P R E F A C E

"IIT JAM MATHEMATICS (2005 to 2025 Topic Wise Questions with Detailed Solutions)" is not only used for IIT JAM aspirants but also useful for CUET, TIFR, NBHM, GATE, Autonomous, Deemed, Private, State, and Central University Entrance Exams.

This book contains all the previous year question of IIT JAM MATHEMATICS which is topic-wise & year-wise sorted with detailed solution. It will help the reader to practice questions from a particular topic at one place. This book relates the present study to its wider context, which would serve as a pedestal upon which the remaining part of the study will be built.

It assumes that the student has had introductory mathematics as a pre-requisite. The book covers all the topic namely Real Analysis, Group Theory, Linear Algebra, Integral Equations & Ordinary Differential Equations.

Simple language is used to clarify important facts. The book aims to develop confidence among the students who are appearing in different examination having common syllabus. Although I have made every effort to make the book error free, I am under no illusion. I welcome comments, criticism and suggestions from the readers to evolve the contents of this book.

A C K N O W L E D G E M E N T

Writing this book **"IIT JAM MATHEMATICS (2005 to 2025 Topic Wise Questions with Detailed Solutions)"** has been an incredible journey. All the chapters have taken a final shape after endless inputs of time and effort. Though many colleagues assisted me in compiling this book, I first of all would like to thank all students at Institute for advanced Studies (IFAS), who have helped me to learn and implement the concepts and ideas of Mathematics in its simplest form.

I would like to specially mention the effort made by Mr. Radheshyam Choudhary, Director IFAS Publications, who extended immense support in myriad ways for bringing out the book in its present form.

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IIT JAM Entrance Exam Details

Joint Admission Test (JAM) is an all India admission test conducted jointly by the Indian Institute of Technology (IITs) and Indian Institute of Science (IISC) since 2004 to provide admissions to M.Sc. (Four Semesters), Joint M.Sc.-Ph.D., M.Sc. - Ph.D. Dual Degree.

To apply for the topmost IIT MSc entrance exam, an aspirant must fulfill the following requirements:

- All candidates admitted through JAM should have a Bachelor's degree.
- The candidates must PASS the qualifying degree examination. In case if a candidate is promoted without a mark sheet, the promotion certificate or a document accompanying the promotion certificate duly signed by the Head of the Institute must mention the subjects taken in that semester/year to evaluate the MEQ requirements for admission to a specific program.
- The medium of instruction is English.
- IISc Bangalore, IISER Bhopal, IISER Pune, IISER Tirupati and Centralized Counselling for M.Sc./M.Sc.(Tech) for NITs (CCMN) Institutes may be using the JAM 2022 results for admission to their postgraduate programmes.

IIT-JAM Exam Pattern

Sections	Section A	Section B	Section C
Number of Questions	30	10	20
Marks	10 questions of 1 mark each and 20 questions of 2 marks each.		10 questions of 1 mark each and 10 questions of 2 marks each.
Negative Marking	For 1 mark questions - $\frac{1}{3}$ marks will be deducted; For 2 marks questions - - $\frac{2}{3}$ marks will be de ducted.	No negative marking	No negative marking
Types of Question	Multiple Choice Questions	Multiple Select Questions	No options will be displayed
Total number of Question	**60**		
Total Marks	**100**		

IIT JAM MATHEMATICS (MA) SYLLABUS

S. No	Real Analysis
1	**Sequences and Series of Real Numbers:** Convergence of sequences, bounded and monotone sequences, Cauchy sequences, Bolzano-Weierstrass theorem, absolute convergence, tests of convergence for series - comparison test, ratio test, root test; Power series (of one real variable), radius and interval of convergence, term-wise differentiation and integration of power series.
2	**Functions of One Real Variable:** Limit, continuity, intermediate value property, differentiation, Rolle's Theorem, mean value theorem, L'Hospital rule, Taylor's theorem, Taylor's series, maxima and minima, Riemann integration (definite integrals and their properties), fundamental theorem of calculus.

S. No	Multivariable Calculus and Differential Equations
1	**Functions of Two or Three Real Variables:** Limit, continuity, partial derivatives, total derivative, maxima and minima.
2	**Integral Calculus:** Double and triple integrals, change of order of integration, calculating surface areas and volumes using double integrals, calculating volumes using triple integrals.
3	**Differential Equations:** Bernoulli's equation, exact differential equations, integrating factors, orthogonal trajectories, homogeneous differential equations, method of separation of variables, linear differential equations of second order with constant coefficients, method of variation of parameters, Cauchy-Euler equation.

S. No	Linear Algebra and Algebra
1	**Matrices:** Systems of linear equations, rank, nullity, rank-nullity theorem, inverse, determinant, eigenvalues, eigenvectors.
2	**Finite Dimensional Vector Spaces:** Linear independence of vectors, basis, dimension, linear transformations, matrix representation, range space, null space, rank-nullity theorem.
3	**Groups:** Cyclic groups, abelian groups, non-abelian groups, permutation groups, normal subgroups, quotient groups, Lagrange's theorem for finite groups, group homomorphisms.

IIT JAM PAPER ANALYSIS

Subject	2024		2023		2022		2021		2020	
	No. of Ques.	Marks	No. of Ques.	Marks	No. of Ques.	Marks	No. of Ques.	Marks	No. of Ques.	Marks
Real Analysis	22	42	20	33	14	23	23	38	19	31
Group Theory	9	14	8	13	7	11	10	17	6	10
Linear Algebra	12	19	12	21	13	23	11	19	9	16
Ordinary Differential Equations	10	14	7	12	9	14	6	10	8	13
Integral Calculus	6	10	7	11	9	14	3	4	5	9
Vector Calculus	0	0	0	0	0	0	4	6	8	13
Function of Several Variable	0	0	6	10	8	13	3	6	5	8

Subject	2019		2018		2017		2016		2015	
	No. of Ques.	Marks	No. of Ques.	Marks	No. of Ques.	Marks	No. of Ques.	Marks	No. of Ques.	Marks
Real Analysis	20	35	20	34	21	35	20	34	21	35
Group Theory	6	11	8	13	6	9	8	13	6	9
Linear Algebra	9	15	8	13	9	15	8	13	9	15
Ordinary Differential Equations	6	10	6	12	6	11	6	12	6	11
Integral Calculus	7	9	7	11	8	13	7	11	8	13
Vector Calculus	6	10	6	9	6	12	6	9	6	12
Function of Several Variable	6	10	5	8	4	5	5	8	4	5

