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ELECTRODYNAMICS

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Chapter 1

Vector Analysis

VECTORS → Physical Quantities having both magnitude & direction and which follows the vector law of addition are known as vectors Eg → Force, Electric field.

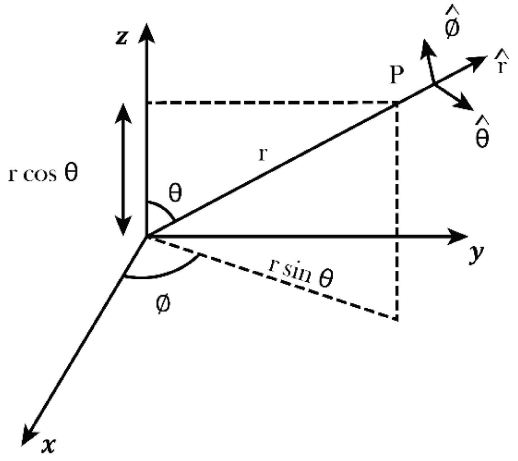
Polar vectors:- Are those vectors whose sense of direction is independent of handness of the coordinate system.
Or
Which change sign under inversion of co-ordinates (parity Inversion).
i.e. Whose sense of direction depends on handedness of the co-ordinate system.
Or
Which does not change sign under Inversion of coordinates.
Polar vector is also called true vector & Pseudo vector is also called Axial vector.
On the basis of parity Inversion we can define Pseudo or true vector.

Parity Inversion:- If some vector changes sign under parity Inversion Then it is called true vector.

$$\frac{Kq}{r^2}(\hat{r}) Parity \rightarrow \frac{Kq}{r^2}(-\hat{r})$$

So Electric field is called a true vector.
Magnetic field does not change sign under parity so it is called pseudo vector.

CURVILINEAR COORDINATES
Spherical Polar Coordinates



The spherical polar coordinates (r, θ, ϕ) at a point p are defined is shown in figure
$r \rightarrow$ Is the distance from the origin (the magnitude of the position vector)
$\theta \rightarrow$ Is called the polar vector (the angle down from the z axis)
$\phi \rightarrow$ Is the azimuthal angle (the angle around from the x axis)
The relation to Cartesian coordinate (x, y, z) can be read from the figure.

$$x = r sin\,\theta\, cos\,\phi$$
$$y = r sin\,\theta\, sin\,\phi$$
$$z = r cos\,\theta$$

And $r = \sqrt{x^2 + y^2 + z^2}$

$$\theta = \cos^{-1}\left(\frac{z}{r}\right)$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right)$$

Figure shows three unit vectors $\hat{r}, \hat{\theta}, \hat{\phi}$

Pointing in the direction of increase of the coordinates. They constitute an orthogonal (mutually perpendicular) basis set (just like $\hat{x}, \hat{y}, \hat{z}$) and any vector $\vec{A}$ can be expressed in term of them in the usual way.

$$\vec{A} = A_r\hat{r} + A_\theta\hat{\theta} + A_\phi\hat{\phi}$$

Where A_r, A_θ and A_ϕ are the radial, polar and Azimuthal components of A.

Length Element θ

An Infinitesimal displacement in the $\hat{r}$ direction is simply dr (figure a), just as an Infinitesimal element of length in the x direction is $xdl_r = dr$.

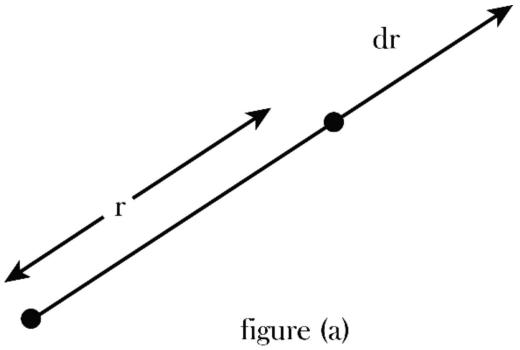


figure (a)

