$ is the set of all real numbers, and $\mathbb{R}^3=\{(x,y,z): x,y,z\in\mathbb{R}\}$

(a) $\vec{\nabla}\phi$ is solenoidal but not irrotational

(b) $\vec{\nabla}\phi$ is irrotational but not solenoidal.

(c) $\vec{\nabla}\phi$ is both solenoidal and irrotational

(d) $\vec{\nabla}\phi$ is neither solenoidal nor irrotational

7. The work done by the force $F=(x+y)\hat{\imath}-(x^2+y^2)\hat{\jmath}$, where i and j are unit vectors in $\overrightarrow{OX}$ and $\overrightarrow{OY}$ directions, respectively, along the upper half of the circle $x^2+y^2=1$ from $(1,0)$ to $(-1,0)$ in the xy-plane is

(a) $-\pi$ (b) $\frac{-\pi}{2}$

(c) $\frac{\pi}{2}$ (d) π

8. Let $F:\mathbb{R}^2\longrightarrow\mathbb{R}^2$ be defined by $F(x,y)=(e^{x+y},e^{x-y})$. The area of the image of the region $\{(x,y)\in\mathbb{R}^2: 0<x,y<1\}$ under the mapping F is

(a) 1 (b) $e-1$

(c) e^2 (d) e^2-1

9. For a differential surjective function $f:(0,1)\longrightarrow(0,1)$, consider the function $F:(0,1)\times(0,1)\longrightarrow(0,1)\times(0,1)$ given by $F(x,y)=\big(f(x),f(y)\big), x,y\in(0,1)$. If $f'(x)\neq 0$ for every $x\in(0,1)$, then which of the following statements are false?

(a) f is injective
(b) f is increasing
(c) For every $(x', y') \in (0,1) \times (0.1)$, there exist a unique $(x, y) \in (0,1) \times (0,1)$ such that $F(x, y) = (x', y')$.
(d) The total derivative $DF(x, y)$ is invertible for all $(x, y) \in (0,1) \times (0,1)$

10. The flux of the vector field $\overrightarrow{u} = x\hat{\imath} + y\hat{\jmath} + 2\hat{k}$ flowing out through the surface of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, a > b > c > 0$ is
(a) πabc (b) $2\pi abc$
(c) $3\pi abc$ (d) $4\pi abc$

11. Let L denote the value of the line integral $\oint (3x - 4x^2y)dx + (4xy^2 + 2y)dy$, where C, a circle of radius 2 with centre at origin of the xy plane, is traversed once in anti-clockwise direction. The $\frac{L}{\pi}$ is equal to
(a) 12 (b) 16
(c) 24 (d) 32

12. Let $S_n = \sum_{k-1}^{n} \frac{1}{k}$ and $I_n = \int_1^n \frac{x-[x]}{x^2} dx$, then $S_{10} + I_{10}$ is equal to
(a) $\ln 10 + 1$ (b) $\ln 10 - 1$
(c) $\ln 10 - \frac{1}{10}$ (d) $\ln 10 + \frac{1}{10}$

13. The line integral of the vector function $u(x, y) = 2y\hat{\imath} + x\hat{\imath}$ along the straight line from (0, 0) to (2, 4) is
(a) 10 (b) 11
(c) 12 (d) 0

14. The divergence of the vector field $\vec{u} = e^x(\cos x\, \hat{\imath} + \sin x\, \hat{\jmath})$ is
(a) 0 (b) $e^x \cos y + e^x \sin y$
(c) $2e^x \cos y$ (d) $2e^x \sin y$

15. Find the value of surface integral $\int_S \vec{F}.ds$ where $\vec{F}$ is given by $(x\hat{\imath} + y\hat{\jmath} + z\hat{k})$ and S is the surface of solid bounded by $x^2 + y^2 = a^2$ and $z = 1,\ z = -1$ planes
(a) $3\pi a^2$ (b) $6\pi a^2$
(c) $9\pi a^2$ (d) $12\pi a^2$