IIT JAM CUT-OFF					
Year	**Candidates Appeared**	**General**	**EWS/OBS-NCL**	**SC / ST / PwD**	**Organizing Institute**
2025	12079	19.9	17.91	9.95	IIT DELHI
2024	14146	11.22	10.09	5.61	IIT MADRAS
2023	13333	24.56	22.10	12.28	IIT GUWAHATI
2022	12716	22.91	20.62	11.46	IIT ROORKEE
2021	13186	24.69	22.22	12.35	IIT BANGALORE
2020	14734	33.65	30.29	16.82	IIT KANPUR
2019	12863	22.96	20.66	11.48	IIT KHARAGPUR
2018	11583	16.84	15.16	08.42	IIT BOMBAY
2017	10834	19.51	17.56	09.76	IIT DELHI
2016	9999	27.67	24.90	13.84	IIT MADRAS

I N D E X

IIT JAM MATHEMATICS - PYQ

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SECTION - I
REAL ANALYSIS

IIT JAM MATHEMATICS: NUMBER OF QUESTIONS ASKED IN PREVIOUS EXAMS (2005-2025)																					
REAL ANALYSIS		2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024	2025
1	Point Set Topology	-	1	-	2	1	1	1	1	1	3	4	4	2	3	4	-	7	-	-	-	-
2	Sequences of Real Numbers	1	1	1	1	-	1	-	-	1	2	3	3	4	3	4	4	3	2	4	3	5
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4	Function & Their Properties	-	-	-	-	-	-	1	1	-	1	2	1	-	1	2	2	7	3	8	2	1
5	Continuity & Uniform Continuity	1	-	1	-	-	1	-	-	2	-	1	1	2	2	-	2	-	-	2	4	3
6	Differentiability	1	2	1	-	-	-	-	-	-	3	3	2	4	1	3	-	1	-	4	4	4
7	Sequence & Series of Functions & Power Series	1	-	1	2	2	1	1	1	-	2	1	-	1	-	2	2	-	5	1	-	-
8	Function of Several Variables	1	-	1	-	-	-	-	1	-	3	3	4	4	2	4	5	-	5	4	3	4

CHAPTER-1 POINT SET TOPOLOGY

(MCQ) Multiple Choice Questions

[2006: 6 Marks]

1. Let G be the set of all irrational numbers. The interior and the closure of G are denoted by $G°$ and $\overline{G}$, respectively. Then
   (a) $G° = \phi,\ \overline{G} = G$ (b) $G° = \mathbb{R},\ \overline{G} = \mathbb{R}$
   (c) $G° = \phi,\ \overline{G} = \mathbb{R}$ (d) $G° = G,\ \overline{G} = \mathbb{R}$

[2008: 6 Marks]

2. The set of all boundary point of $\mathbb{Q}$ in $\mathbb{R}$ is
   (a) $\mathbb{R}$ (b) $\mathbb{R}/\mathbb{Q}$
   (c) $\mathbb{Q}$ (d) ϕ

[2008: 6 Marks]

3. The set $U = \left\{x \in \mathbb{R} | \sin x = \frac{1}{2}\right\}$ is
   (a) Open
   (b) Closed
   (c) Both open and closed
   (d) Neither open nor closed

[2009: 6 Marks]

4. The set of all limit points of the sequence $1, \frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}, \frac{1}{16}, \frac{3}{16}, \frac{5}{16}, \frac{7}{16}, \frac{9}{16}, \ldots$ is
   (a) $[0,1]$
   (b) $(0,1]$
   (c) The set of all rational numbers in $[0,1]$
   (d) The set of all rational numbers in $[0,1]$ of the form $\frac{m}{2^n}$ where m and n are integers.

[2010: 6 Marks]

5. Let S be an infinite subset of $\mathbb{R}$ such that $S \cap \mathbb{Q} = \phi$. Which of the following statements is true?
   (a) S must have a limit point which belongs to $\mathbb{Q}$
   (b) S must have a limit point which belongs to $\mathbb{R}/\mathbb{Q}$
   (c) S cannot be a closed set in $\mathbb{R}$
   (d) $\mathbb{R}/S$ must have a limit point which belongs to S

[2011: 6 Marks]

6. Consider the following subsets of $\mathbb{R}$:
   $E = \left\{\frac{n}{n+1} : n \in \mathbb{N}\right\}, F = \left\{\frac{1}{1-x} : 0 \le x < 1\right\}$
   Then
   (a) Both E and F are closed
   (b) E is closed and F is NOT closed
   (c) E is NOT closed and F is closed
   (d) Neither E nor F is closed

[2012: 6 Marks]

7. If $Y = \left\{\frac{x}{1+|x|} : x \in \mathbb{R}\right\}$ then the set of all limit points of Y is
   (a) $(-1,1)$ (b) $(-1,1]$
   (c) $[0,1]$ (d) $[-1,1]$

[2013: 2 Marks]

8. Let A and B be subsets of $\mathbb{R}$. Which of the following is NOT necessarily true?
   (a) $(A \cap B)° \subseteq A° \cap B°$
   (b) $A° \cup B° \subseteq (A \cup B)°$
   (c) $\overline{A} \cup \overline{B} \subseteq \overline{A \cup B}$
   (d) $\overline{A} \cap \overline{B} \subseteq \overline{A \cap B}$

[2014: 2 Marks]

9. The se $\left\{\frac{x^2}{1+x^2} : x \in \mathbb{R}\right\}$ is
   (a) Connected but NOT compact in $\mathbb{R}$
   (b) Compact but NOT connected in $\mathbb{R}$
   (c) Compact and connected in $\mathbb{R}$
   (d) Neither compact nor connected in $\mathbb{R}$

[2014: 2 Marks]

10. The set of all limit points of the set $\left\{\frac{2}{x+1} : x \in (-1,1)\right\}$ in $\mathbb{R}$ is
    (a) $[1,\infty)$ (b) $(1,\infty)$
    (c) $[-1,1]$ (d) $[-1,\infty)$

[2014: 2 Marks]

11. Let $S = \{x \in \mathbb{R} : x^6 - x^5 \le 100\}$ and $T = \{x^2 - 2x : x \in (0,\infty)\}$. The set $S \cap T$ is
    (a) Closed and bounded in $\mathbb{R}$
    (b) Closed but NOT bounded in $\mathbb{R}$
    (c) Bounded but NOT closed in $\mathbb{R}$
    (d) Neither closed nor bounded in $\mathbb{R}$

[2015: 1 Mark]

12. Let A be a non-empty subset of $\mathbb{R}$. Let $I(A)$ denote the set of interior points of A. Then $I(A)$ can be
    (a) Empty
    (b) Singleton
    (c) A finite set containing more than one element
    (d) Countable but not finite

