FOR 2027 EXAM

IIT JAM

(JOINT ADMISSION TEST FOR MASTERS)

PHYSICS

22 YEARS - (2005 TO 2026)

TOPIC WISE SORTED PREVIOUS SOLVED PAPERS

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Preface

The IIT JAM Physics examination is one of the most competitive gateways to postgraduate study at India's finest institutes and there is no better preparation than mastering the questions that have shaped it over the years.

Solving previous years' question papers is one of the most effective ways to prepare for the IIT JAM Physics examination. It helps aspirants understand the exam pattern, recognize important topics, and improve their problem-solving skills with confidence.

Salient Features

- Fully aligned with the latest IIT JAM Physics syllabus and examination pattern
- 22 years (2005–2026) of previous solved papers
- Topic-wise classification for structured, focused preparation
- Detailed, concept-oriented solutions for deeper understanding
- 5 full-length online mock tests with convenient QR code access

We encourage every aspirant to attempt each question independently before turning to the solutions, and to make full use of the online mock tests to build genuine exam readiness.

Practice with purpose, understand deeply, and evaluate often that is the path to success.

We wish every aspirant the very best for IIT JAM 2027.

ANALYSIS OF CUT OFF FOR IIT JAM PHYSICS EXAMINATIONS					
Year	**Candidates Appeared**	**General**	**EWS/OBC-NCL**	**SC/ST/PwD**	**Organizing Institute**
2026	9123	12.04	10.83	6.02	IIT BOMBAY
2025	10877	14.66	13.19	7.33	IIT DELHI
2024	13157	19.63	17.66	9.81	IIT MADRAS
2023	12350	21.29	19.16	10.64	IIT GUWAHATI
2022	12740	26.05	23.45	13.03	IIT ROORKEE
2021	12522	19.91	17.92	9.96	IISc Bangalore
2020	13000	21.7	19.53	10.85	IIT KANPUR
2019	15501	24.99	22.49	12.49	IIT KHARAGPUR
2018	13759	17.05	15.35	8.53	IIT BOMBAY
2017	11902	14.79	13.31	7.39	IIT DELHI
2016	10989	18.74	16.87	9.37	IIT MADRAS

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NUMBER OF QUESTION ASKED IN IIT JAM - PHYSICS (2005 to 2026)

Mathematical Methods in Physics		2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024	2025	2026
1.1	Vector Algebra and Vector Calculus	-	-	-	1	-	1	1	1	-	3	2	3	2	2	5	2	1	3	5	5	5	1
1.2	Matrices	1	1	1	1	-	-	1	-	1	-	-	1	1	1	-	1	-	1	2	1	1	1
1.3	Eigenvalues and Eigenvectors	-	-	-	-	-	1	-	-	-	-	1	1	-	-	1	-	1	-	-	-	-	1
1.4	Linear Ordinary Differential Equations of First & Second Order	-	-	-	-	-	-	-	-	2	-	1	-	2	1	-	1	1	1	-	-	-	1
1.5	Fourier series	1	-	1	-	1	-	-	-	-	-	-	-	3	1	-	-	1	-	1	-	-	1
1.6	Complex Analysis	-	-	-	-	-	-	-	-	1	1	1	1	-	-	1	-	1	1	1	1	1	1
1.7	Taylor & Laurent series	-	-	-	-	-	-	-	-	-	-	-	-	-	1	-	-	1	1	-	1	-	1

Mathematical Methods in Physics

CHAPTER 1

Unit 1.1: Vector Algebra and Vector Calculus

[IIT JAM 2008]

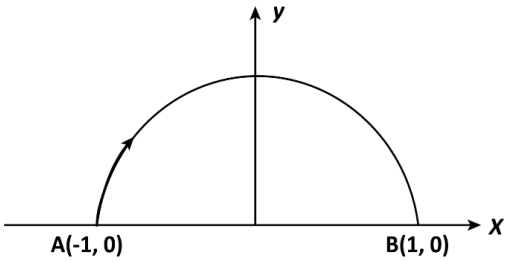
1. The work done by a force in moving a particle of mass m from any point (x, y) to a neighbouring point $(x+dx, y+dy)$is given by $= 2xydx + x^2dy$. The work done for a complete cycle around a unit circle is
   (a) 0 (b) 1
   (c) 3 (d) 2π

[IIT JAM 2010]

2. The equation of a surface of revolution is $z = \pm\sqrt{\frac{3}{2}x^2 + \frac{3}{2}y^2}$. The unit vector normal to the surface at the point $A\left(\sqrt{\frac{2}{3}}, 0, 1\right)$ is
   (a) $\sqrt{\frac{3}{5}}\hat{\imath} + \frac{2}{\sqrt{10}}\hat{k}$ (b) $\sqrt{\frac{3}{5}}\hat{\imath} - \frac{2}{\sqrt{10}}\hat{k}$
   (c) $\sqrt{\frac{3}{5}}\hat{\imath} + \frac{2}{\sqrt{5}}\hat{k}$ (d) $\sqrt{\frac{3}{10}}\hat{\imath} + \frac{2}{\sqrt{10}}\hat{k}$

[IIT JAM 2011]

3. The line integral $\int_A^B \vec{F}.d\vec{l}$, where $\vec{F} = \frac{x}{\sqrt{x^2+y^2}}\hat{x} + \frac{y}{\sqrt{x^2+y^2}}\hat{y}$, along the semi-circular path as shown in the figure below is



   (a) -2 (b) 0
   (c) 2 (d) 4

[IIT JAM 2012]

4. If $\vec{F}$ is a constant vector and $\vec{r}$ is the position vector then $\vec{\nabla}(\vec{F}.\vec{r}\,)$ would be
   (a) $(\vec{\nabla}.\vec{r})\vec{F}$ (b) $\vec{F}$
   (c) $(\vec{\nabla}.\vec{F})\vec{r}$ (d) $|\vec{r}|\vec{F}$

[IIT JAM 2014]

5. For vector $\vec{a} = \hat{\jmath} + \hat{k}, \vec{b} = 2\hat{\imath} + 3\hat{\jmath} - 5\hat{k}$ and $\vec{c} = \hat{\jmath} - \hat{k}$ the vector product $\vec{a} \times (\vec{b} \times \vec{c})$is
   (a) in the same direction as $\vec{c}$
   (b) in the direction opposite to $\vec{c}$
   (c) in the same direction as $\vec{b}$
   (d) in the direction opposite to $\vec{b}$

[IIT JAM 2014]

6. If the surface integral of field $\vec{A}(x,y,z) = 2\alpha x\hat{\imath} + \beta y\hat{\jmath} - 3yz\hat{k}$ over the closed surface of an arbitrary unit sphere is to be zero then the relationship between α, β and γ is
   (a) $\alpha + \beta/6 - \gamma = 0$ (b) $\alpha/3 + \beta/6 - \gamma/2 = 0$
   (c) $\alpha/2 + \beta - \gamma/3 = 0$ (d) $2/\alpha + 1/\beta - 3/\gamma = 0$

[IIT JAM 2014]

7. The line integral $\oint \vec{A}.d\vec{l}$ of a vector field $\vec{A}(x,y) = \frac{1}{r^2}(-y\hat{\imath} + x\hat{\jmath})$ where $r^2 = x^2 + y^2$ is taken around a square (see figure) of side of unit length and centered at (x_0, y_0) with $|x_o| > \frac{1}{2}$ and $|y_o| > \frac{1}{2}$. If the value of the integral is L, then
   (a) L depends on (x_o, y_o)
   (b) L is independent of (x_o, y_o) and its value is -1
   (c) L is independent of (x_o, y_o) and its value is 0
   (d) L is independent of (x_o, y_o) and its value is 2

[IIT JAM 2015]

8. Consider the coordinate transformation $x' = \frac{x+y}{\sqrt{2}}, y' = \frac{x-y}{\sqrt{2}}$. The relation between the area elements $dx'dy'$ and $dxdy$ is given by $dx'dy' = j(x', y')\, dxdy$. The value of j is
   (a) 2 (b) 1
   (c) -1 (d) -2

[IIT JAM 2015]

9. Consider a vector field$\vec{F} = y\hat{\imath} + xz^3\hat{\jmath} - zy\hat{k}$. Let C be the circle $x^2 + y^2 = 4$ on the plane $z = 2$, oriented counter-clockwise. The value of the integral $\oint_C \vec{F}.d\vec{r}$ is
   (a) 28π (b) 4π
   (c) -4π (d) -28π

[IIT JAM 2016]

10. The tangent line to the curve $x^2 + xy + 5 = 0$ at $(1,1)$ is represented by
    (a) $y = 3x - 2$ (b) $y = -3x + 4$
    (c) $x = 3y - 2$ (d) $x = -3y + 4$

[IIT JAM 2016]

11. Consider a closed triangular contour traversed in counter-clockwise direction, as shown in the figure. The value of the integral $\oint \vec{F}.\overrightarrow{dl}$ evaluated along this contour, for a vector field, $\vec{F} = y\hat{e}_x - x\hat{e}_y$, is …… ($\hat{e}_x, \hat{e}_y$ and $\hat{e}_z$ are unit vectors in Cartesian-coordinate system.)