Similarly, an Infinitesimal element of length in the $\hat{\theta}$ direction (figure b) is $r\ d\theta$

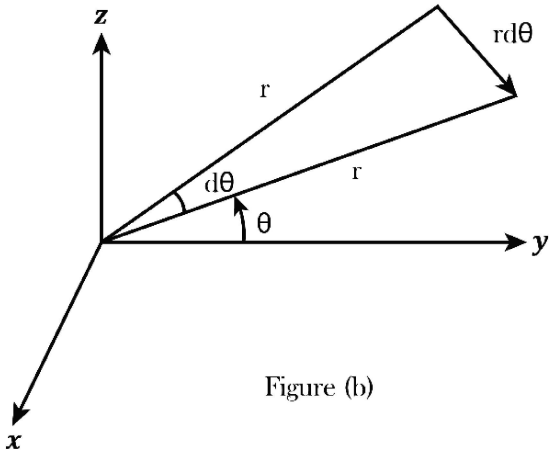


Figure (b)

Similarly, an infinitesimal element of length in the $\hat{\phi}$ direction (figure c) is $r = sin\ \theta\ d\phi$

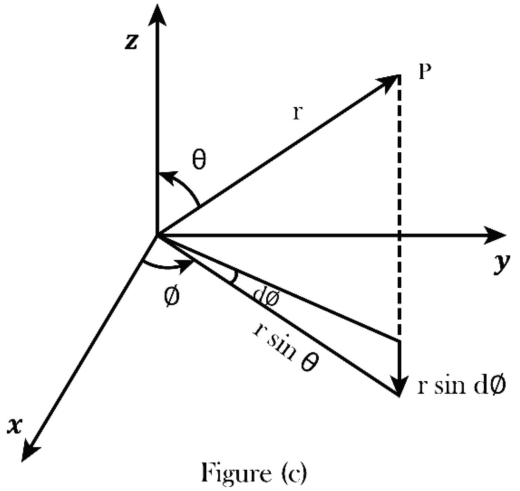


Figure (c)

Thus the general infinitesimal displacement $\hat{d}l$ is

$$\hat{d}l = dr\,\hat{r} + rd\theta\hat{\theta} + rsin\,\theta\,d\phi\hat{\phi}$$

The range of r is $0 \to \infty$, θ goes from $0 \to \pi$ and ϕ goes from 0 to 2π.

Area Element

Just like the line element we also have three Area element.

$\overrightarrow{dS_r}$ = surface which does not change in r will be fix

$$\overrightarrow{dS_r} = \overrightarrow{dl}_\theta \times \overrightarrow{dl}_\phi$$

$= r^2 sin\,\theta\;\; d\theta\, d\phi\,\hat{r}$

$\overrightarrow{dS_\theta} \to$ surface where θ will not change 2 dimension version of sphere is disc, for a dis c θ will not change.

$$\overrightarrow{dS_\theta} = \overrightarrow{dl}_\phi \times \overrightarrow{dl}_r$$

$= r\,dr\,d\phi\hat{\theta}$

~ $\overrightarrow{dS_\phi} \to$ In this area element ϕ will not change

$$\overrightarrow{dS_\phi} = \overrightarrow{dl}_r \times \overrightarrow{dl}_\theta$$

$= r\,dr\,d\,\theta\hat{\phi}$

Volume element:- The volume element in the spherical polar coordinate is given by

$$d\tau = d\vec{l}_r.\left(\overrightarrow{dl}_\theta \times \overrightarrow{dl}_\phi\right)$$

$$d\tau = r^2 sin\,\theta\,dr\,d\theta d\phi$$

Relation between Unit Vectors.