[2015: 1 Mark]

13. Let S be a non-empty subset of $\mathbb{R}$. If S is a finite union of disjoint bounded intervals, then which one of the following is true?
    (a) If S is not compact, then sup $S \notin S$ and inf $S \notin S$
    (b) Even if sup $S \in S$ and inf $S \in S$, S need not be compact
    (c) If sup $S \in S$ and inf $S \in S$, then S is compact
    (d) Even if S is compact, it is not necessary that sup $S \in S$ and inf $S \in S$

[2015: 2 Marks]

14. Let $S = \bigcap_{n=1}^{\infty} \left(\left[0, \frac{1}{2n+1}\right] \cup \left[\frac{1}{2n}, 1\right] \right)$
    Which one of the following statements is FALSE?
    (a) There exist sequences $\{a_n\}$ and $\{b_n\}$ in $[0,1]$ such that $S = [0,1] \setminus \cup_{n=1}^{\infty}(a_n, b_n)$
    (b) $[0,1] \setminus S$ is an open set
    (c) If A is an infinite subset of S, then A has a limit point
    (d) There exists an infinite subset of S having no limits points

[2015: 2 Marks]

15. Let G and H be non-empty subsets of $\mathbb{R}$, where G is connected and $G \cup H$ is not connected. Which one of the following statements is true for all such G and H?
    (a) If $G \cap H = \phi$, then H is connected
    (b) If $G \cap H = \phi$, then H is not connected
    (c) If $G \cap H \neq \phi$ then H is connected
    (d) If $G \cap H \neq \phi$ then H is not connected

[2016: 1 Mark]

16. Let S be a closed subset of $\mathbb{R}$, T a compact subset of $\mathbb{R}$ such that $S \cap T \neq \phi$. Then $S \cap T$ is
    (a) Closed but not compact
    (b) Not closed
    (c) Compact
    (d) Neither closed nor compact

[2016: 2 Marks]

17. Let $S \subset \mathbb{R}$ and ∂S denote the set of points x in $\mathbb{R}$ such that every neighbourhood of x contains some points of S as well as some points of complement of S. Further, let $\overline{S}$ denote the closure of S. Then which one of the following is FALSE?
    (a) $\partial\mathbb{Q} = \mathbb{R}$
    (b) $\partial(\mathbb{R}/T) = \partial T, T \subset \mathbb{R}$
    (c) $\partial(T \cup V) = \partial T \cup \partial V$, $T, V \subset \mathbb{R}, T \cap V \neq \phi$
    (d) $\partial T = \overline{T} \cap \left(\overline{\mathbb{R}/T}\right), T \subset \mathbb{R}$

[2017: 2 Marks]

18. Let S be an infinite subset of $\mathbb{R}$ such that $S \setminus \{\alpha\}$ is compact for some $\alpha \in S$.
    Then which one of the following is TRUE?
    (a) S is a connected set
    (b) S contains no limit points
    (c) S is a union of open intervals
    (d) Every sequence in S has a subsequence converging to an element in S.

[2019: 1 Mark]

19. Let S be the set of all limit points of the set $\left\{\frac{n}{\sqrt{2}} + \frac{\sqrt{2}}{n} : n \in \mathbb{N}\right\}$. Let $\mathbb{Q}^+$ be the set of all positive rational numbers. Then
    (a) $\mathbb{Q}^+ \subseteq S$ (b) $S \subseteq \mathbb{Q}^+$
    (c) $S \cap (\mathbb{R}/\mathbb{Q}^+) \neq \phi$ (d) $S \cap \mathbb{Q}^+ \neq \phi$

[2019: 2 Marks]

20. The set $\left\{\frac{x}{1+x} : -1 < x < 1\right\}$, as a subset of $\mathbb{R}$ is
    (a) Connected and compact
    (b) Connected but not compact
    (c) Not connected but compact
    (d) Neither connected nor compact

[2019: 2 Marks]

21. The set $\left\{\frac{1}{m} + \frac{1}{n} : m, n \in \mathbb{N}\right\} \cup \{0\}$, as a subset of $\mathbb{R}$ is
    (a) Compact and open
    (b) Compact but not open
    (c) Not compact but open
    (d) Neither compact nor open

[2021: 2 Marks]

22. Define
    $S = \lim_{n\to\infty} \left(1 - \frac{1}{2^2}\right) \left(1 - \frac{1}{3^2}\right) \ldots \left(1 - \frac{1}{n^2}\right)$.
    Then
    (a) $S = 1/2$. (b) $S = 1/4$.
    (c) $S = 1$. (d) $S = 3/4$.

(MSQ) Multiple Select Questions

[2016: 2 Marks]

1. Which of the following statement(s) is (are) TRUE?
    (a) There exists a connected set in $\mathbb{R}$ which is not compact
    (b) Arbitrary union of closed intervals in $\mathbb{R}$ need not be compact
    (c) Arbitrary union of closed intervals in $\mathbb{R}$ is always closed
    (d) Every bounded infinite subset V of $\mathbb{R}$ has a limit point in V itself

[2016: 2 Marks]

2. Let $S = \left\{\frac{1}{3^n} + \frac{1}{7^m} \mid n, m \in \mathbb{N}\right\}$
Then which of the following statement(s) is (are) TRUE?
(a) S is closed
(b) S is not open
(c) S is connected
(d) 0 is a limit point of S

[2017: 2 Marks]

3. Let S be the set of all rational numbers in (0,1). Then which of the following statements is/are TRUE?
(a) S is a closed subset of $\mathbb{R}$
(b) S is not a closed subset of $\mathbb{R}$
(c) S is an open subset of $\mathbb{R}$
(d) Every $x \in (0,1)\backslash S$ is a limit point of S

[2018: 2 Marks]

4. Which of the following subsets of $\mathbb{R}$ is (are) connected?
(a) $\{x \in \mathbb{R} \mid x^2 + x > 4\}$
(b) $\{x \in \mathbb{R} \mid x^2 + x < 4\}$
(c) $\{x \in \mathbb{R} \mid |x| < |x - 4|\}$
(d) $\{x \in \mathbb{R} \mid |x| > |x - 4|\}$

[2018: 2 Marks]

5. Let P and Q be two non-empty disjoint subsets of $\mathbb{R}$. Which of the following is (are) FALSE?
(a) If P and Q are compact, then $P \cup Q$ is also compact
(b) If P and Q are not connected, then $P \cup Q$ is also not connected
(c) If $P \cup Q$ and P are closed, then Q is closed
(d) If $P \cup Q$ and P are open, then Q is open

[2018 2 Marks]