16. Let $F = xi + yj + zk$ then find the integral of F around the simple closed curve C over the boundary of surface?
(a) 2 (b) 0
(c) 3 (d) 1

17. Let $f: \mathbb{R} \longrightarrow \mathbb{R}$ be a continuous function with $f(1) = 5$ and $f(3) = 11$. If $g(x) = \int_1^3 f(x + t)dt$, then $g'(0)$ is equal to
(a) 3 (b) 6
(c) -1 (d) 2

18. Find $\lim_{(x,y)\to(0,0)} f(x, y)$ where $f(x, y) = \begin{cases} x \sin \frac{1}{y} \\ 0 \end{cases}$
(a) 0 (b) 2
(c) 4 (d) Does not exist

19. If C is the circle $x^2 + y^2 = 1$ taken in anti-clockwise direction then
$\int_C [(x^{2015}y^{2016} + 2014\, y)dy + (x^{2016}y^{2015} + 2017\, x)dy]$
will be
(a) 0 (b) π
(c) $\dot{2}\pi$ (d) 3π

20. Which of the following is correct?
(a) Green's theorem is a particular case of Stoke's theorem
(b) Stoke's theorem is a particular case of Green's theorem
(c) Both Stoke's theorem and Green's theorem are same
(d) None of the above

21. Evaluate $\oint_C\ y^3dx - x^3dy$ where C positively oriented circle of radius 2 is centered at origin
(a) 24π (b) 4π
(c) -24π (d) -4π

22. Integral $\int_0^{\pi} 2\frac{1}{1+\tan x} dx$ is equal to
(a) 0.68 (b) 0.78
(c) 0.15 (d) 1.78

23. If $E = \{(x, y) \in R^2: 0 \le x \le 1, 0 \le\ y\ \le x\}$, then the value of $\iint_B (x\ +\ y)dx\, dy$ is equal to
(a) 0.50 (b) 0.30
(c) 1.5 (d) None

24. The area of the region bounded by the curves $x\ =\ y^2$ and $y\ =\ x^2$ is
(a) $\frac{1}{2}$ (b) $\frac{1}{4}$
(c) $\frac{1}{3}$ (d) $\frac{-1}{3}$

25. The value of the triple integral $\int_{x=0}^{1}\int_{y=0}^{x^2}\int_{z=0}^{y}(y+2z)\,dxdydz$ is
(a) $\frac{1}{22}$ (b) $\frac{2}{21}$
(c) $\frac{1}{21}$ (d) $\frac{3}{21}$

26. The value of the double integral $\iint \frac{xy}{\sqrt{1-y^2}}dx\,dy$ over the first quadrant of the circle $x^2+y^2=1$ is
(a) $\frac{1}{6}$ (b) $\frac{1}{3}$
(c) $\frac{1}{2}$ (d) 1

27. The value of the triple integral $\int_{x=0}^{1}\int_{y=0}^{1}\int_{z=0}^{1}(x^2+y^2+z^2)\,dzdydx$ is
(a) 0 (b) 1
(c) -1 (d) 2

28. Let $\boldsymbol{f}:\mathbb{R}^{\mathbf{2}}\to\mathbb{R}$ be defined by
$$f(x,y)=\begin{cases}\frac{y}{\sin y}, & y\neq 0\\ 1, & y=0\end{cases}$$
Then the integral in $\frac{1}{\pi^2}\int_{x=0}^{1}\int_{y=\sin^{-1}x}^{\frac{\pi}{2}}f(x,y)dydx$ correct upto three decimal places, is
(a) $\frac{1}{2}$ (b) $\frac{1}{8}$
(c) $\frac{1}{7}$ (d) $\frac{1}{6}$

29. The volume of the portion of the solid cylinder $x^2+y^2\leq 2$ bounded above by the surface $z=x^2+y^2$ and bounded below by the xy-plane is
(a) $\frac{1}{2}$ (b) $\frac{1}{3}$
(c) -1 (d) 1

30. If the volume of the solid in $\mathbb{R}^3$ bounded by the surfaces $x=-1,x=1,y=-1,y=1,z=2,y^2+z^2=2$ is $\alpha-\pi$, then $\alpha=$
(a) 3 (b) 1
(c) 2 (d) 6

