P R E F A C E

This **"M.sc Physics Entrance Book Volume 3" -** has been carefully designed to provide a comprehensive overview of topics from Electrodynamics, Wave and Optics and Thermodynamics for undergraduate and graduate students in the field of Physical Science.

The aim of this book is to provide a clear and concise understanding of the subject matter, without overwhelming the reader with unnecessary technical jargon. The language used throughout the book is simple and easy to understand, making it accessible to students with varying levels of knowledge and experience.

In addition to the text, the book includes a range of tables and diagrams that illustrate key concepts for better understanding. One of the key features of this book is its relevance to various competitive exams, such as IIT JAM, CUET (PG), JEST, TIFR, (Physical Science) exams. The book has been designed keeping in mind the syllabus and requirements of these exams, making it an ideal resource for students who are preparing for these tests.

We hope that this book will serve as a valuable resource for students studying physics and will help them to deepen their understanding of this fascinating subject. If you come across any misprints or errors, please do inform us, as we are committed to providing accurate and up-to-date information to our readers.

A C K N O W L E D G E M E N T

Writing this book **"M.Sc Physics Entrance Book Volume 3"** - has been an incredible journey. All the chapters have taken a final shape after endless inputs of time and effort. Though many colleagues assisted me in compiling this book, I first of all would like to thank all students at Institute for advanced Studies (IFAS), who have helped me to learn and implement the concepts and ideas of mathematical physics in its simplest form.

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UNIT - 1

ELECTRODYNAMICS

Chapter 1

Mathematics for EMT

Vectors→ Physical Quantities having both magnitude & direction and which follows the vector law of addition are known as vectors Eg → Force, Electric field.

Polar vectors:- Are those vectors whose sense of direction is independent of handness of the coordinate system.

Or

Which change sign under inversion of co-ordinates (parity Inversion).

i.e. Whose sense of direction depends on handedness of the co-ordinate system.

OR

Which does not change sign under Inversion of coordinates.

Polar vector is also called true vector & Pseudo vector is also called Axial vector.

On the basis of parity Inversion we can define Pseudo or true vector.

Parity Inversion:- If some vector changes sign under parity Inversion Then it is called true vector.

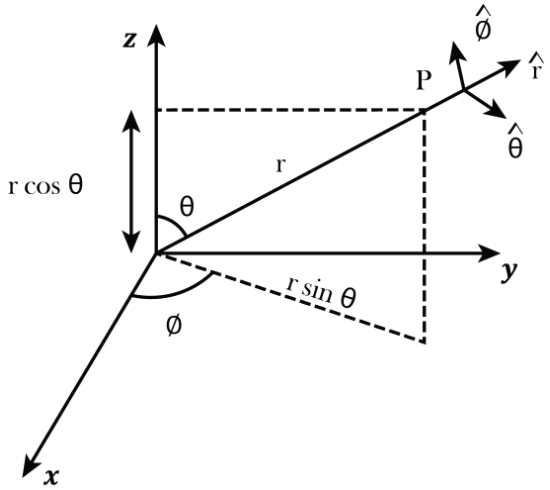
$\frac{Kq}{r^2}(\hat{r}) Parity \rightarrow \frac{Kq}{r^2}(-\hat{r})$

So Electric field is called a true vector.

Magnetic field does not change sign under parity so it is called pseudo vector.

Curvilinear Coordinates

Spherical Polar Coordinates



The spherical polar coordinates (r, θ, ϕ) at a point p are defined is shown in figure

$r \rightarrow$ Is the distance from the origin (the magnitude of the position vector)

$\theta \rightarrow$ Is called the polar vector (the angle down from the z axis)

$\phi \rightarrow$ Is the azimuthal angle (the angle around from the x axis)

The relation to Cartesian coordinate (x, y, z) can be read from the figure.