6. Let S be a subset of $\mathbb{R}$ such that 2018 is an interior point of S. Which of the following is (are) TRUE?
(a) S contains an interval
(b) There is sequence in S which does not converge to 2018
(c) There is element $y \in S$, $y \neq 2018$ such that y is also an interior point of S
(d) There is a point $z \in S$, such that $|z - 2018| = 0.002018$

[2019: 2 Marks]

7. Consider the intervals $S = (0,2]$ and $T = [1,3)$. Let $S°$ and $T°$ be the sets of interior points of S and T, respectively. Then the set of interior points of $S\backslash T$ is equal to
(a) $S\backslash T°$ (b) $S\backslash T$
(c) $S°\backslash T°$ (d) $S°\backslash T$

[2021: 2 Marks]

8. Which of the following subsets of $\mathbb{R}$ is/are connected?
(a) The set $\{x \in \mathbb{R}: x \text{ is irrational}\}$.
(b) The set $\{x \in \mathbb{R}: x^3 - 1 \geq 0\}$.
(c) The set $\{x \in \mathbb{R}: x^3 + x + 1 \geq 0\}$.
(d) The set $\{x \in \mathbb{R}: x^3 - 2x + 1 \geq 0\}$.

[2021: 2 Marks]

9. Consider the four functions from $\mathbb{R}$ to $\mathbb{R}$:
$f_1(x) = x^4 + 3x^3 + 7x + 1$, $f_2(x) = x^3 + 3x^2 + 4x$, $f_3(x) = \arctan(x)$
and
$$f_4(x) = \begin{cases} x & if\ x \in \mathbb{Z}. \\ 0 & if\ x \in \mathbb{Z}. \end{cases}$$
Which of the following subsets of $\mathbb{R}$ are open?
(a) The range of f_1. (b) The range of f_2.
(c) The range of f_3. (d) The range of f_4.

NAT (Numerical Answer Type)

[2021: 1 Mark]

1. Consider the set $A = \{a \in \mathbb{R}: x^2 = a(a+1)(a+2)$ has a real root$\}$. The number of connected components of A is _____.

[2021: 1 Mark]

2. The value of
$$\frac{\pi}{2} \lim_{n\to\infty} \cos\left(\frac{\pi}{4}\right) \cos\left(\frac{\pi}{8}\right) \ldots \cos\left(\frac{\pi}{2^{n+1}}\right)$$
is _____.

[2021: 2 Marks]

3. The value of
$$\lim_{n\to\infty} \int_0^1 e^{x^2} \sin(nx)\, dx$$
is _____.

[2021: 1 Mark]

4. The value of
$$\lim_{n\to\infty} (3^n + 5^n + 7^n)^{1/n}$$
is _____.

Answer Key									
Multiple Choice Questions (MCQ)									
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
(c)	(c)	(b)	(a)	(d)	(c)	(d)	(d)	(a)	(a)
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
(a)	(a)	(b)	(d)	(d)	(c)	(c)	(d)	(b)	(b)
21	**22**								
(b)	(a)								
Multiple Select Questions (MSQ)									
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	
(a)(b)	(b)(d)	(b)(d)	(b)(c)(d)	(b)(c)(d)	(a)(b)(c)	(b)(d)	(c)(b)	(b)(c)(d)	
Numerical Answer Type (NAT)									
1	**2**	**3**	**4**						
2	1	0	7						

:: SOLUTIONS ::

(MCQ) Multiple Choice Questions

1. Solution: (c)

Given,

$G = \mathbb{Q}^c$

As we know,

For any neighbourhood of point $p \in \mathbb{Q}^c$

$\exists$ infinitely many rational numbers belong to that neighbourhood.

$\therefore \mathbb{Q}^c$ does not contain any neighbourhood of point $p \in \mathbb{Q}^c$

$\therefore G^\circ = \phi$.

Also, we know that,

$(\mathbb{Q}^c)' = \mathbb{R}$

$\therefore \overline{G} = G \cup \mathbb{R} = \mathbb{R}$

2. Solution: (c)

As we know,

$Fr(\mathbb{Q}) = \mathbb{R}$

$\therefore \delta(\mathbb{Q}) = Fr(\mathbb{Q}) \cap \mathbb{Q}$

$= \mathbb{R} \cap \mathbb{Q}$

$= \mathbb{Q}$

3. Solution: (b)

$\sin x = \frac{1}{2}$

$\Rightarrow \sin x = \sin\frac{\pi}{6}$

$\therefore x = n\pi + (-1)^n \frac{\pi}{6}, n \in \mathbb{Z}$

$\therefore$ set U contains isolated points.

$\therefore (U)' = \phi$

$\therefore U$ is closed

4. Solution: (a)

For any point $p \in [0,1]$ infinitely many points in neighbourhood of p lies in $[0,1]$

$\therefore$ limit points $= [0,1]$

5. Solution: (d)

Discard by examples

For (a) $S = \left\{\sqrt{2} + \left\{\frac{1}{n}\right\}\right\}, n \in \mathbb{N}$

For (b) $S = \left\{1 + \frac{\sqrt{2}}{n}, n \in \mathbb{N}\right\}$

For (c) If $S = \left\{\sqrt{2} + \frac{1}{n}\right\} \cup \left\{\{\sqrt{2}\}\right\}, n \in \mathbb{N}$

6. Solution: (c)

Given, $E = \left\{\frac{n}{n+1}, n \in \mathbb{N}\right\}$

1 is limit point which not belongs to E

$\therefore E$ is not closed.

Also, $F = \left\{\frac{1}{1-x}, 0 \le x < 1\right\}$

$\because 0 \leq x < 1$

$\Rightarrow 0 \geq -x > -1$

$\Rightarrow 1 \geq 1 - x > 0$

$\Rightarrow 1 \leq \frac{1}{1-x} < \infty$

$f = [1, \infty]$

f contains all of its limit points

Hence, f is closed set

7. Solution: (d)

$$Y = \begin{cases} \frac{x}{1+x}, & x > 0 \\ \frac{x}{1-x}, & x < 0 \end{cases}$$

$\therefore Y = (-1,1)$

$\therefore$ limit point of $Y = [-1,1]$

8. Solution: (d)

(d) is not necessarily true.

Take example,

$A = (0,1), B = (1,2)$

$\therefore \overline{A} \cap \overline{B} = [0,1] \cap [1,2] = \{1\}$

But, $A \cap B = \phi$

$\therefore \overline{A} \cap \overline{B} \nsubseteq \overline{(A \cap B)}$

9. Solution: (a)

Given, $S = \left\{\frac{x^2}{1+x^2}, x \in \mathbb{R}\right\}$

As we know, $x^2 \geq 0$

$\therefore 1 + x^2 \geq 1$

$\therefore \frac{x^2}{1+x^2} \geq 0$

Also, $\lim_{x \to \infty} \frac{x^2}{1+x^2} = 1$

$\therefore 0 \leq S < 1$

$\therefore S$ is connected but not compact in $\mathbb{R}$

$\therefore (a)$ is true.