[IIT JAM 2016]

12. A hemispherical shell is placed on the x-y plane centered at the origin. For a vector field $\vec{E} = \left(-y\hat{e}_x - x\hat{e}_y\right)/(x^2 +$

$y^2)$, the value of the integral $\int_S (\vec{\nabla} \times \vec{E}).d\vec{a}$ over the hemispherical surface is……π
($d\vec{a}$ is the elemental surface area, $\hat{e}_x, \hat{e}_y$ and $\hat{e}_z$ are unit vector in Cartesian-coordinate system)

[IIT JAM 2017]

13. The integral of the vector $\vec{A}(\rho, \varphi, z) = \frac{40}{\rho} cos\varphi \hat{p}$ (standard notation for cylindrical coordinates is used) over the volume of a cylinder of height L and radius R_o is:
(a) $20\pi R_o L(\hat{\imath} + \hat{\jmath})$ (b) 0
(c) $40\pi R_o L\hat{\jmath}$ (d) $40\pi R_o L\hat{\imath}$

[IIT JAM 2017]

14. The volume integral of the function $f(r, \theta, \phi) = r^2 cos\theta$ over the region $(0 \le r \le 2, 0 \le \theta \le \pi/3 and 0 \le \phi \le 2\pi)$ is

[IIT JAM 2018]

15. Let $f(x, y) = x^3 - 2y^3$. The curve along which $\nabla^2 f = 0$ is
(a) $x = \sqrt{2}y$ (b) $x = 2y$
(c) $x = \sqrt{6}y$ (d) $x = -\frac{y}{2}10$

[IIT JAM 2018]

16. A curve is given by $\vec{r}(t) = t\hat{\imath} + t^2\hat{\jmath} + t^3\hat{k}$. The unit vector of the tangent to the curve at $t = 1$ is
(a) $\frac{\hat{\imath}+\hat{\jmath}+\hat{k}}{\sqrt{3}}$ (b) $\frac{\hat{\imath}+\hat{\jmath}+\hat{k}}{\sqrt{6}}$
(c) $\frac{\hat{\imath}+2\hat{\jmath}+2\hat{k}}{3}$ (d) $\frac{\hat{\imath}+2\hat{\jmath}+3\hat{k}}{\sqrt{14}}$

[IIT JAM 2019]

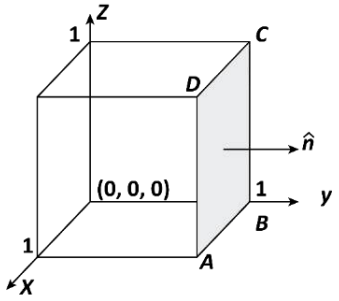
17. If $\phi(x, y, z)$ is a scalar function which satisfies the Laplace equation, then the gradient of ϕ is
(a) Solenoidal and irrotational
(b) Solenoidal but not irrotational
(c) Irrotational but not solenoid
(d) Neither Solenoidal nor irrotational

[IIT JAM 2019]

18. The gradient of scalar field S (x, y,z) has the following characteristic(s)
(a) Line integral of a gradient is path-independent
(b) Closed line integral of a gradient of scalar field S (x, y,z) is zero
(c) Gradient of S is a measure of the maximum rate of change in the field S
(d) Gradient of S is a scalar quantity

[IIT JAM 2019]

19. The flux of the function $\vec{F} = (y^2)\hat{x} + (3xy - z^2)\hat{y} + (4yz)\hat{z}$ passing through the surface ABCD along $\hat{n}$is_____ (Round off to 2 decimal places)



[IIT JAM 2019]

20. Consider the transformation to a new set of coordinates (ξ, η) from rectangular Cartesian coordinates (x, y), where $\xi = 2x + 3y$ and $\eta = 3x - 2y$. In the (ξ, η) coordinate system, the area element dxdy is
(a) $\frac{1}{13}d\xi d\eta$ (b) $\frac{2}{13}d\xi d\eta$
(c) $5d\xi d\eta$ (d) $\frac{3}{5}d\xi d\eta$

[IIT JAM 2019]

21. A unit vector perpendicular to the plane containing $\vec{A} = \hat{\imath} + \hat{\jmath} - 2\hat{k}$ and $\vec{B} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$ is
(a) $\frac{1}{\sqrt{26}}(-\hat{\imath} + 3\hat{\jmath} - 4\hat{k})$ (b) $\frac{1}{\sqrt{19}}(-\hat{\imath} + 3\hat{\jmath} - 3\hat{k})$
(c) $\frac{1}{\sqrt{35}}(-\hat{\imath} + 5\hat{\jmath} - 3\hat{k})$ (d) $\frac{1}{\sqrt{35}}(-\hat{\imath} - 5\hat{\jmath} - 3\hat{k})$

[MSQ JAM 2020]

22. Consider a vector function $\vec{u}(\vec{r})$ and two scalar functions $\Psi(\vec{r})$ and $\phi(\vec{r})$. The unit vector $\hat{n}$ is normal to the elementary surface dS, dV is an infinitesimal volume, $\overrightarrow{dl}$ is an infinitesimal line element and $\frac{\partial}{\partial n}$ denotes the partial derivative along $\hat{n}$. Which of the following identities is/are correct?
(a) $\int_V \vec{\nabla}.\vec{u}dV = \oint_S \vec{u}.\hat{n}dS$, where surface S bounds the volume V
(b) $\int_V [\Psi\nabla^2\phi - \phi\nabla^2\Psi]dV = \oint_S \left[\Psi\frac{\partial\phi}{\partial n} - \phi\frac{\partial\Psi}{\partial n}\right]dS$, where surface S bounds the volume V
(c) $\int_V [\Psi\nabla^2\phi - \phi\nabla^2\Psi]dV = \oint_S \left[\Psi\frac{\partial\phi}{\partial n} + \phi\frac{\partial\Psi}{\partial n}\right]ds$, where surface S bounds the volume V
(d) $\oint_C \vec{u} \cdot \overrightarrow{dt} = \iint_S (\vec{\nabla} \times \vec{u}) \cdot \hat{n}dS$, where C is the boundary of surface S

[IIT JAM 2020]

23. The volume integral $\int_v e^{-\left(\frac{r}{R}\right)^2} \vec{\nabla} \cdot \left(\frac{\hat{r}}{r^2}\right) d^3r$, where V is the volume of a sphere of radius R centered at the origin, is equal to
(a) 4π (b) 0
(c) $\frac{4}{3}\pi R^3$ (d) 1

[IIT JAM 2021]

24. Let (x, y)denote the coordinates in a rectangular Cartesian coordinate system C. Let (x', y') denote the coordinates in another coordinate system C'. Defined by
$x' = 2x + 3y$
$y' = -3x + 4y$
The area element in C', is
(a) $\frac{1}{17}dx'dy'$ (b) $12dx'dy'$
(c) $dx'dy'$ (d) $x'dx'dy'$

[NAT] [IIT JAM 2022]

25. Consider a unit circle C in the x-y plane with center at the origin. The line integral of the vector field, $\vec{F}(x, y, z) = -2y\hat{x} - 3z\hat{y} + x\hat{z}$, taken anticlockwise over C is _____ π.