$x = r sin\,\theta\, cos\,\phi$

$y = r sin\,\theta\, sin\,\phi$

$z = r cos\,\theta$

And $r = \sqrt{x^2 + y^2 + z^2}$

$\theta = \cos^{-1}\left(\frac{z}{r}\right)$

$\phi = \tan^{-1}\left(\frac{y}{x}\right)$

Figure shows three unit vectors $\hat{r}, \hat{\theta}, \hat{\phi}$

Pointing in the direction of increase of the coordinates. They constitute an orthogonal (mutually perpendicular) basis set (just like $\hat{x}, \hat{y}, \hat{z}$) and any vector $\vec{A}$ can be expressed in term of them in the usual way.

$\vec{A} = A_r\hat{r} + A_\theta\hat{\theta} + A_\phi\hat{\phi}$

Where A_r, A_θ and A_ϕ are the radial, polar and Azimuthal components of A.

Length Element $\boldsymbol{\theta}$

An Infinitesimal displacement in the $\hat{r}$ direction is simply dr (figure a), just as an Infinitesimal element of length in the x direction is $xdl_r = dr$.

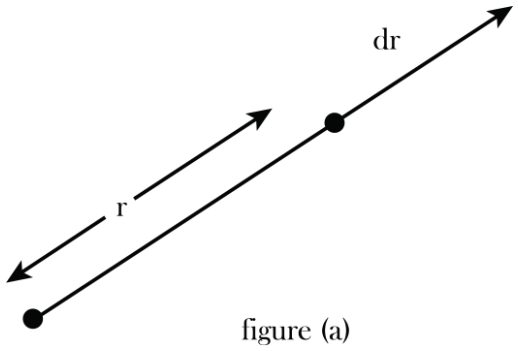


figure (a)

Similarly, an Infinitesimal element of length in the $\hat{\theta}$ direction (figure b) is $r\ d\theta$

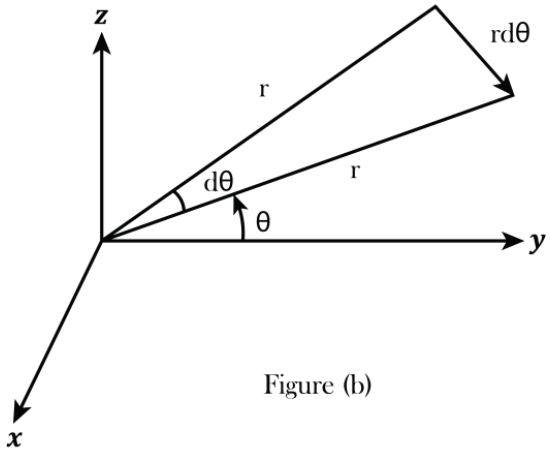


Figure (b)

Similarly, an infinitesimal element of length in the $\hat{\phi}$ direction (figure c) is $r = sin\ \theta\ d\phi$

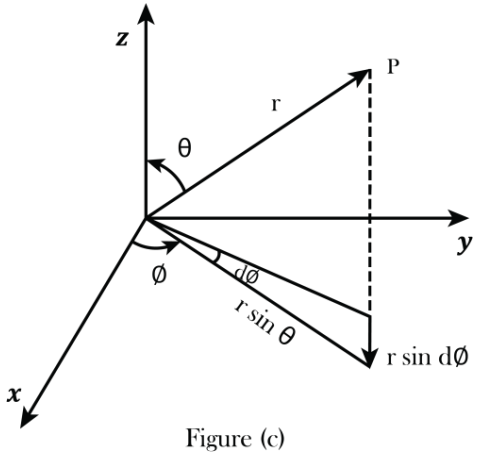


Figure (c)

Thus the general infinitesimal displacement $\hat{dl}$ is

$\hat{dl} = dr\ \hat{r} + rd\theta\hat{\theta} + rsin\ \theta\ d\phi\hat{\phi}$

The range of r is $0 \rightarrow \infty, \theta$ goes from $0 \rightarrow \pi$ and ϕ goes from 0 to 2π.

Area Element

Just like the line element we also have three Area element.