10. Solution: (a)

$x \in (-1,1)$

$-1 < x < 1$

$0 < x + 1 < 2$

$\infty > \frac{2}{x+1} > 1$

$S \in (1, \infty)$

$\therefore S' = [1, \infty]$

11. Solution: (a)

Given, $S = \{x \in \mathbb{R}: x^6 - x^5 \geq 100\}$

Clearly S is closed and bounded above.

And

$T = \{x^2 - 2x: x \in (0, \infty)\}$

$= \{(x-1)^2 - 1: x \in (0, \infty)\}$

$\because 0 < x < \infty$

$-1 < x - 1 < \infty$

$0 \leq (x-1)^2 < \infty$

$-1 \leq (x-1)^2 - 1 < \infty$

$\therefore T = [-1, \infty]$

$\therefore T$ is closed and bounded below

$\therefore S \cap T$ is closed and bounded.

12. Solution: (a)

For any point $p \in A, A \neq \phi$ if p is interior of A then, $(p - \delta, p + \delta) \subset A$, for some $\delta > 0$

$\therefore A$ must be uncountable

13. Solution: (b)

If, $S = [0,1] \cup (2,3]$

Sup $(S) = 3 \in S$

And inf $(S) = 0 \in S$

But S is not compact

$\therefore (a)$ and (c) are incorrect and (b) is correct

Also, if S is compact then supremum and infimum must belong to the set

$\therefore (d)$ is incorrect.

14. Solution: (d)

Given, $S_1 = \bigcap_{n=1}^{\infty} T_n$

Where, $T_n = \left[0, \frac{1}{2n+1}\right] \cup \left[\frac{1}{2n}, 1\right]$

$\therefore T_1 = \left[0, \frac{1}{3}\right] \cup \left[\frac{1}{2}, 1\right]$

$T_2 = \left[0, \frac{1}{5}\right] \cup \left[\frac{1}{4}, 1\right]$

$T_3 = \left[0, \frac{1}{7}\right] \cup \left[\frac{1}{6}, 1\right]$

.

.

$T_n = \left[0, \frac{1}{2n+1}\right] \cup \left[\frac{1}{2n}, 1\right]$

as n tends to ∞, T_n become

$T = \{0\} \cup [0,1]$

$\therefore \cap T_n = \{0\} \cup \left[\frac{1}{2}, 1\right]$

$\therefore S = \{0\} \cup \left[\frac{1}{2}, 1\right]$

Clearly S is infinite but bounded subset of $\mathbb{R}$

$\therefore$ By Bolzano weirstrass theorem. Every infinite subset of S has a limit point

$\therefore (d)$ is false.

15. Solution: (d)

Discard by example

(a) $G = \{1\}, H = \{2\} \cup \{3\}$

(b) $G = \{1\}, H = \{2\}$

(c) $G = \{1\}, H = [0,1] \cup \{2\}$

$\therefore$ option d is only correct

16. Solution: (c)

Given,

S is a closed subset of $\mathbb{R}$

And T is compact $\Rightarrow T$ is closed

$\therefore S \cap T$ is closed

Also T is bounded

$\therefore S \cap T$ is bounded also

$\therefore (S \cap T)$ is closed + bounded

$\Rightarrow S \cap T$ is compact

17. Solution: (c)

Given definition is definition of Frontier points

And $\partial\mathbb{Q} = \mathbb{R}$

$\therefore$ option (a) is true

Option (b) is also true

But if we take,

$T = [0,1]$ and $V = [1,2]$

Then option (c) is false.

18. Solution: (d)

As, $S \backslash \alpha$ is compact for some $\alpha \in \mathbb{R}$

$\therefore \alpha$ is isolated point

$\therefore S$ can't be connected

$\therefore (a)$ is incorrect

If, $S = [a, b] \cup \{\alpha\}, a, b, \in \mathbb{R}\ a \neq b$

Then (b) is incorrect

And (c) is also incorrect

$\therefore$ (d) is correct

19. Solution: (b)

Given, $A = \left\{\frac{n}{\sqrt{2}} + \frac{\sqrt{2}}{n}, n \in \mathbb{N}\right\}$

$\therefore A' = \phi = S$

$\therefore$ option (b) is true.

Discard other options by taking examples

20. Solution: (b)

$S = \frac{n}{n+1}$

$= 1 - \frac{1}{n+1}$

Given, $-1 < n < 1$

$\Rightarrow 0 < n + 1 < 2$

$\Rightarrow \infty > \frac{1}{n+1} > \frac{1}{2}$

$\Rightarrow -\infty < -\frac{1}{n+1} < -\frac{1}{2}$

$\Rightarrow -\infty < 1 - \frac{1}{n+1} < \frac{1}{2}$

$\therefore S = \left(-\infty, \frac{1}{2}\right)$

Which is connected but not compact

21. Solution: (b)

Given,

$S = \left\{\frac{1}{m} + \frac{1}{n} : m, n \in \mathbb{N}\right\} \cup \{0\}$

$\therefore S' = \left\{\frac{1}{n}, n \in \mathbb{N}\right\} \cup \{0\}$

Clearly, $S' \subseteq S$

$\therefore S$ is closed

Also, $0 < S \leq \frac{1}{2}$

$\therefore S$ is closed + Bounded$\Rightarrow S$ is compact

But S can't contain any neighbourhood as S is countable

$\therefore S$ can't be open

$\therefore S$ is compact but not open

$\therefore$ (b) is correct.

22. Solution: (a)

$S = \lim_{n\to\infty} \left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right) \ldots \left(1 - \frac{1}{n^2}\right)$

$\lim_{n\to\infty} \left(\frac{3^2-1}{2^2}\right)\left(\frac{3^2-1}{3^2}\right) \ldots \left(\frac{n^2-1}{n^2}\right)$

$= \lim_{n\to\infty} \left(\frac{1\cdot3}{2^2}\right)\left(\frac{2\cdot4}{3^2}\right) \ldots \left(\frac{(n-1)(n+1)}{n^2}\right)$

$= \lim_{n\to\infty} \frac{1}{2} \cdot \frac{(n+1)}{n}$

$= \frac{1}{2}$

$\therefore$ (a) is correct.