[IIT JAM 2022]

26. Consider a unit circle C in the x-y plane, centered at the origin. The value of the integral $\oint [(sinx - y)dx -$

$(siny - x)dy]$ over the circle C, traversed anticlockwise, is

(a) 0 (b) 2π

(c) 3π (d) 4π

[IIT JAM 2022]

27. Consider a two-dimensional force field $\vec{F}\,(x,y) = (5x^2 + ay^2 + bxy)\,\hat{x} + (4x^2 + 4xy + y^2)\,\hat{y}$ If the force field is conservative, then the values of 'a' and 'b' are

(a) a = 2 and b = 4 (b) a = 2 and b = 8

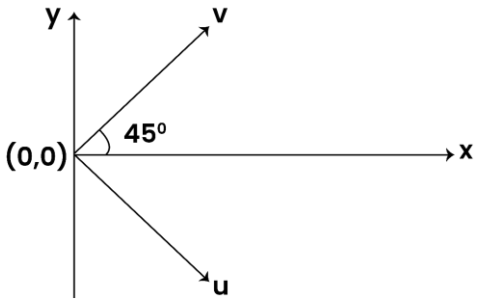
(c) a = 4 and b = 2 (d) a = 8 and b = 2

[IIT JAM 2023]

28. The sum of the x-components of unit vectors $\hat{r}$ and $\hat{\theta}$ for a particle moving with angular speed 2 rad s^{-1} at angle $\theta = 215°$ is _____ (rounded off to two decimal places)

[IIT JAM 2023]

29. The Jacobian matrix for transforming from (x, y) to another orthogonal coordinates system (u, v) as shown in the figure is

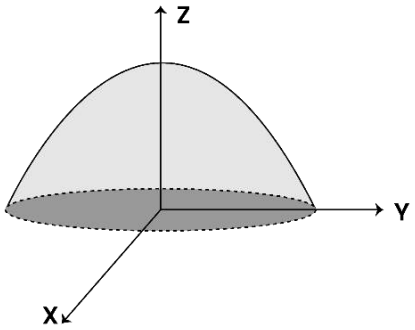


(a) $\frac{1}{\sqrt{2}}\begin{bmatrix}1 & -1\\ 1 & -1\end{bmatrix}$ (b) $\frac{1}{\sqrt{2}}\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}$

(c) $\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ -1 & 1\end{bmatrix}$ (d) $\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$

[IIT JAM 2023]

30. For a given vector$\vec{F} = -y\hat{\imath} + z\hat{\jmath} + x^2\hat{k}$, the surface integral $\int_S (\vec{\nabla} \times \vec{F}) \cdot \hat{r}\,dS$ over the surface S of a hemisphere of radius R with the centre of the base at the origin is



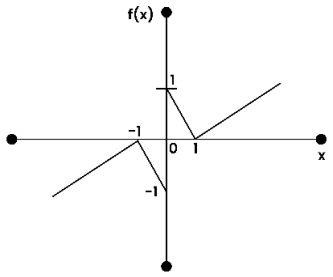
(a) πR^2 (b) $-\frac{2\pi R^2}{3}$

(c) πR^2 (d) $\frac{2\pi R^2}{3}$

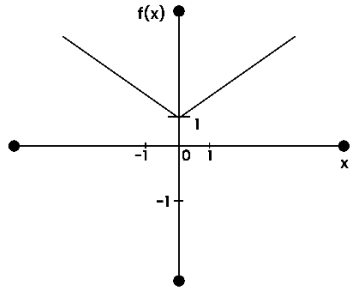
[IIT JAM 2023]

31. The plot of the function $f(x) = \big||x| - 1\big|$ is

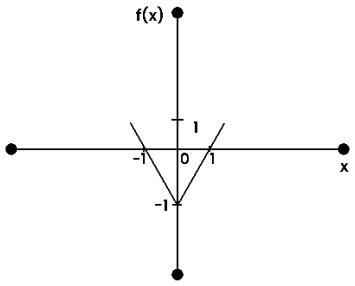
(a)



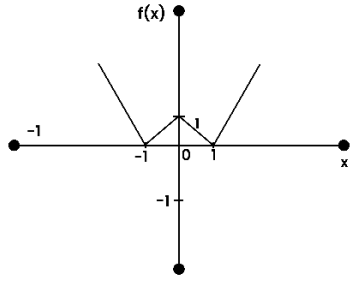
(b)



(c)



(d)



[IIT JAM 2023]

32. Which of the following fields has non-zero curl?

(a) $y^2\hat{\imath} + (2xy + z^2)\hat{\jmath} + 2yz\hat{k}$

(b) $(y + z)\hat{\imath} + (x + z)\hat{\jmath} + (x + y)\hat{k}$

(c) $x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

(d) $xy\hat{\imath} + 2yz\hat{\jmath} + 3xz\hat{k}$

[IIT JAM 2024]

33. The divergence of a 3-dimensional vector $\frac{\hat{r}}{r^3}$ (is the unit radial vector) is:

(a) $-\frac{1}{r^4}$ (b) Zero

(c) $\frac{1}{r^3}$ (d) $-\frac{3}{r^4}$

[IIT JAM 2024]

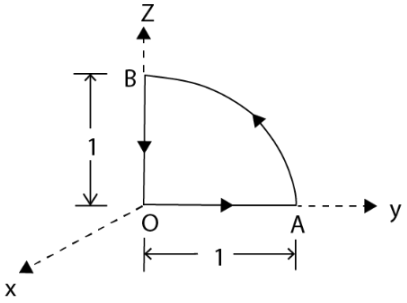
34. A vector field is expressed in the cylindrical coordinate system (s, ϕ, z) as, $\vec{F} = \frac{A}{s}\hat{S} + \frac{B}{s}\hat{Z}$. If this field represents an electrostatic field, then the possible values of A and B, respectively, are:

(a) 1 and 0 (b) 0 and 1

(c) -1 and 1 (d) 1 and -1

[IIT JAM 2024]

35. The value of the line integral for the vector, $\vec{v} = 2\hat{x} + yz^2\hat{y} + (3y + z^2)\hat{z}$ along the closed path OABO (as shown in the figure) is:



(Path AB is the arc of a circle of unit radius)

(a) $\frac{1}{4}(3\pi - 1)$ (b) $3\pi - \frac{1}{4}$

(c) $\frac{3\pi}{4} - 4$ (d) $3\pi - 1$

[IIT JAM 2024]

36. The co-ordinate system (x, y, z) is transformed to the system (u, v, w), as given by:

$u = 2x + 3y - z$

$v = x - 4y + z$

$w = x + y$

The Jacobian of the above transformation is _____

[IIT JAM 2024]

37. Two sides of a triangle OAB are given by:

$\overrightarrow{OA} = \hat{x} + 2\hat{y} + \hat{z}$

$\overrightarrow{OB} = 2\hat{x} - \hat{y} + 3\hat{z}$

The area of the triangle is _____.

(Rounded off to one decimal place).

[IIT JAM 2025]

38. Consider a volume V enclosed by a closed surface S having unit surface normal $\hat{n}$. For $\mathrm{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, the value of the surface integral $\frac{1}{9}\oiint \mathrm{r} \cdot \hat{n} dS$ is

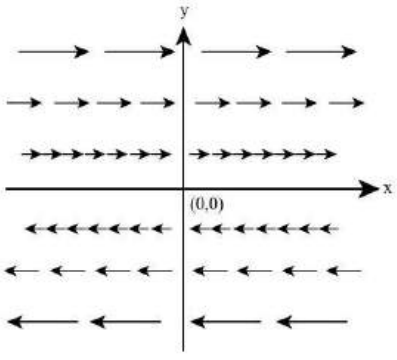
(a) v (b) $3\,V$

(c) $\frac{V}{3}$ (d) $\frac{V}{9}$

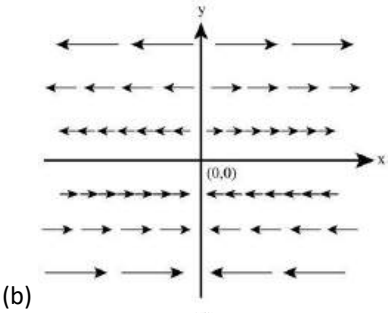
[IIT JAM 2025]

39. Which one of the following figures represents the vector field $A = y\hat{\imath}$?

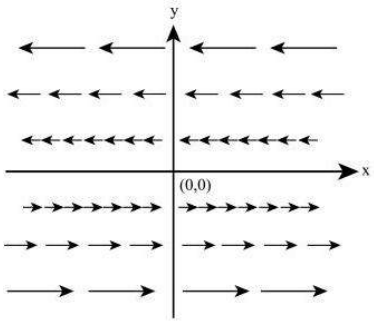
($\hat{\imath}$ is the unit vector along the x-direction)



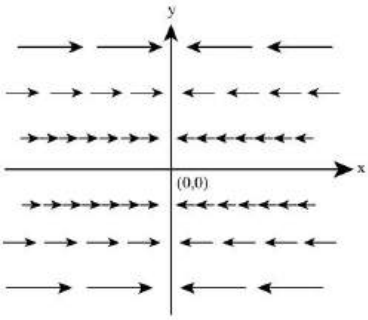
(a)



(b)



(c)



(d)

[IIT JAM 2025]

40. Given a function $f(x, y) = \frac{x}{a}e^y + \frac{y}{b}e^x$, where $x = at$ and $y = bt$(a and b are non-zero constants), the value of $\frac{df}{dt}$ at $t = 0$ is

(a) -1 (b) 0

(c) 1 (d) 2

[IIT JAM 2025]

41. In Cartesian coordinates, consider the functions $u(x, y) = \frac{1}{2}(x^2 - y^2)$ and $v(x, y) = xy$. If (r, θ) are the polar coordinates, the Jacobian determinant $\left|\frac{\partial(u,v)}{\partial(r,\theta)}\right|$ is

(a) r (b) $\frac{1}{r}$

(c) r^2 (d) r^3

[IIT JAM 2025]

42. Consider a vector $F = \frac{1}{\pi}[-\sin y\hat{\imath} + x(1 - \cos y)\hat{\jmath}]$. The value of the integral $\oint \mathrm{F} \cdot d\mathrm{r}$ over a circle $x^2 + y^2 = 1$ evaluated in the anti-clockwise direction is_____. (in integer)

[IIT JAM 2026]

43. Consider the following vector $\vec{V} = xyz\hat{k}$.

Which of the following statements is correct for the resultant vector $\vec{\nabla} \times \vec{\nabla} \times \vec{V}$?