Just like the $\hat{x}, \hat{y}, \hat{z}$ are the unit vectors in Cartesian coordinates similarly $\theta\,\hat{r}, \hat{\theta}\&\hat{\phi}$ are the unit vectors in spherical polar coordinates.

$$z = r\,cos\,cos\,\theta$$

$$x = rsin\,\theta\,cos\,\phi$$

$$y = rsin\,\theta\,sin\,\phi$$

$$\theta = \left(\frac{z}{r}\right)$$

$$\theta = \left(\frac{z}{\sqrt{x^2 + y^2 + z^2}}\right)$$

$$\phi = \frac{y}{x} = tan\,\phi$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$$

$$\vec{r} = rsin\,\theta\,cos\,\phi\,\hat{x} + rsin\,\theta\,sin\,\phi\,\hat{y} + rcos\,\theta\,\hat{z}$$

$\hat{r} = \frac{\frac{\partial\vec{r}}{\partial r}}{\left|\frac{\partial\vec{r}}{\partial r}\right|} = sin\,\theta\,cos\,\phi\,\hat{x} + sin\,\theta\,sin\,\phi\,\hat{y} + cos\,\theta\,\hat{z}$ ..... (1)

$$\hat{r} = \left|\frac{\partial\vec{r}}{\partial r}\right| = \sqrt{\theta\;\phi\;+\theta\;\phi\;+\theta}$$

$\frac{\frac{\partial\vec{r}}{\partial\theta}}{\left|\frac{\partial\vec{r}}{\partial\theta}\right|} = \hat{\theta} = cos\,\theta cos\,\phi\;\;\hat{x} + cos\,\theta\,sin\,\phi\,\hat{y} - sin\,\theta\,\hat{z}.$ ..... (2)

$\hat{\phi} = \frac{\frac{\partial\vec{r}}{\partial\phi}}{\left|\frac{\partial\vec{r}}{\partial\phi}\right|} = -sin\,\phi\,\hat{x} + cos\,\phi\,\hat{y}$ ..... (3)

$$\left|\frac{\partial\vec{r}}{\partial\phi}\right| = rsin\,\theta$$

Using these (1), (2) & (3) convert spherical to Cartesian

$$1 \times sin\,\theta\;\; 2 \times cos\,\theta\}odd$$

$$sin\,\theta\,\hat{r} = \theta cos\,\phi\,\hat{x} + \theta sin\,\phi\,\hat{y} + cos\,\theta\,sin\,\theta\,\hat{z}$$

$$cos\,\theta\,\hat{\theta} = cos\,\phi\;\hat{x} + \theta sin\,\phi\,\hat{y} - cos\,\phi\,sin\,\theta\,\hat{z}$$

$$sin\,\theta\,\hat{r} + cos\,\theta\,\hat{\theta} = cos\,\phi\,\hat{x} + sin\,\phi\,\hat{y} \text{ ..... (4)}$$

Eq. $(4) \times cos\,\phi\; - (3) \times sin\,\phi$

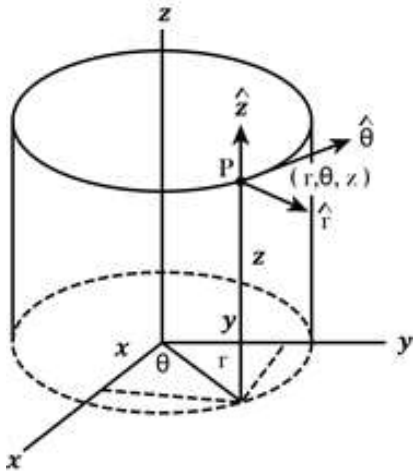
$$\hat{x} = sin\,\theta\,cos\,\phi\,\hat{r} + cos\,\theta\,cos\,\phi\,\hat{\theta} - sin\,\phi\,\hat{\phi}$$

$(4) \times sin\,\phi\; + (3) cos\,\phi$

$$\hat{y} = sin\,\theta\,sin\,\phi\,\hat{r} + sin\,\phi\,cos\,\theta\,\hat{\theta} + cos\,\phi\,\hat{\phi}$$

$(1) \times cos\,\theta\; - (2) \times sin\,\theta$

$$\hat{z} = cos\,\theta\,\hat{r} - sin\,\theta\,\hat{\theta}$$



(B) Cylindrical co-ordinate system

(Image 1)

Let the point P (in above figure) has Cartesian coordinates (x, y, z) and cylindrical coordinates (r, ϕ, z) so the relation between Cartesian and cylindrical co-ordinates.

$$x = rcos\,\phi$$

$$y = rsin\,\phi$$

$$z = z$$

In Cylindrical co-ordinate

$$r = \sqrt{x^2 + y^2}$$

$\phi = \tan^{-1}\left(\frac{y}{x}\right)$ angle which Q makes with positive x-axis

$z =$ height of that point above the x-y plane.