31. The value of the integral $\int_0^1\int_x^1 y^4e^{xy^2}\,dydx$ is
(a) $\frac{e-2}{3}$ (b) $\frac{e-1}{2}$
(c) $\frac{e+2}{3}$ (d) $\frac{e+1}{3}$

32. Let p and t be positive real numbers. Let D_t be the closed disc of radius t centered at (0,0), i.e., $D_1=\{(x,y)\in\mathbb{R}^2: x^2+y^2\leq t^2\}$. Define $I(p,t)=\iint_{D_t}\frac{dxdy}{(p^2+x^2+y^2)^p}$. Then $\lim_{t\to\infty}I(p,t)$ is finite
(a) only if $p<1$. (b) only if $p>1$.
(c) for no value of p. (d) only if $p=1$.

33. For $(x,y)\in\mathbb{R}^2$ with $(x,y)\neq(0,0)$. Let $\theta=\theta(x,y)=(r\cos\theta, r\sin\theta)$, where $r=\sqrt{x^2+y^2}$. Then the resulting function $\theta:\mathbb{R}^2\backslash\{(0,0)\}\to\mathbb{R}$ is
(a) Differentiable
(b) Continuous but not differentiable
(c) Bounded, but not continuous
(d) Neither bounded, nor continuous

34. Let $f(x,y)=\log(cos^2(e^{x^2}))+\sin(x+y)$. Then $\frac{\partial}{\partial y}\frac{\partial}{\partial x}f(x,y)$ is
(a) $\frac{\cos(e^{x^2})}{1+sin^2(e^{x^2})}-\cos(x+y)$ (b) 0
(c) $-\sin(x+y)$ (d) $\cos(x+y)$

35. The value of $\int_{-1}^{1}xe^{|x|}dx$
(a) -1 (b) 0
(c) 2 (d) 1

36. Let $f:\mathbb{R}^m\to\mathbb{R}^m$ be a differential function. Let $Df(x)$ be the derivative of f at $x\in\mathbb{R}^m$. Which of the following is /are correct?
(a) $Df(0)(u)=0$ all u in $\mathbb{R}^m$
(b) $Df(x)(u)=0$ for all u in $\mathbb{R}^m$ and some $x\in\mathbb{R}^m$ only if f is a constant
(c) $Df(x)(u)=0$ for all $u\in\mathbb{R}^m$ and all $x\in\mathbb{R}^m$ only if f is a constant
(d) If f is not a constant function, then $Df(x)$ in a one-to-one function for some $x\in\mathbb{R}^m$

37. $G=\left\{(x,y)\in\mathbb{R}^2: x^2+y^2<1, \frac{x}{\sqrt{3}}<y<\sqrt{3}x, x>0, y>0\right\}$ Then the value of $\frac{200}{\pi}\iint_G x^2\,dx\,dy$ is equal to
(a) 4.16 (b) 3.57
(c) 2 (d) 5.32

38. The rate at which the radius of a sphere changes over time is $\frac{1}{2\pi}$. When the radius is 5 cm, then the rate of change of the lightning surface area of the sphere will be
(a) 15 (b) 24
(c) 12 (d) 20

39. Suppose that $f:\mathbb{R}^n \to \mathbb{R}$ is given by $f(\underline{x}) = a_1x_1^2 + a_2x_2^2 + .. + a_nx_n^2$, where $\underline{x} = (x_1, x_2, \dots, x_n)$ and at least one a_j, is not zero. Then we can conclude that
(a) f is not everywhere differentiable
(b) The gradient $(\nabla f)(\underline{x}) \neq 0$ for every $\underline{x} \in \mathbb{R}^n$
(c) If $\underline{x} \in \mathbb{R}^n$ is such that $(\nabla f)(x) = 0$ then $f(x) = 0$
(d) If $\underline{x} \in \mathbb{R}^n$ is such that $f(x) = 0$ then $(\nabla f)(x) = 0$