(a) It lies in xy - plane

(b) It lies in yz – plane

(c) It lies in xz – plane

(d) It is along the x direction

ANSWER KEY									
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
a	b	b	b	a	b	c	c	a	B
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
-2	2	d	15.07	b	d	a	a,b,c	1.17	A
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
d	a,b,d	a	a	4π	d	b	2.7	b	A
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
d	d	a	a	a	0.25	4.33	c	a	d
41	**42**	**43**							
d	1	a							

:: Solutions ::

1. Solution: (a)

Let the x and y coordinate be given as

$$x = (1)cos\theta$$
$$y = (1)sin\theta$$

$\Rightarrow x = cos\theta \; and \; y = sin\theta.$

Therefore, $dx = -sin\theta d\theta$

$dy = cos\theta d\theta$

Thus work done can be written as $dW = 2(cos\theta)(sin\theta)(-sin\theta)d\theta + (cos\theta)^2 cos\theta d\theta$

$-2sin^2\theta . cos\theta d\theta + cos^3\theta d\theta$

Thus the total work done along the complete circle is

$W = -2\int_0^{2\pi} sin^2\theta \, . cos\theta d\theta + \int_0^{2\pi} cos^3\theta d\theta$

The value of both these integrals is zero.

2. Solution: (b)

Given that $z = \pm\sqrt{\frac{3}{2}x^2 + \frac{3}{2}y^2}$

$\Rightarrow z^2 = \frac{3}{2}x^2 + \frac{3}{2}y^2$

$\Rightarrow 3x^2 + 3y^2 - 2z^2 = 0$

$V = 3x^2 + 3y^2 - 2z^2,$

Gradient V is, $\vec{\nabla}V = 6x\hat{x} + 6y\hat{y} - 4z\hat{z}$

The unit normal to the surface at the point $A\left(\sqrt{\frac{2}{3}}, 0, 1\right)$ is

$\hat{n} = \frac{\vec{\nabla}V}{|\vec{\nabla}V|}.$

$$\hat{n} = \frac{6\sqrt{\frac{2}{3}}\hat{x} + 60\hat{y} - 4 \times 1\hat{z}}{\sqrt{36 \times \frac{2}{3} + 16}}$$

$$\hat{n} = \frac{6\sqrt{\frac{2}{3}}\hat{x} - 4\hat{z}}{\sqrt{40}}$$

$$\hat{n} = \sqrt{\frac{3}{5}}\hat{x} - \frac{2}{\sqrt{10}}\hat{z}$$

3. Solution: (b)

Since $x^2 + y^2 = 1$

$\Rightarrow xdx + ydy = 0$

Also $\overrightarrow{dl} = dx\hat{x} + dy\hat{y}$

$\Rightarrow \vec{F} . d\vec{l} = \frac{x\,dx}{\sqrt{x^2+y^2}} + \frac{y\,dy}{\sqrt{x^2+y^2}} = 0$

$since \; xdx = -ydy$

$\Rightarrow \int_A^B \vec{F} . d\vec{l} = 0$

4. Solution: (b)

Let us consider a vector $\vec{F} = (F_x\hat{x} + F_y\hat{y} + F_z\hat{z})$, where $F_x, F_y \; and \; F_z$ are some arbitrary constants

And $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$

Such that $\overrightarrow{F} . \vec{r} = F_o \; (x + y + z)$

Therefore $\vec{\nabla}(\vec{F} . \vec{r}) = (F_x\hat{x} + F_y\hat{y} + F_z\hat{z})$

$$\vec{\nabla}(\vec{F} . \vec{r}) = \vec{F}$$

5. Solution: (a)

The cross product of b and c is given as , $\vec{b} \times \vec{c} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 3 & -5 \\ 0 & 1 & -1 \end{vmatrix} = \hat{\imath}(-3+5) - \hat{\jmath}(-2+0) + \hat{k}(2-0) =$

$2\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$

Now, $\vec{a} \times (\vec{b} \times \vec{c}) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 1 & 1 \\ 2 & 2 & 2 \end{vmatrix} = \hat{\imath}(2-2) - \hat{\jmath}(0-2) +$

$\hat{k}(0-2)$

$$\vec{a} \times (\vec{b} \times \vec{c}) = 2\hat{\jmath} - 2\hat{k}$$
$$\vec{a} \times (\vec{b} \times \vec{c}) = 2\vec{c}$$

6. Solution: (b)

Since it is given that, $\oint_S \vec{A} . d\,\vec{a} = 0$

Therefore, from divergence theorem,

$\oint_S \vec{A} . d\,\vec{a} = \int_v (\vec{\nabla} . \vec{A}) d\tau = 0$

$$\int_V (\vec{\nabla} . \vec{A}) d\tau = 0$$

Where $d\tau$ is the volume element, which can't be zero

$\Rightarrow 2\alpha + \beta - 3\gamma = 0$

$\Rightarrow \frac{\alpha}{3} + \frac{\beta}{6} - \frac{\gamma}{2} = 0$

7. Solution: (c)

Curl of vector A is given as

$$\vec{\nabla} \times \vec{A} = \begin{pmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -\frac{y}{x^2+y^2} & \frac{x}{x^2+y^2} & 0 \end{pmatrix}$$

$= \hat{x}(0-0) - \hat{y}(0-0) + \hat{z}\left[\frac{\partial}{\partial x}\left(\frac{x}{x^2+y^2}\right) + \frac{\partial}{\partial y}\left(\frac{y}{x^2+y^2}\right)\right]$

$$\vec{\nabla} \times \vec{A} = \hat{z}\left[\frac{(x^2+y^2) - x \times 2x}{(x^2+y^2)^2} + \frac{(x^2+y^2) - y \times 2y}{(x^2+y^2)^2}\right]$$

$$\vec{\nabla} \times \vec{A} = \hat{z}\left[\frac{(y^2-x^2) + x^2 - y^2}{(x^2+y^2)^2}\right]$$

$$\vec{\nabla} \times \vec{A} = 0$$

Thus, $\oint \vec{A} . d\vec{l} = \oint \vec{\nabla} \times \vec{A} \, dv = 0$

8. Solution: (c)

It is given that $x' = \frac{x+y}{\sqrt{2}}$ and

$$y' = \frac{x-y}{\sqrt{2}}$$

Since $dx'dy' = \boldsymbol{j}(\boldsymbol{x}', \boldsymbol{y}')\, dxdy$

$$\Rightarrow J = \begin{pmatrix} \frac{\partial x'}{\partial x} & \frac{\partial x'}{\partial y} \\ \frac{\partial y'}{\partial x} & \frac{\partial y'}{\partial y} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$J = -\frac{1}{2} - \frac{1}{2}$$

$$J = -1$$

9. Solution: (a)

Using stokes theorem $\oint_C \vec{F}.d\vec{r} = \int_s (\vec{\nabla} \times \vec{F}).d\vec{a}$

Curl of F is , $\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/z \\ y & xz^3 & -zy \end{vmatrix}$

$\Rightarrow \vec{\nabla} \times \vec{F} = \hat{x}\left(\frac{\partial(-yz)}{\partial y} - \frac{\partial(xz^3)}{\partial z}\right) + \hat{y}\left(\frac{\partial y}{\partial z} - \frac{\partial(-zy)}{\partial x}\right) + \hat{z}\left(\frac{\partial(xz^3)}{\partial x} - \frac{\partial y}{\partial y}\right)$

$\Rightarrow \vec{\nabla} \times \vec{F} = \hat{x}(-z - 3xz^2) + \hat{y}(0 - 0) + \hat{z}(z^3 - 1)$

Since $z = 2$ is given,

$\Rightarrow \vec{\nabla} \times \vec{F} = -(2 + 12x)\hat{x} + 7\hat{z}$

$\because d\vec{a} = rdrd\phi\hat{z}$

$\Rightarrow (\vec{\nabla} \times \vec{F}).d\vec{a} = [-(2 + 12x)\hat{x} + 7\hat{z}].rdrd\phi\hat{z}$

$$(\vec{\nabla} \times \vec{F}) = 7rdrd\phi$$

$$\Rightarrow \int_s (\vec{\nabla} \times \vec{F}).d\vec{a} = 7\int_0^2 rdr \int_0^{2\pi} d\phi$$

$$\oint_C \vec{F}.d\vec{r} = \int_s (\vec{\nabla} \times \vec{F}).d\vec{a} = 28\pi$$