Unit vector in cylindrical coordinates

The position vector of the point is $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

Putting the value of (x, y, z) in the above relation.

$$\vec{r} = rcos\,\phi\,\hat{\imath} + rsin\,\phi\,\hat{\jmath} + z\hat{k}$$

The unit vector along r is given by

$$\hat{r} = \frac{\partial\vec{r}/\partial\delta}{\left|\frac{\partial r}{\partial\delta}\right|} = cos\,\phi\,\hat{\imath} + sin\,\phi\,\hat{\jmath}$$

$$\hat{\phi} = -sin\,\phi\,\hat{\imath} + cos\,\phi\,\hat{\jmath}$$

$$\hat{z} = \hat{k}$$

Here $\hat{r}, \hat{\phi}$ and $\hat{z}$ are orthogonal to each other and they follow.

$$\hat{r} \times \hat{\phi} = \hat{z}$$

$$\hat{\phi} \times \hat{z} = \hat{r}$$

$\hat{z} \times \hat{r} = \hat{\phi}$.

Line element

Line element between r to r +dr is

$\overrightarrow{dl}_r = dr\,\hat{r}$ (along r curve)

Line element between ϕ to $\phi + d\phi$

$\overrightarrow{dl}_\phi = r\,d\phi\,\hat{\phi}$ (alongϕ curve)

Line element z to z+dz is

$\overrightarrow{dl}_z = dz\,\hat{z}$ (along z curve).

Surface Element

Constant r surface: $\overrightarrow{dS}_r = \overrightarrow{dl}_\phi \times \overrightarrow{dl}_z$

$$\overrightarrow{dS}_r = r\,d\phi dz\,\hat{r}$$

(curved surface of circular cylinder)

Constant ϕ surface: $\overrightarrow{dS}_z = \overrightarrow{dl}_r \times \overrightarrow{dl}_\phi$

$= r\,dr\,d\phi\hat{z}$ (plane surface of a circular cylinder parallel to x-y plane)

Volume Element

The volume element in the cylindrical polar coordinate is given by

$$dv = \overrightarrow{dl}_r.\left(\overrightarrow{dl}_\phi \times \overrightarrow{dl}_z\right)$$

$= r\,dr\,d\phi\,dz.$

Gradient:- It is defined for a scalar.

We cannot take gradient of any vector suppose a function varry with $x\ f\ (x)$ then

$$df = \left(\frac{\partial f}{\partial x}\right)_{slope} dx$$

This derivative shows how f increases with x. This is the slope of the graph.

Now consider a function $T\ (x, y, z)$ now T is changing with x, y, z. It can't describe by one variable.

Its differential

$$dT = \left(\frac{\partial T}{\partial x}\right)dx + \left(\frac{\partial T}{\partial y}\right)dy + \left(\frac{\partial T}{\partial z}\right)dz$$

We know that

$$\nabla = \left(\frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} + \frac{\partial}{\partial z}\hat{k}\right)$$

$$\overrightarrow{dr} = dx\hat{\imath} + dy\hat{\jmath} + dz\hat{k}$$

So we can write

$$dT = (\nabla T).\overrightarrow{dr}$$

Gradient of a function.

Gradient of a scalar is a vector quantity.

$$dT = \nabla T\ dr\cos\theta$$

Magnitude of grad of scalar

If $\theta = 0$

Then change is maximum

If $\theta = 90^\circ$, then change is minimum direction of grad of scalar shows direction of maximum Increases of the function.