(MSQ) Multiple Select Questions

1. Solution: (a), (b)

$\mathbb{R}$ is connected but not bounded hence not compact

$\therefore$ (a) is true

Also, Arbitrary union of closed set may be open

$\therefore$ option (b) is true and (c) is false

Now, to discard option (d)

Take, $V = \left\{\frac{1}{n}, n \in \mathbb{N}\right\}$

2. Solution: (b), (d)

Given,

$S = \left\{\frac{1}{3^n} + \frac{1}{7^n}, m, n \in \mathbb{N}\right\}$

0 is limit point which is not in S

$\therefore S$ is not closed.

$\therefore$ (d) is true (a) is false.

Also, S is countable and disconnected

$\therefore S$ can't be open

$\therefore$ (b) is true

And (c) is false

3. Solution: (b), (d)

$S = \mathbb{Q} \cap (0,1)$

$\therefore S' = [0,1] \subseteq S$

$\therefore S$ is not closed

$\therefore$ (b) is true and (a) is false.

Also s is countable

$\therefore$ S can't be open

$\therefore$ (c) is false and (d) is true

4. Solution: (b), (c), (d)

If, $x^2 + x - 4 = 0$ then,

$x = \frac{-1 \pm \sqrt{17}}{2} = \propto, \beta$ (say)

Where, $\propto < \beta$

$\therefore x^2 + x > 4$

$\Rightarrow x^2 + x - 4 > 0$

$\Rightarrow (x - \propto)(x - \beta) > 0$

$\therefore x > \beta$ or $x < \propto$

$\therefore x \in (-\infty, \propto) \cup (\beta, \infty)$

$\therefore S\{x \in \mathbb{R}: x^2 + x > 4\}$

Is disconnected

But, $\{x \in \mathbb{R}: x^2 + x < 4\}$

Is connected

Also,

If, $|x| < |x - 4|$

Then, $x < x - 4$ (case-I)

$\Rightarrow 0 < -4 \rightarrow$ contradiction

$x < -(x - 4)$ (Case-II)

$\Rightarrow 2x < 4$

$\Rightarrow x < 2$

$x \in (-\infty, 2)$ which is connected

But if $|x| > |x - 4|$

Case –I

$x > x - 4$

$\Rightarrow 0 > -4$ satisfied for all $x \in \mathbb{R}$

Case-II: $x > -(x - 4)$

$\Rightarrow 2x > 4$

$\Rightarrow x > 2$

$x \in (2, \infty)$ which is connected

5. Solution: (b), (c), (d)

Given P and Q are disjoint subsets of $\mathbb{R}$

(a) If P and Q are compact $\Rightarrow$

P and Q both are closed and bounded

$\therefore (P \cup Q)$ is also closed and bounded

$\therefore (P \cup Q)$ is compact

$\therefore$ (a) is correct.

(b) If, $P = [1,2] \cup [3,4]$

& $Q = [2,3] \cup [4,5]$ then P and Q are disconnected

But, P$\cup Q = [1,5]$, which is connected

$\therefore$ (b) is incorrect

(c) If, $P = \{1\}, Q = (1,2]$ then, P and $P \cup Q$ both are closed But, Q is not closed.

$\therefore$ Option (c) is incorrect

(d) If, $P = (1,2), Q = \left\{\frac{1}{2}\right\}$ then, $(P \cup Q)$ and P are open but Q is not open

$\therefore$ (d) is also incorrect

6. Solution: (a), (b), (c)

As, 2018 is an interior point of S

$\therefore S$ contains an interval

$\therefore$ (a), (b) and (c) are correct

But, if we take $|\delta| < |0.002018|$

Then, (d) is incorrect

7. Solution: (b), (d)

Given $S = (0,2]$

$T = [1,3)$

$\therefore S \backslash T = (0,1)$

$\therefore (S \backslash T)^0 = (0,1)$

Also, $S \backslash T = (0,1) = S^0 / T$

$\therefore$ (b) and (d) are correct

8. Solution: (b), (c)

(a) as we know,

Infinitely many rational

Number between two irrational

$\therefore$ not connected

(b) $x^3 - 1 \geq 0$

$(x - 1)(x^2 + x + 1) \geq 0$

As $x^2 + x + 1 > 0$

$\therefore (x - 1) > 0$

$\Rightarrow x > 1$

$\therefore S = (1, \infty)$, which is connected

(c) $f(x) = x^3 + x + 1 \geq 0$

$\therefore f'(x) = 3x^2 + 1$

∴ f is strictly increasing

∴ $f(x)$ cuts only once the x-axis

∴ $S = \{x \in \mathbb{R} : x^3 + x + 1 \geq 0\}$ is connected

(d) $f(x) = x^3 - 2x + 1$

$\therefore f'(x) = 3x^2 - 2 = 0$

$\Rightarrow x = \pm\sqrt{\frac{2}{3}}$

∴ f has two critical points

Also, $f(x) = (x-1)(x^2 + x - 1)$

$= (x-1)\left(x + \frac{\sqrt{5}+1}{2}\right)\left(x - \frac{\sqrt{5}-1}{2}\right)$

∴ $f(x) \geq 0$ has two disconnected components

9. Solution: (b), (c), (d)

(a) $f_1(x)$ = even degree polynomial

∴ f_1 is closed and bounded beiow

∴ Range $(f_1) = [\alpha, \infty)$ of some $\alpha \in \mathbb{R}$

∴ Range (f_1) is not open

(b) $f_2(x)$ = odd degree polynomial

∴ Range $\left(f_2(x)\right) = (-\infty, \infty)$, which is open

(c) $f_3(x) = \tan^{-1}(x)$

∴ Range $\left(f_3(x)\right) = \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$, which is open

(d) Range $\left(f_4(x)\right) = (-1,1) \cup (1,2) \cup (2,3) \ldots \cup (-1,-2) \cup (-2,-3) \cup \ldots$ which is open.