10. Solution: (b)

The Given curve is $x^2 + xy + 5 = 0$

$$\Rightarrow 2x + y + x\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-(2x+y)}{x}$$

Now At (1,1), $\frac{dy}{dx} = -\frac{3}{1}$

$$\frac{dy}{dx} = -3$$

Therefore the equation of tangent lines is

$$y - 1 = -3(x - 1)$$

$$\Rightarrow y = -3x + 4$$

11. Solution: (-2)

The given vector field is $\vec{F} = y\hat{e}_x - x\hat{e}_y$

Using stokes theorem, $\int_s (\vec{\nabla} \times \vec{E}).d\vec{a} = \oint_{line} \vec{E}.\overrightarrow{dl}$

Curl of this vector field is

$\Rightarrow \vec{\nabla} \times \vec{F} = -2\hat{z}$ and $d\vec{a} = dxdy\hat{z}$

$\Rightarrow (\vec{\nabla} \times \vec{F}).d\vec{a} = -2dxdy$

$\oint \vec{F}.\overrightarrow{dl} = \int (\vec{\nabla} \times \vec{F}).d\vec{a}$

$\oint \vec{F}.\overrightarrow{dl} = \iint (-2)dxdy$

$\oint \vec{F}.\overrightarrow{dl} = (-2)\frac{1}{2}(2 \times 1)$

$\oint \vec{F}.\overrightarrow{dl} = -2$

12. Solution: (2)

Given vector field is $\vec{E} = \frac{(-y\hat{e}_x - x\hat{e}_y)}{x^2+y^2}$

Here, on the z axis i.e. for x=0 and y=0, the given vector field $\vec{E} = \frac{(-y\hat{e}_x - x\hat{e}_y)}{x^2+y^2}$ is not defined. So, calculation of $\int_s (\vec{\nabla} \times \vec{E}).\boldsymbol{d\vec{a}}$ **i.e** over the surface will be having a point with co-ordinate (0, 0, r) where the given vector field will be not defined. So, the calculation of surface integral will give the wrong results. So, solving this $\int_s (\vec{\nabla} \times \vec{E}).\boldsymbol{d\vec{a}}$ using Using stokes theorem, $\int_s (\vec{\nabla} \times \vec{E}).d\vec{a} = \oint_{line} \vec{E}.\overrightarrow{dl}$

Using stokes theorem, $\int_s (\vec{\nabla} \times \vec{E}).d\vec{a} = \oint_{line} \vec{E}.\overrightarrow{dl}$

Taking line integral around circle $x^2 + y^2 = r^2$ in $z = 0$ plane. We will use cylinder coordinate and use $x = rcos\phi, y = rsin\phi$

$$\Rightarrow dx = -rsin\phi d\phi, dy = rcos\phi d\phi$$

$$\vec{E}.\overrightarrow{dl} = \frac{(-ydx + xdy)}{x^2 + y^2}$$

$$\vec{E}.\overrightarrow{dl} = \frac{-rsin\phi(-rsin\phi)d\phi + rcos\phi(rcos\phi)d\phi}{r^2} = d\phi$$

$$\oint_{line} \vec{E}.\overrightarrow{dl} = \int_0^{2\pi} d\phi = 2\pi$$

13. Solution: (d)

$\int_V \vec{A}\, d\tau = \int_0^{R_o} \int_0^{2\pi} \int_0^L \frac{40}{\rho} cos\varphi\ \hat{\rho}\ [\rho\, d\rho d\varphi dz]$

$[\, where, \hat{\rho} = cos\phi\hat{\imath} + sin\phi\hat{\jmath}]$

$$\int_V \vec{A}\, d\tau = 40\pi R_o L \int_0^{2\pi} cos\varphi(cos\varphi\hat{\imath} + sin\varphi\hat{\jmath})\, d\varphi$$

$$\int_V \vec{A}\, d\tau = 40R_o L\pi\hat{\imath}$$

14. Solution: (15.07)

$$I = \int_V f\ d\tau = \int_0^2 \int_0^{\frac{\pi}{3}} \int_0^{2\pi} (r^2 cos\theta)\, r^2 sin\theta dr d\theta d\phi$$

Where, $d\tau$ is the volume element given by: $r^2 sin\theta dr d\theta d\phi$

$$I = \frac{2^5}{5} \times \frac{1}{2}\left[-\frac{cos2\theta}{2}\right]_0^{\pi/3} \times 2\pi$$

$$\Rightarrow I = \frac{32}{5} \times \frac{1}{4}\left[-\frac{cos2\pi}{3} + cos0\right]_0^{\frac{\pi}{3}} \times 2\pi$$

$$I = \frac{24}{5}\pi$$

$$I = 15.07$$

15. Solution: (b)

Since $\nabla^2 f = \frac{\partial^2}{\partial x^2}\boldsymbol{f} + \frac{\partial^2}{\partial y^2}\boldsymbol{f} + \frac{\partial^2}{\partial Z^2}\boldsymbol{f}$

Therefore $\nabla^2 f = \frac{\partial^2}{\partial x^2}(x^3 - 2y^3) + \frac{\partial^2}{\partial y^2}(x^3 - 2y^3) + 0$

$\nabla^2 f = 6x - 12y$

$\because given\ that\ \nabla^2 f = 0$
$\Rightarrow 6x - 12 = 0$
$\Longrightarrow x = 2y$

16. Solution: (d)

Let a unit vector tangent to the curve at t be $\hat{n}$.

So $\hat{n} = \frac{\frac{d\vec{r}}{dt}}{\left|\frac{d\vec{r}}{dt}\right|}$

$= \frac{\hat{\imath} + 2t\hat{\jmath} + 3t\hat{k}}{\sqrt{1 + 4t^2 + 5t^2}}$

$\Rightarrow at\ t = 1, \qquad \hat{n} = \frac{\hat{\imath} + 2j + 2\hat{k}}{\sqrt{14}}$

17. Solution: (a)

Laplace equation means:

$\vec{\nabla}^2\phi$ =0 and grad of $\boldsymbol{\phi(x, y, z}$ **is** $\vec{\nabla}\phi$

Since$\vec{\nabla} \times \vec{\nabla}\phi$ = 0 is always true. Hence we can call the grad of $\boldsymbol{\phi(x, y, z)}$ **as Irrotational field.**

Also, since $\vec{\nabla}.\vec{\nabla}\phi = \vec{\nabla}^2\phi$ =0 (given). So, $\vec{\nabla}\phi\ is\ Solenoidal\ field\ as\ well.$

This could also be understood by the help of knowledge of EMT: Laplace equation is $\nabla^2\phi = 0 \Rightarrow$

$$\rho = 0$$

$\Rightarrow \vec{E} = \vec{\nabla}\phi = 0,$

$$\vec{E} = 0,$$
$$\vec{\nabla} \times \vec{E} = 0$$

18. Solution: (a), (b), (c)

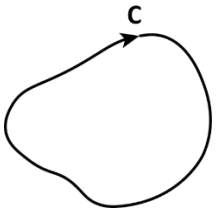
(a) The line integral of gradient of a scalar field only depends on initial and final points.

$\therefore \int_I \vec{\nabla} S \cdot d\vec{\ell} = \int_{II} \vec{\nabla} S \cdot d\vec{\ell} = \int_{III} \vec{\nabla} S \cdot d\vec{\ell}$

Therefore, line integral of the gradient of a scalar field are path-independent.

(**Note**: this concept is valid only till when, there is no singular point (not-defined) between the paths which we are calling path-independent here corresponding to the conservative or irrotational field)

(b) $\oint \vec{\nabla} S \cdot d\vec{\ell} = \iint (\vec{\nabla} \times \vec{\nabla} S) \cdot d\vec{A} = 0 \quad (\because \vec{\nabla} \times \vec{\nabla} S = 0)$



(**Note**: this concept is valid only till when, there is no singular point (not-defined) between the paths which we are calling path-independent here corresponding to the conservative or irrotational field)

(c) Gradient of S is a measure of the maximum rate of change in the field S. This can be understood from the concept of directional derivative. Directional derivative is maximum in the direction normal to the surface at that point.

(d) Gradient of S is a vector quantity. Since, Grad operator or the Del operator is a vector operator, so gradient of a scalar function will be a vector quantity.