E.g. $E = -\nabla V$

Direction of electric field is the direction of maximum decrease of potential (because there is-ve sign) Let us consider a conducting sphere on which electric field is the normal to the surface. It is equipotential surface so electric field is the direction of maximum decrease in potential.

$$\vec{\nabla}T = \frac{\partial T}{\partial x}\hat{\imath} + \frac{\partial T}{\partial y}\hat{\jmath} + \frac{\partial T}{\partial z}\hat{k}$$

In Cartesian

$$\vec{\nabla}T = \frac{\partial T}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial T}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial T}{\partial \phi}\hat{\phi}$$

In spherical polar coordinate.

$$\vec{\nabla}T = \frac{\partial T}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial T}{\partial \phi}\hat{\phi} + \frac{\partial T}{\partial z}\hat{z}$$

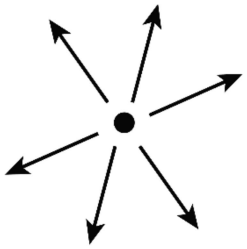
In Cylindrical

Divergence → It is defined for a vector divergence means spread (how much a vector can spread in space). How much a vector diverges from a point in the space.

For source → $+ve$ divergence.

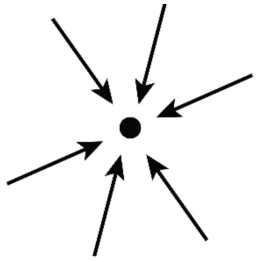
For $sink \rightarrow -ve$ divergence.

Outward direction is +ve



Outward direction is +ve

Inward direction is –ve



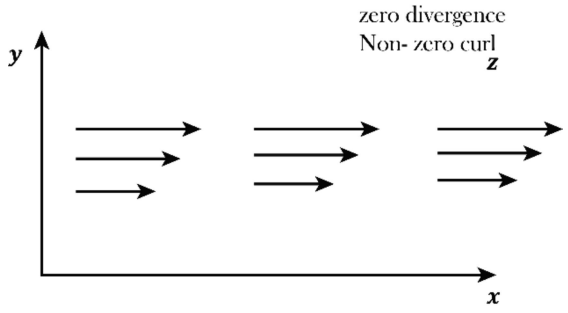
Inward direction is -ve

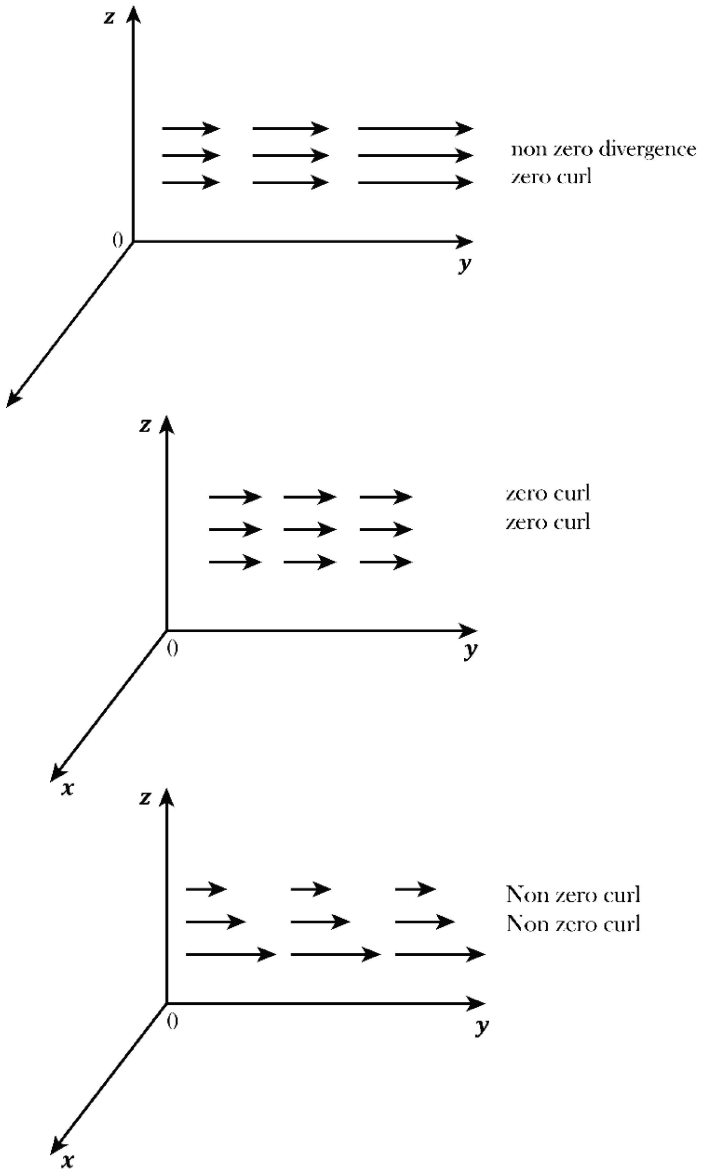
If $\vec{A} = \frac{\partial}{\partial x}A_x + \frac{\partial}{\partial y}A_y + \frac{\partial}{\partial z}A_z$