40. Let $f:\mathbb{R}^2 \to \mathbb{R}$ be defined as $f(x,y) = x|x| + |y|$. Then which of the following are true?
(a) f is differentiable at (0,0)
(b) f is not differentialble at (0,0) but all its directinal derivatives at (0,0) exist
(c) $\frac{\partial f}{\partial x}(0,0)$ exists and equals 0
(d) $\frac{\partial f}{\partial y}(0,0)$ exists and equals 0

Answer Key									
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
(a)	(a)	(c)	(a)	(a)	(c)	(b)	(d)	(b)	(d)
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
(d)	(a)	(c)	(c)	(b)	(b)	(b)	(a)	(d)	(a)
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
(c)	(b	(a)	(c)	(b)	(a)	(b)	(b)	(a)	(d)
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
(a)	(b	(c)	(c)	(b)	(c)	(a)	(d)	(c)	(c)

MSQ (Multiple Select Questions)

1. Consider the map $f:\mathbb{R}^2 \to \mathbb{R}^2$ defined by $f(x,y) = (3x - 2y + x^2,\ 4x + 5y + y^2)$.Then
(a) f is discontinuous at (0,0)
(b) f is continuous at (0, 0) and all directional derivatives exists at (0, 0)
(c) f is differentiable at (0, 0) but the derivatives $Df(0,0)$ is not invertible
(d) f is differentiable at (0, 0) and the derivative $Df(0,0)$ is invertible

2. Let $f:\mathbb{R}^2 \to \mathbb{R}^2$ be given by the formula $f(x,y) = (3x + 2y + y^2 + |xy|, 2x + 3y + x^2 + |xy|)$ Then,
(a) f is discontinuous at (0, 0)
(b) f is continuous at (0, 0) but not differentiable at $(0,0)$
(c) f is differentiable at (0, 0)
(d) f is differentiable at (0, 0) and the derivative $Df(0,0)$ is invertible

3. Suppose that $f:\mathbb{R}^n \to \mathbb{R}, n \geq 2$ *is a* C^2 function satisfying
$$f(y) \geq f(x) + \nabla(f)(x)\,(y - x)$$
for every x, y *in* $\mathbb{R}^n$. Here ∇ denotes the gradient. Which of the following statements are true?
(a) f is constant.
(b) f is convex.
(c) f is convex and bounded.
(d) f is constant if ƒ is bounded.

4. The volume generated when the region surrounded by $y = \sqrt{x}$ and $y = 0$ is revolved about $y -$ axis then required volume is
(a) 20.10 (b) 12.10
(c) 14.15 (d) 18.20

5. Define
$$f(x,y) = \begin{cases} \frac{x^3}{x^2+y^2} for\ (x,y) \neq (0,0) \\ 0 \quad for\ (x,y) = (0,0) \end{cases}$$
Which of the following statements are true?
(a) f is discontinuous at (0,0)
(b) f is continuous at (0,0)
(c) all directional derivatives of f at (0,0) exist
(d) f is not differentiable at (0,0)

6. Let $f(x,y) = \frac{1-\cos(x+y)}{x^2+y^2}$ if $(x,y) \neq (0,0)$,
$f(0,0) = \frac{1}{2}$ and
$g(x,y) = \frac{1-\cos(x+y)}{(x+y)^2}$ *if* $x + y \neq 0$ *and*
$g(x,y) = \frac{1}{2}$ *if* $x + y = 0$ then

(a) f is continuous at (0, 0)
(b) f is continuous everywhere except at (0, 0)
(c) g is continuous at (0, 0)
(d) g is continuous everywhere

7. Let $f:\mathbb{R}^2 \to \mathbb{R}$ is defined as $f(x,y) = \begin{cases} xy\ \frac{x^2-y^2}{x^2+y^2} & if\ x^2 + y^2 \neq 0 \\ 0 & if\ x = y = 0 \end{cases}$. Then which of the following is/are true?
(a) $f_x(0,0) = 0 = f_y(0,0)$
(b) f_x is continuous at (0,0).
(c) f is differentiable at (0,0).
(d) None of the above