Correct option are (a), (b) and (c)

19. Solution: (1.17)

The given ABCD plane is nothing but $y = 1$ plane bounded with the constraints 0 ≤x≤ 1 and 0 ≤ z ≤ 1

Flux of the function F is given as $\oint \vec{F}.d\vec{a} = \iint \vec{F}.(dxdz\hat{y})$

$= \iint (3xy - z^2)dxdz$

$= \int_0^1 \int_0^1 (3x - z^2)\, dxdz = \int_0^1 \left[3xz - \frac{z^3}{3}\right]_{z=0}^{1} dx$

$= \int_0^1 \left[3z - \frac{1}{3}\right] dx$

$= \left[3\frac{x^3}{2} - \frac{x}{3}\right]_0^1$

$= \frac{3}{2} - \frac{1}{3}$

$= \frac{7}{6}$

$= 1.17$

20. Solution: **(a)**

Jacobian is

$$J\left(\frac{\boldsymbol{\xi, \eta}}{x, y}\right) or\ J(\boldsymbol{\xi, \eta}) = \begin{vmatrix} \frac{\partial\xi}{\partial x} & \frac{\partial\xi}{\partial y} \\ \frac{\partial\eta}{\partial x} & \frac{\partial\eta}{\partial y} \end{vmatrix} = \begin{vmatrix} 2 & 3 \\ 3 & -2 \end{vmatrix} = -13$$

We will prefer to solve it, since we know that $J\left(\frac{\xi,\eta}{x,y}\right) = \frac{1}{J\left(\frac{x,y}{\xi,\eta}\right)}$ and the given transformation is $(\boldsymbol{\xi, \eta})$ in terms of (x, y)

Therefore, $J\left(\frac{x,y}{\xi,\eta}\right) = -\frac{1}{13} = J$

Since area element in $\xi - \eta$ system is,

$$dA = \left|J\left(\frac{x, y}{\boldsymbol{\xi, \eta}}\right)\right| d\xi d\eta$$

$= \frac{1}{13} d\xi d\eta$

21. Solution: **(d)**

Method 1: Since $\vec{A}.\hat{n} = 0$ and $\vec{B}.\hat{n} = 0$

Checking the above conditions $only$ for option $(d), the\ given\ conditions\ are\ satisfied$:

$\vec{A}.\hat{n} = \frac{1}{\sqrt{35}}(-1 - 5 - 6) = 0$ and $\vec{B}.\hat{n} = \frac{1}{\sqrt{35}}(-2 + 5 - 3) = 0$

Method 2: $\vec{A} \times \vec{B} \times \vec{C}$ =0 will also give us the condition for the normal vector, where $\vec{C}\ is\ the\ normal\ vector$

Method 3: $\vec{A} \times \vec{B}$ will give normal vector, so dividing it by it's magnitude will give the unit normal vector

22. Solution: (a), (b) and (d)

$\int \vec{\nabla}.\vec{u}\, dV = \oint \vec{u}.\hat{n}\, dS$

(Mathematical From of Gauss Divergence Theorem)

Hence, option (a) is correct.

$\int_V [\psi\nabla^2\phi - \phi\nabla^2\psi]\ dV = \int_V [\vec{\nabla}\phi.\vec{\nabla}\psi + \psi\vec{\nabla}.\vec{\nabla}\phi - \vec{\nabla}\phi.\vec{\nabla}\psi - \phi\vec{\nabla}.\vec{\nabla}\psi]\ dV$

$= \int_V \vec{\nabla}.(\psi\vec{\nabla}\phi - \phi\vec{\nabla}\psi)dV$

$= \oint_s (\psi\vec{\nabla}\phi - \phi\vec{\nabla}\psi)\,.\,dS\,\hat{n}$ (Using Gauss Divergent Theorem)

$\left[Here, \vec{\nabla}\phi.\hat{n} = \left(\frac{\partial\phi}{\partial n}\hat{n}\right).\hat{n} = \frac{\partial\phi}{\partial n}; \vec{\nabla}\psi.\hat{n} = \left(\frac{\partial\psi}{\partial n}\hat{n}\right).\hat{n} = \frac{\partial\psi}{\partial n}\right]$

$= \oint_s \left(\psi\frac{\partial\phi}{\partial n} - \phi\frac{\partial\psi}{\partial n}\right)dS$

Hence, option (b) is correct and option (c) is incorrect.

$\oint_C \vec{u}.d\vec{l} = \iint(\vec{\nabla}\times\vec{u})\,.\,\hat{n}dS$ (Mathematical Form of Stoke's theorem)

Correct option are (a), (b) and (d)

23. Solution: (a)

Since, we know that: $\vec{\nabla}.\left(\frac{\hat{r}}{r^2}\right) = 4\pi\delta^3(\vec{r})$

Therefore, Volume integral is given by

$\iiint e^{-\left(\frac{r}{R}\right)^2}\vec{\nabla}.\left(\frac{\hat{r}}{r^2}\right)d^3r = \iiint e^{-\left(\frac{r}{R}\right)^2}4\pi\delta^3(\vec{r})d^3r$

Also by the identity: $\iiint f(r)\,\delta^3(\vec{r}-\vec{r}')d^3r = f(\vec{r}')$

Therefore, $\iiint e^{-\left(\frac{r}{R}\right)^2}\vec{\nabla}.\left(\frac{\hat{r}}{r^2}\right)d^3r =$

$\iiint e^{-\left(\frac{r}{R}\right)^2}4\pi\delta^3(\vec{r})d^3r = 4\pi e^{-\left(\frac{0}{r}\right)^2} = 4\pi$

24. Solution: (a)

The transformation $C \to C'$ is given by,

$x' = 2x + 3y$

$y' = 3x + 4y$

Supposes, the area element in C coordinate system is $dx\,dy = dA$ then the area element in C coordinate system can be expressed by $dA' = J(\frac{x,y}{x',y'})\,dx'\,dy'$.

So, $dx\,dy = J\left(\frac{x,y}{x',y'}\right)dx'dy' = \frac{1}{J\left(\frac{x',y'}{x,y}\right)}dx'dy'$

Where $J\left(\frac{x',y'}{x,y}\right)$ is the jacobian of (x',y') w.r.t (x,y). We will prefer to solve it, since we know that $J\left(\frac{x,y}{x',y'}\right) = \frac{1}{J\left(\frac{x',y'}{x,y}\right)}$ and the given transformation is (x',y') in terms of (x,y)

Where, $J\left(\frac{x',y'}{x,y}\right) = \begin{vmatrix}\partial x'/\partial x & \partial x'/\partial y\\ \partial y'/\partial x & \partial y'/\partial y\end{vmatrix} = \begin{vmatrix}2 & 3\\ -3 & 4\end{vmatrix} = 8 - (-9) = 17$

Therefore, the are element in C' coordinate system is $\frac{1}{17}\,dx'dy'$

Correct option is (a)

25. Solution: (4π)

Given: $\vec{F} = -2y\,\hat{x} - 3z\hat{y} + x\hat{z}$

Using stoke's theorem: $\oint\vec{A}\cdot\overrightarrow{dL} = \iint(\vec{\nabla}\times\vec{A})\,.\,\overrightarrow{dS}$

Where $\vec{F} = -2y\,\hat{x} - 3z\hat{y} + x\hat{z}$

and $\overrightarrow{dl} = dx\ \hat{\imath} + dy\ \hat{\jmath}]$ and $\overrightarrow{dS}$ = dx dy $\hat{k}$

$\vec{\nabla}\times\vec{F} = \begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ -2y & -3z & x\end{vmatrix} = |2|\hat{z}$

$\iint\vec{\nabla}\times\vec{F}\,dxdy = \iint 2\,dx\,dy = 2(2\pi) = 4\pi$

26. Solution: (d)

$\oint[(\sin x - y)dx - (\sin y - x)dy] = \oint[(\sin x - y)\hat{\imath} - (\sin y - x)\hat{\jmath}]\,.\,[dx\ \hat{\imath} + dy\ \hat{\jmath}]$

Using stoke's theorem: $\oint\vec{A}\cdot\overrightarrow{dL} = \iint(\vec{\nabla}\times\vec{A})\,.\,\overrightarrow{dS}$

Where $\vec{A} = (\sin x - y)\hat{\imath} - (\sin y - x)\hat{\jmath}$ and $\overrightarrow{dl} = dx\ \hat{\imath} + dy\ \hat{\jmath}]$ and $\overrightarrow{dS}$ = dx dy $\hat{k}$

$\vec{\nabla}\times\vec{A}\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ \sin x - y & -\sin y + x & 0\end{vmatrix} = (+1+1)\hat{k} = 2\,\hat{k}$

$= \iint 2\,dx\,dy = 2(2\pi) = 4\pi$

27. Solution: (b)

Given that the field is conservative i.e. $\vec{\nabla}\times\vec{F} = 0$

$\vec{\nabla}\times\vec{F} = \begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ (5x^2 + 0y^2 + bxy) & (4x^2 + 4xy + y^2) & 0\end{vmatrix}$

=0

$= \hat{\imath}(0-0) - \hat{\jmath}(0-0) + \hat{k}(8x + 4y - 2ay - bx) = 0$

comparing coefficients on both the side

$\Rightarrow 8x + 4y - 2ay - bx = 0$

$\Rightarrow b = 8\ \&\ a = 2$

$\therefore$ Option (b) is correct.