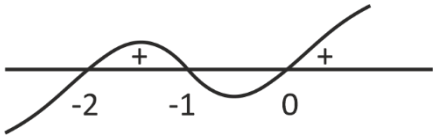
(NAT) Numerical Answer Type

1. Solution: (2)

$A = \{a \in \mathbb{R} : x^2 = a(a+1)(a+2)\ has\ a\ real\ root\}$

$\because x^2 \geq 0$

$\therefore a(a+1)(a+2) \geq 0$



∴ $a \in [-2,-1] \cup [0,\infty)$

∴ two connected components

∴ A has two connected components

2. Solution: (1)

$L = \frac{\pi}{2}\lim_{n\to\infty} \cos\left(\frac{\pi}{2^2}\right).\cos\left(\frac{\pi}{2^3}\right) \ldots . \cos\left(\frac{\pi}{2^{n+1}}\right)$

Result:- $\boldsymbol{\cos(\alpha).\cos(2\alpha).\cos(2^2\alpha) \ldots . \cos(2^n\alpha) = \frac{\sin(2^n\alpha)}{2^n \sin\alpha}}$

$L = \frac{\pi}{2}\frac{\sin\left(2^n.\frac{\pi}{2^{n+1}}\right)}{2^n.\sin\left(\frac{\pi}{2^{n+1}}\right)}$

$= \frac{\sin\left(\frac{\pi}{2}\right)}{\sin\left(\frac{\pi}{2^{n+1}}\right)}\frac{\pi}{2^{n+1}}$

$= \frac{\left(\frac{\pi}{2^{n+1}}\right)}{\sin\left(\frac{\pi}{2^{n+1}}\right)} \cdot \sin\frac{\pi}{2}$

$= 1.1$

$= 1$

3. Solution: (0)

$L = lim_{n\to\infty} \int_0^1 e^{x^2} \cdot \sin(nx)\, dx$

$I = \int_0^1 e^{x^2} \cdot \sin(nx)\, dx$

Using integration by parts,

$I = \left[e^{x^2} \cdot \left(-\frac{\cos n x}{n}\right)\right]_0^1 - \int_0^1 2x \cdot e^{x^2} . \left(-\frac{\cos nx}{n}\right) dx$

$\Rightarrow I = -\left[\frac{e^{x^2}\cdot \cos n x}{n}\right]_0^1 + \frac{2}{n}\int_0^1 x \cdot e^{x^2} \cdot \cos nx\, dx$

$\Rightarrow I = -\frac{1}{n}[e\cos n - 1] + \frac{2}{n}\int_0^1 x \cdot e^{x^2} \cdot \cos nx\, dx$

$I = \frac{1}{n}\left[(1 - e\cos n) + 2\int_0^1 x \cdot e^{x^2} . \cos nx\, dx\right]$

As, $\left[(1 - e\cos n) + 2\int_0^1 x \cdot e^{x^2} \cdot \cos nx\, dx\right]$ is bounded

$\therefore I = \frac{k}{n}$, where k = bounded

∴ as $n \to \infty$

$L = 0$

4. **Solution: (7)**

$\lim_{n\to\infty} (3^n + 5^n + 7^n)^{1/n}$

$\lim_{n\to\infty} 7\left[1 + \left(\frac{3}{7}\right)^n + \left(\frac{5}{7}\right)^n\right]^{1/n}$

$= 7(1 + 0 + 0) = 7$

CHAPTER-2 SEQUENCES OF REAL NUMBERS

(MCQ) Multiple Choice Questions

[2005: 6 Marks]

1. Let $\{a_n\}, \{b_n\}$ and $\{c_n\}$ be sequences of real numbers such that $b_n = a_{2n}$ and $\{c_n\} = a_{2n-1}$. Then $\{a_n\}$ is convergent
   (a) Implies $\{b_n\}$ is convergent but $\{c_n\}$ need not be convergent
   (b) Implies $\{c_n\}$ is convergent but $\{b_n\}$ need not be convergent
   (c) Implies both $\{b_n\}$ and $\{c_n\}$ are convergent
   (d) If both $\{b_n\}$ and $\{c_n\}$ are convergent

[2006: 6 Marks]

2. $\lim_{n\to\infty} \frac{2^{n+1}+3^{n+1}}{2^n+3^n}$ equals
   (a) 3 (b) 2
   (c) 1 (d) 0

[2007: 6 Marks]

3. Let (a_n) be an increasing sequence of positive real numbers such that the series $\sum_{k=1}^{\infty} a_k$ is divergent. Let $S_n = \sum_{k=1}^{\infty} a_k$, for $n = 1, 2, \ldots$ and $t_n = \sum_{k=2}^{\infty} \frac{a_k}{S_{k-1}S_k}$ for $n = 2,3, \ldots$. then $\lim_{n\to\infty} t_n$ is equal to
   (a) $\frac{1}{a_1}$ (b) 0
   (c) $\frac{1}{(a_1+a_2)}$ (d) $a_1 + a_2$

[2008: 6 Marks]

4. Let $\{a_n\}$ and $\{b_n\}$ be sequences of real numbers defined as $a_1 = 1$ and for $n \geq 1$, $a_{n+1} = a_n + (-1)^n 2^{-n}, b_n = \frac{2a_{n+1}-a_n}{a_n}$.
   Then
   (a) $\{a_n\}$ converges to zero and $\{b_n\}$ is a Cauchy sequence
   (b) $\{a_n\}$ converges to a non-zero number and $\{b_n\}$ is a Cauchy sequence
   (c) $\{a_n\}$ converges to zero and $\{b_n\}$ is not a convergent sequence
   (d) $\{a_n\}$ converges to a non-zero number and $\{b_n\}$ is not a convergent sequence

[2010: 6 Marks]

5. Which of the following conditions does NOT ensure the convergence of a real sequence $\{a_n\}$?
   (a) $|a_n - a_{n+1}| \to 0$ as $n \to \infty$
   (b) $\sum_{n=1}^{\infty} |a_n - a_{n+1}|$ is convergent
   (c) $\sum_{n=1}^{\infty} n a_n$ is convergent
   (d) The sequences $\{a_{2n}\}, \{a_{2n+1}\}$ and $\{a_{3n}\}$ are convergent.

[2013: 2 Marks]

6. Let $x_n = \left(1-\frac{1}{3}\right)^2 \left(1-\frac{1}{6}\right)^2 \left(1-\frac{1}{10}\right)^2 \ldots \left(1-\frac{1}{\frac{n(n+1)}{2}}\right)^2$, $n \geq 2$. Then $\lim_{n\to\infty} x_n$ is
   (a) $\frac{1}{3}$ (b) $\frac{1}{9}$
   (c) $\frac{1}{81}$ (d) 0

[2014: 2 Marks]

7. Let $x_n = 2^{2n}\left(1-\cos\left(\frac{1}{2^n}\right)\right)$ for all $n \in \mathbb{N}$. Then the sequence $\{x_n\}$
   (a) Does NOT converge (b) Converges to 0
   (c) Converges to $\frac{1}{2}$ (d) Converges to $\frac{1}{4}$

[2014: 2 Marks]

8. Let $\{x_n\}$ be a sequence of real numbers such that $\lim_{n\to\infty}(x_{n+1} - x_n) = c$, where c is a positive real number. Then the sequence $\left\{\frac{x_n}{n}\right\}$
   (a) is NOT bounded
   (b) is bounded but NOT convergent
   (c) converges to c
   (d) converges to 0