If the divergence of a vector is zero then it is called solenoidal vector

$\vec{\nabla} \cdot \vec{A} = 0.$

Graphical





Curl → It is defined for a vector. Curl of a vector is also a vector. Curl is related to rotation (how much a vector is rotates about a point in the space) It curl of a vector is zero then vector is irrotational.

$$\vec{\nabla} \times \vec{A} = \left| \hat{\imath}\hat{\jmath}\hat{k} \frac{\partial}{\partial x} \frac{\partial}{\partial y} \frac{\partial}{\partial z} A_x A_y A_z \right|$$

This is the curl in Cartesian co-ordinate system. If vector in x-direction then it will change in perpendicular direction i.e. A_x change with y & z axis then curl is non zero but if A_x is changes with x-axis then we have zero curl.

Table for general formula

System	u	v	w	f	g	h
				1	1	1
C	x	y	y			
				1	r	r sin θ
S.P	r	θ	∅			
Cy	S	∅	y	1	r	1

Gradient

$$\underline{\nabla} T = \frac{1}{F}\frac{\partial T}{\partial u}\hat{u} + \frac{1}{g}\frac{\partial T}{\partial v}\hat{v} + \frac{1}{h}\frac{\partial T}{\partial w}\hat{w}$$

$$\vec{\nabla}.\vec{A} = \frac{1}{fgh}\left[\frac{\partial}{\partial u}(gh\,A_u) + \frac{\partial}{\partial v}(fh\,A_v) + \frac{\partial}{\partial w}(fg\,A_w)\right]$$

Curl

$$\vec{\nabla} \times \vec{A} = \frac{1}{fgh}\left|f\hat{u}g\hat{v}h\hat{w}\frac{\partial}{\partial u}\frac{\partial}{\partial v}\frac{\partial}{\partial w}fA_u gA_v hA_w\right|$$

Laplacian:

Laplacian is defined for both scalar and vector

$$\vec{\nabla}.\vec{\nabla} = \nabla^2$$

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$

$$\nabla^2 A = \frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} + \frac{\partial^2 A}{\partial z^2}$$

If $\nabla^2 A = 0$ Laplacian equation

Spherical polar

$$\nabla^2 A = \frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r}A + \frac{1}{r^2 sin\,\theta}\frac{\partial}{\partial \theta}sin\,\theta\,\frac{\partial}{\partial \theta}A + \frac{1}{r^2\theta}\frac{\partial^2 A}{\partial \phi^2}$$