28. Solution: (2.7)

$r\hat{\imath} + \theta\hat{\imath}$. Given that $\theta = 215$ and $\theta = 2rad/s \Rightarrow \hat{r} = \cos(\theta)\,\hat{\imath} + \sin(\theta)\,\hat{\jmath}$

$\hat{\theta} = -\sin(\theta)\,\hat{\imath} + \cos(\theta)\,\hat{\jmath}$

$\frac{d\hat{r}}{dt}\hat{\imath} = \dot{\theta}(-\sin\theta)\ \&\ \frac{d\hat{\theta}}{dt}\hat{\imath} = -\hat{\theta}(\cos\theta)$

$\frac{d\hat{r}}{dt}\hat{\imath} + \frac{d\hat{\theta}}{dt}\hat{\imath} = -\dot{\theta}(\sin\theta + \cos\theta)$

At $\theta = 215$ and $\theta = 2\,rad/s$

$\frac{d\hat{r}}{dt}\hat{\imath} + \frac{d\hat{\theta}}{dt}\hat{\imath} = -\dot{\theta}(\sin\theta + \cos\theta) = 2.7$

29. Solution: (b)

$\begin{bmatrix}u\\ v\end{bmatrix} = \begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}$

$\begin{bmatrix}u\\ c\end{bmatrix} = \begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}\frac{u}{\sqrt{2}} + \frac{v}{\sqrt{2}}\\ -\frac{u}{\sqrt{2}} + \frac{v}{\sqrt{2}}\end{bmatrix}$

$\begin{bmatrix}u\\ c\end{bmatrix} = \begin{bmatrix}\frac{au+av}{\sqrt{2}} + \frac{-bu+bv}{\sqrt{2}}\\ \frac{cu+cv}{\sqrt{2}} + \frac{-du+dv}{\sqrt{2}}\end{bmatrix}$

$u = \frac{(a-b)u}{\sqrt{2}} + \left(\frac{a+b}{\sqrt{2}}\right)v\ \&\ v = \frac{(c-d)u}{\sqrt{2}} + \frac{(c+d)v}{\sqrt{2}}$

$\frac{a-b}{\sqrt{2}} = 1, \frac{a+b}{\sqrt{2}} = 0, \frac{c-d}{\sqrt{2}} = 0, \frac{c+d}{\sqrt{2}} = 1$

$\frac{2a}{\sqrt{2}} = 1, a = \frac{1}{\sqrt{2}}, \frac{-2b}{\sqrt{2}} = 1, b = -\frac{1}{\sqrt{2}}, c = \frac{1}{\sqrt{2}}, \frac{-2d}{\sqrt{2}} = -1$

Thus, $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

30. Solution: (a)

Given that, $\vec{F} = -y\hat{\imath} + z\hat{\jmath} + x^2\hat{k}$

$\therefore \int_s (\nabla \times F) \cdot \hat{r} ds$

$= \int_l \vec{F} \cdot d\vec{l}$

$= \int \left(-y\hat{\imath} + z\hat{\jmath} + x^2\hat{k}\right) \cdot (dx\hat{\imath} + dy\hat{\jmath})$

$= -\int y dx|_{z=0}$

$= -\int R\sin\theta\, R(-\sin\theta) d\omega$

$= +R^2 \int_0^{2\pi} \sin^2\theta\, d\omega$

$= \pi R^2$

31. Solution: (d)

We will check value of function at x=1 and x=0,

At $x = 1, |x| = 1 \Rightarrow f(x) = |1-1| = 0$ & at $x = 0$, $|x| = o \Rightarrow f(0) = |0-1| = 1$

We can see only option (4) is satisfying the condition.

32. Solution: (d)

Here we will check curl of each function

(a) $\nabla \times E = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix} = i(0-0) - j(0-0) + \hat{k}(0-0) = 0$

(b) $\nabla \times E = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y+z & x+z & x+z \end{vmatrix} = i(1-1) - j(1-1) + \hat{k}(1-1) = 0$

(c) $\nabla \times E = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 & (2xy+z^2) & 2yz \end{vmatrix} = \hat{\imath}(2z-2z) - \hat{\jmath}(0-0) + \hat{k}(2y-2y) = 0$

(d) $\nabla \times E = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & 2yz & 3xz \end{vmatrix} = i(0-2y) - j(3z-0) + \hat{k}(0-x) \neq 0$

33. Solution: $-\frac{\hat{r}}{r^3} = \frac{\vec{r}}{r^4}$

$\vec{\nabla}.\left(\phi.\vec{A}\right) = \phi\left(\vec{\nabla}.\vec{A}\right) + \vec{A}(\vec{\nabla}.\phi)$

$= \frac{1}{r^4}\left(\vec{\nabla}.\vec{r}\right) + \vec{r}(\vec{\nabla}.\frac{1}{r^4})$

$= \frac{3}{r^4} + \vec{r}\left(-\frac{4}{r^5}\right) \Rightarrow \frac{3}{r^4} + r\hat{r}\left(-\frac{4}{r^5}\right)$

$= \frac{3}{r^4} - \frac{4}{r^4} = -\frac{1}{r^4}$

Therefore, correct option is (a).

34. Solution:- For field to be conservative $\left(\vec{\nabla} \times \vec{F}\right) = 0$

$\frac{1}{s}\begin{vmatrix} \hat{s} & s\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial s} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ \frac{A}{s} & 0 & \frac{B}{s} \end{vmatrix}$

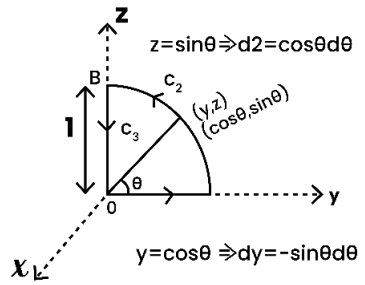
$\Rightarrow \frac{1}{s}\left[\hat{s}(0) - s\hat{\phi}\left(-\frac{B}{s} - 0\right) + \hat{z}(0)\right] = \frac{1}{s} \times \hat{\phi}\frac{B}{s} = \frac{B}{s^2}\hat{\phi}$

$= 0$

$\because B = 0$

Therefore, correct option is (a).

35. Solution:-



$\vec{v} = 2\hat{x} + yz^2\hat{y} + (3y + z^2)\hat{z}$

$\oint_c \vec{v}.d\vec{r} = \int_{c_1} + \int_{c_2} + \int_{c_3} dr$

$\vec{v}.d\vec{r} = 2dx + yz^2dy + (3y + z^2)dz$

$on\ c_1, x = 0, z = 0 \Rightarrow dx = 0, dz = 0$

$\vec{v}.d\vec{r} = 0$

$on\ c_2, \Rightarrow x = 0, dx = 0$

$\vec{F}.d\vec{r} = yz^2dy + (3y + z^2)dz$

$= cos\theta \sin^2\theta\,(-sin\theta)d\theta + (3cos\theta + \sin^2\theta)\, cos\theta d\theta$

$= \sin^3\theta\ cos\theta d\theta + 3\cos^2\theta + \sin^2\theta\ cos\theta d\theta$

$\int_{c_2} \vec{v}.d\vec{r} = -\int_0^{\pi/2} \sin^3\theta\ cos\theta d\theta + 3\int_0^{\pi/2} \cos^2\theta\, d\theta +$

$\int_0^{\pi/2} \sin^2\theta\, cos\theta\, d\theta$

$-\frac{\sin^4\theta}{4}\Big|_0^{\frac{\pi}{2}} + \frac{3\pi}{4} + \frac{\sin^3\theta}{3}\Big|_0^{\pi/2}$

$-\frac{1}{4} + \frac{3\pi}{4} + \frac{1}{3}$

$on\ c_3, x = 0, dx = 0, y = 0\ dy = 0$

$\vec{F}.d\vec{r} = z^2dz$

$\int_{c_3} \vec{v}.d\vec{r} = \int_1^0 z^2dz$

$\left[\frac{z^3}{3}\right]_1^0 = -\frac{1}{3}$

$\Rightarrow c_1 + c_2 + c_3 = 0 - \frac{1}{4} + \frac{3\pi}{4} + \frac{1}{3} - \frac{1}{3} = \frac{1}{4}(3\pi - 1)$

Therefore option (a) is correct.