[2015: 1 Mark]

9. Let $\{x_n\}$ be a convergent sequence of real numbers. If $x_1 > \pi + \sqrt{2}$ and $x_{n+1} = \pi + \sqrt{x_n - \pi}$ for $n \geq 1$, then which one of the following is the limit of this sequence?
   (a) $\pi + 1$ (b) $\pi + \sqrt{2}$
   (c) π (d) $\pi + \sqrt{\pi}$

[2015: 2 Marks]

10. The sequence $\left\{\cos\left(\frac{1}{2}\tan^{-1}\left(-\frac{n}{2}\right)^n\right)\right\}$ is
    (a) Monotone and convergent
    (b) Monotone but not convergent
    (c) Convergent but not monotone
    (d) Neither monotone nor convergent

[2016: 1 Mark]

11. The sequence $\{S_n\}$ of real numbers given by $S_n = \frac{\sin\frac{\pi}{2}}{1.2} + \frac{\sin\frac{\pi}{2^2}}{2.3} + \ldots + \frac{\sin\frac{\pi}{2}}{n.(n+1)}$ is
    (a) A divergent sequence
    (b) An oscillatory sequence
    (c) Not a Cauchy sequence
    (d) Cauchy sequence

[2016: 2 Marks]

12. Let $\{a_n\}$ be a sequence of positive real numbers satisfying $\frac{4}{a_{n+1}} = \frac{3}{a_n} + \frac{a_n^3}{81}, n \geq 1, a_1 = 1$. Then all the terms of the sequence lie in
    (a) $\left[\frac{1}{2}, \frac{3}{2}\right]$
    (b) $[0,1]$
    (c) $[1,2]$
    (d) $[1,3]$

[2017: 2 Marks]

13. Let $0 < a_1 < b_1$. For $n \geq 1$, define $a_{n+1} = \sqrt{a_n b_n}$ and $b_{n+1} = \frac{a_n + b_n}{2}$.
    Then which one of the followings is NOT TRUE?
    (a) Both $\{a_n\}$ and $\{b_n\}$ converge, but the limits are not equal
    (b) Both $\{a_n\}$ and $\{b_n\}$ converge and the limits are equal
    (c) $\{b_n\}$ is a decreasing sequence
    (d) $\{a_n\}$ is an increasing sequence

[2017: 2 Marks]

14. $\lim_{n\to\infty} \frac{1}{\sqrt{n}}\left(\frac{1}{\sqrt{3}+\sqrt{6}} + \frac{1}{\sqrt{6}+\sqrt{9}} + \cdots + \frac{1}{\sqrt{3n}+\sqrt{3n+3}}\right) =$
    (a) $1 + \sqrt{3}$
    (b) $\sqrt{3}$
    (c) $\frac{1}{\sqrt{3}}$
    (d) $\frac{1}{1+\sqrt{3}}$

[2017: 2 Marks]

15. Which one of the following is TRUE?
    (a) Every sequence that has convergent subsequence is a Cauchy sequence
    (b) Every sequence that has convergent subsequence is a bounded sequence
    (c) The sequence $\{\sin n\}$ has convergent subsequence
    (d) The sequence $\left\{n\cos\frac{1}{n}\right\}$ has a convergent subsequence

[2018: 1 Mark]

16. Let $S_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \ldots + \frac{1}{n!}$ for $n \in \mathbb{N}$. Then which one of the following is TRUE for the sequence $\{S_n\}_{n=1}^{\infty}$
    (a) $\{S_n\}_{n=1}^{\infty}$ converges in $\mathbb{Q}$
    (b) $\{S_n\}_{n=1}^{\infty}$ is a Cauchy sequence but does not converge in $\mathbb{Q}$
    (c) The subsequence $\{S_{k^n}\}_{n=1}^{\infty}$ is convergent in $\mathbb{R}$, only when k is even natural number
    (d) $\{S_n\}_{n=1}^{\infty}$, is not a Cauchy sequence

[2018: 1 Mark]

17. Let $a_n = \frac{b_{n+1}}{b_n}$, where $b_1 = 1, b_2 = 1$ and $b_{n+2} = b_n + b_{n+1}, n \in \mathbb{N}$. Then $\lim_{n\to\infty} a_n$ is
    (a) $\frac{1-\sqrt{5}}{2}$ (b) $\frac{1-\sqrt{3}}{2}$
    (c) $\frac{1+\sqrt{3}}{2}$ (d) $\frac{1+\sqrt{5}}{2}$

[2018: 2 Marks]

18. Let $a_n = \begin{cases} 2 + \frac{(-1)^{\frac{n-1}{2}}}{n} & if\ n\ is\ odd \\ 1 + \frac{1}{2^n} & if\ n\ is\ even \end{cases}, n \in \mathbb{N}$
    Then which one of the following TRUE?
    (a) $\sup\{a_n | n \in \mathbb{N}\} = 3$ and $\inf\{a_n | n \in \mathbb{N}\} = 1$
    (b) $\liminf(a_n) = \limsup(a_n) = \frac{3}{2}$
    (c) $\sup\{a_n | n \in \mathbb{N}\} = 2$ and $\inf\{a_n | n \in \mathbb{N}\} = 1$
    (d) $\liminf(a_n) = 1$ and $\limsup(a_n) = 3$

[2019: 1 Mark]

19. Let $a_1 = b_1 = 0$, and for each $n \geq 2$, let a_n and b_n be real numbers given by
    $a_n = \sum_{m=2}^{n} \frac{(-1)^m m}{(\log(m))^m}$ and $b_n = \sum_{m=2}^{n} \frac{1}{(\log(m))^m}$.
    Then which one of the following is TRUE about the sequences $\{a_n\}$ and $\{b_n\}$?
    (a) Both $\{a_n\}$ and $\{b_n\}$ are divergent
    (b) $\{a_n\}$ is convergent and $\{b_n\}$ is divergent
    (c) $\{a_n\}$ is divergent and $\{b_n\}$ is convergent
    (d) Both $\{a_n\}$ and $\{b_n\}$ are convergent

[2019: 2 Marks]

20. Let $\{a_n\}$ be a sequence of positive real numbers such that $a_1 = 1$,
    $a_{n+1}^2 - 2a_n a_{n+1} - a_n = 0$ for all $n \geq 1$
    Then the sum of the series $\sum_{n=1}^{n} \frac{a_n}{3^n}$ lies in the interval
    (a) $(1,2]$ (b) $(2,3]$
    (c) $(3,4]$ (d) $(4,5]$