Cylindrical

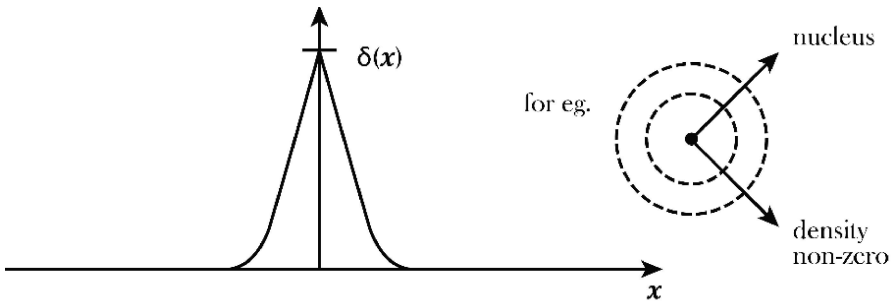
$$\nabla^2 A = \frac{1}{r}\frac{\partial}{\partial r}\,r\frac{\partial A}{\partial r} + \frac{1}{r^2}\frac{\partial^2 A}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}$$

Dirac-Delta Function:-This is not a ordinary function but generalized function or distribution function. It is has the property that it is ∞ when $x = 0$

i.e. $\delta(x) = \infty(x = 0)$

$= 0\ (x \neq 0)$

Where this function is centralized it is ∞ at other point, it is 0.



Physical ex- density

$$\vec{A} = \frac{\hat{r}}{r^2} = \frac{\hat{r}}{r^2}$$

$$\nabla.\vec{A} = \nabla.\left(\frac{\hat{r}}{r^2}\right) = 4\pi\oint^3(r)$$

$4\pi\int_{-1}^{1}\delta^3(r)d\tau \neq 0 \& 4\pi\int_1^2\delta^3(r)dz = 0$

This Integral will be non-zero when limits of r enclose it

Properties

For 1 dim

1. $\int_{-\infty}^{\infty} \delta(x)dx = 1$

2. $f(x)\delta(x) = f(0)\delta(x)$

3. $\int_{-\infty}^{\infty} f(x)\delta(x)dx = f(0)\int_{-\infty}^{\infty} \delta(x)dx = f(0)$

Limit must enclose the origin of dirac delta function.

If centre at $x = a$ then

$$\delta(x-a) = \infty x = a$$
$$= 0\ x \neq a$$

$\int_{-\infty}^{\infty} \delta(x-a)dx = 1$

E.g. $\delta(x-2)$ centre at 2

$$\int_3^4 \delta(x-2)dx = 0$$

Because 2 is not between 3 & 4 i.e. limits are not enclosing $x = 2$.

2. $f(x)\delta(x-a) = f(a)\delta(x-a)$

3. $\int_{-\infty}^{\infty} f(x)\delta(x-a)dx = f(a)\int_{-\infty}^{\infty} \delta(x-a)dx$
$= f(a)$

Area of Dirac-Delta function is 1.

(height × width = finite)

If $\delta(kx) = \frac{1}{|K|}\delta(x)$

E.g.:- $\delta(-3x) = \frac{1}{|3|}\delta(x)$

Dirac Delta Function is Even Function

$$\delta(-x) = \delta(x)$$

Derivative of dirac delta function

$$\frac{1}{dx}\delta(x) = \delta'(x) = \frac{-1}{x}\delta(x)$$

If there is a polynomial is dirac delta function as $\delta[x^2 + 2x + 1]$

First find the roots $x = -1, -1$

If $\delta[(x-a)(x-b)]$ is a dirac delta function who has two centres then.

$$\delta[(x-a)(x-b)] = \frac{1}{|a-b|}[\delta(x-a) + \delta(x-b)]$$

$$\delta(x^2 - a^2) = \frac{1}{2|a|}[\delta(x-a) + \delta(x+a)], a \neq 0$$

 # For nth derivative of dirac-delta function

$$\int_{-\infty}^{\infty} f(x)\delta^n(x)dx = (-1)^n f^n(0)$$

E.g. $\int_{-\infty}^{\infty} x^2\delta'(x)\,dx = (-1)^1\ 2x$
$= 0$

For nth derivative of dirac-delta function

Spherical-polar $\delta^3(r)$

Cartesian $\delta(x)\delta(y)\delta(z)$