36. Solution:- $dxdydz = \left|\frac{\partial(x,y,z)}{\partial(u,v\,w)}\right| dudvdw$

$\frac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix} = \begin{vmatrix} 2 & 3 & -1 \\ 1 & -4 & 1 \\ 1 & 1 & 0 \end{vmatrix}$

$2(-1) - 3(-1) - 1(1+4) = 0 \Rightarrow -4$

$\left|\frac{\partial(x,y,z)}{\partial(u,v\,w)}\right| = \frac{1}{\frac{\partial(u,v,w)}{\partial(x,y,z)}} = -\left(\frac{1}{4}\right) = -0.25$

Therefore, the answer is -0.25.

37. Solution: - *Two sides of triangle OAB are given*

$\overrightarrow{OA} = \hat{x} + \hat{y} + \hat{z}$ *and* $\overrightarrow{OB} = 2\hat{x} - \hat{y} + 3\hat{z}$

Area of triangle $= \frac{1}{2}\left|\overrightarrow{OA} \times \overrightarrow{OB}\right|$ … (1)

$$\overrightarrow{OA} \times \overrightarrow{OB} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & 2 & 1 \\ 2 & -1 & 3 \end{vmatrix}$$

$\hat{x}(6+1) - \hat{y}(3-2) + \hat{z}(-1-4) = 7\hat{x} - \hat{y} - 5\hat{z}$

$\left|\overrightarrow{OA} \times \overrightarrow{OB}\right| = \sqrt{49+1+25} = \sqrt{75}$

Put this value in equation (1) then we get,

$= \frac{1}{2}\sqrt{75} = 4.33$

Therefore, correct answer is 4.33

38. Solution: (c)

$$\mathrm{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$$

Gauss's Divergence Theorem.

$$\oiint_S \mathrm{r} \cdot \hat{n}\, dS = \int_V (\nabla \cdot \mathrm{r})\, dV$$

$$\nabla \cdot \mathrm{r} = \frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial z} = 1 + 1 + 1 = 3$$

$$\oiint_S \mathrm{r} \cdot \hat{n}\, dS = \int_V 3\, dV = 3\mathrm{v}$$

$$\oiint_S \mathrm{r} \cdot \hat{n}\, dS = 3V$$

$$\frac{1}{9}\oiint_S \mathrm{r} \cdot \hat{n}\, dS = \frac{1}{9} \times 3V = \frac{V}{3}$$

39. Solution: (a)

The vector field A = y î means: All vectors point horizontally (along the x-axis, since î is the x-direction unit vector). Magnitude the length of each vector depends on y.

If y > 0, vectors point right (+x), If y < 0, vectors point left (−x)

At y = 0 (x-axis), vectors vanish (length = 0)

As we move upward along the positive y-axis, the vector field grows stronger, meaning the magnitude (length) of the vectors increases.

Option (a) is correct.

40. Solution: (d)

$$\mathrm{f}(\mathrm{x},\mathrm{y}) = \frac{\mathrm{x}}{\mathrm{a}}\mathrm{e}^{\mathrm{y}} + \frac{\mathrm{y}}{\mathrm{b}}\mathrm{e}^{\mathrm{x}}, \qquad \mathrm{x} = \mathrm{at} \text{ and } \mathrm{y} = \mathrm{bt}$$

$$\frac{\mathrm{df}}{\mathrm{dt}} = \frac{\partial \mathrm{f}}{\partial \mathrm{x}}\frac{\mathrm{dx}}{\mathrm{dt}} + \frac{\partial \mathrm{f}}{\partial \mathrm{y}}\frac{\mathrm{dy}}{\mathrm{dt}}$$

$$\frac{\partial \mathrm{f}}{\partial \mathrm{x}} = \frac{1}{\mathrm{a}}\mathrm{e}^{\mathrm{y}} + \frac{\mathrm{y}}{\mathrm{b}}\mathrm{e}^{\mathrm{x}}, \qquad \frac{\partial \mathrm{f}}{\partial \mathrm{y}} = \frac{\mathrm{x}}{\mathrm{a}}\mathrm{e}^{\mathrm{y}} + \frac{1}{\mathrm{b}}\mathrm{e}^{\mathrm{x}}$$

$$\frac{\mathrm{dx}}{\mathrm{dt}} = \mathrm{a}, \qquad \frac{\mathrm{dy}}{\mathrm{dt}} = \mathrm{b}$$

$$\frac{\mathrm{df}}{\mathrm{dt}} = \left(\frac{1}{\mathrm{a}}\mathrm{e}^{\mathrm{y}} + \frac{\mathrm{y}}{\mathrm{b}}\mathrm{e}^{\mathrm{x}}\right)\mathrm{a} + \left(\frac{\mathrm{x}}{\mathrm{a}}\mathrm{e}^{\mathrm{y}} + \frac{1}{\mathrm{b}}\mathrm{e}^{\mathrm{x}}\right)\mathrm{b}$$

$$\frac{\mathrm{df}}{\mathrm{dt}} = \mathrm{e}^{\mathrm{bt}} + \mathrm{ate}^{\mathrm{at}} + \mathrm{bte}^{\mathrm{bt}} + \mathrm{e}^{\mathrm{at}}$$

$$\frac{\mathrm{df}}{\mathrm{dt}}\Big|_{\mathrm{t}=0} = 1 + 0 + 0 + 1 = 2$$

41. Solution: (d)

$$u(x,y) = \frac{1}{2}(x^2 - y^2) \quad , v(x,y) = xy$$

$$\frac{\partial(u,v)}{\partial(r,\theta)} = \frac{\partial(u,v)}{\partial(x,y)} \cdot \frac{\partial(x,y)}{\partial(r,\theta)}$$

$$\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} x & -y \\ y & x \end{vmatrix} = x^2 + y^2 = r^2$$

$$\frac{\partial(x,y)}{\partial(r,\theta)} = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} = r\cos^2\theta + r\sin^2\theta = r$$

$\frac{\partial(u,v)}{\partial(r,\theta)} = r^2 . r = r^3$

42. Solution: (1)

For a closed curve C enclosing region D, Green's Theorem states:

$$\oint \mathrm{F} \cdot d\mathrm{r}\, \mathrm{F} = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\, dy$$

$$P = -\frac{\sin y}{\pi}, Q = \frac{x(1-\cos y)}{\pi}$$

$$\frac{\partial Q}{\partial x} = \frac{1-\cos y}{\pi}, \frac{\partial P}{\partial y} = -\frac{\cos y}{\pi}$$

$$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = \frac{1-\cos y}{\pi} - \left(-\frac{\cos y}{\pi}\right) = \frac{1}{\pi}$$

$$\iint_D \frac{1}{\pi}\, dx\, dy = \frac{1}{\pi} \times \text{Area of } circle$$

For the unit circle $x^2 + y^2 \leq 1$, the area is π:

$$\frac{1}{\pi} \times \pi = 1$$

43. Solution: (a)

For $\vec{V} = (0, 0, xyz)$: $\vec{\nabla} \times \vec{V} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & xyz \end{vmatrix}$

$$= \hat{\imath}\left(\frac{\partial}{\partial y}(xyz) - \frac{\partial}{\partial z}(0)\right) - \hat{\jmath}\left(\frac{\partial}{\partial x}(xyz) - \frac{\partial}{\partial z}(0)\right) + \hat{k}\left(\frac{\partial}{\partial x}(0) - \frac{\partial}{\partial y}(0)\right)$$

$$= \hat{\imath}(xz) - \hat{\jmath}(yz) + \hat{k}(0) = xz\,\hat{\imath} - yz\,\hat{\jmath}$$

Apply the determinant curl formula again, to $\vec{W} = (xz, -yz, 0)$:

$$\vec{\nabla} \times \vec{W} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xz & -yz & 0 \end{vmatrix}$$

$$= \hat{\imath}\left(\frac{\partial}{\partial y}(0) - \frac{\partial}{\partial z}(-yz)\right) - \hat{\jmath}\left(\frac{\partial}{\partial x}(0) - \frac{\partial}{\partial z}(xz)\right) + \hat{k}\left(\frac{\partial}{\partial x}(-yz) - \frac{\partial}{\partial y}(xz)\right)$$

$$= \hat{\imath}(0 - (-y)) - \hat{\jmath}(0 - x) + \hat{k}(0 - 0) = y\,\hat{\imath} + x\,\hat{\jmath} + 0\,\hat{k}$$

$\vec{\nabla} \times (\vec{\nabla} \times \vec{V}) = y\,\hat{\imath} + x\,\hat{\jmath}$

Answer: (A) It lies in the xy -plane.