$\overrightarrow{dS_r}$ = surface which does not change in r will be fix

$\overrightarrow{dS_r} = \overrightarrow{dl_\theta} \times \overrightarrow{dl_\phi}$

$= r^2 sin\,\theta \;\; d\theta\, d\phi\, \hat{r}$

$\overrightarrow{dS_\theta}$ → surface where θ will not change 2 dimension version of sphere is disc, for a dis c θ will not change.

$\overrightarrow{dS_\theta} = \overrightarrow{dl_\phi} \times \overrightarrow{dl_r}$

$= r\, dr\, d\phi\hat{\theta}$

~ $\overrightarrow{dS_\phi}$ → In this area element ϕ will not change

$\overrightarrow{dS_\phi} = \overrightarrow{dl_r} \times \overrightarrow{dl_\theta}$

$= r\, dr\, d\,\theta\hat{\phi}$

Volume element:- The volume element in the spherical polar coordinate is given by

$d\tau = d\vec{l}_r.(\overrightarrow{dl_\theta} \times \overrightarrow{dl_\phi})$

$d\tau = r^2 sin\,\theta\, dr\, d\theta d\phi$

Relation between Unit Vectors.

Just like the $\hat{x}, \hat{y}, \hat{z}$ are the unit vectors in Cartesian coordinates similarly $\theta\,\hat{r}, \hat{\theta} \& \hat{\phi}$ are the unit vectors in spherical polar coordinates.

$z = r\,cos\,cos\,\theta$

$x = rsin\,\theta\,cos\,\phi$

$y = rsin\,\theta\,sin\,\phi$

$\theta = \left(\frac{z}{r}\right)$

$\theta = \left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)$

$\phi = \frac{y}{x} = tan\,\phi$

$\phi = \tan^{-1}\left(\frac{y}{x}\right)$

$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$\vec{r} = rsin\,\theta\,cos\,\phi\,\hat{x} + rsin\,\theta\,sin\,\phi\,\hat{y} + rcos\,\theta\,\hat{z}$

$\hat{r} = \frac{\frac{\partial\vec{r}}{\partial r}}{\left|\frac{\partial\vec{r}}{\partial r}\right|} = sin\,\theta\,cos\,\phi\,\hat{x} + sin\,\theta\,sin\,\phi\,\hat{y} + cos\,\theta\,\hat{z}$ ..... (1)

$\hat{r} = \left|\frac{\partial\vec{r}}{\partial r}\right| = \sqrt{\theta\;\phi\; + \theta\;\phi\; + \theta}$

$\frac{\frac{\partial\vec{r}}{\partial\theta}}{\left|\frac{\partial\vec{r}}{\partial\theta}\right|} = \hat{\theta} = cos\,\theta cos\,\phi\;\;\hat{x} + cos\,\theta\,sin\,\phi\,\hat{y} - sin\,\theta\,\hat{z}.$ ..... (2)

$\hat{\phi} = \frac{\frac{\partial\vec{r}}{\partial\phi}}{\left|\frac{\partial\vec{r}}{\partial\phi}\right|} = -sin\,\phi\,\hat{x} + cos\,\phi\,\hat{y}$ ..... (3)

$\left|\frac{\partial\vec{r}}{\partial\phi}\right| = rsin\,\theta$

Using these (1), (2) & (3) convert spherical to Cartesian

$1 \times sin\,\theta\;\; 2 \times cos\,\theta\}odd$

$sin\,\theta\,\hat{r} = \theta cos\,\phi\,\hat{x} + \theta sin\,\phi\,\hat{y} + cos\,\theta\,sin\,\theta\,\hat{z}$

$cos\,\theta\,\hat{\theta} = cos\,\phi\;\;\hat{x} + \theta sin\,\phi\,\hat{y} - cos\,\phi\,sin\,\theta\,\hat{z}$

$sin\,\theta\,\hat{r} + cos\,\theta\,\hat{\theta} = cos\,\phi\,\hat{x} + sin\,\phi\,\hat{y}$ ..... (4)

Eq. $(4) \times cos\,\phi\; - (3) \times sin\,\phi$

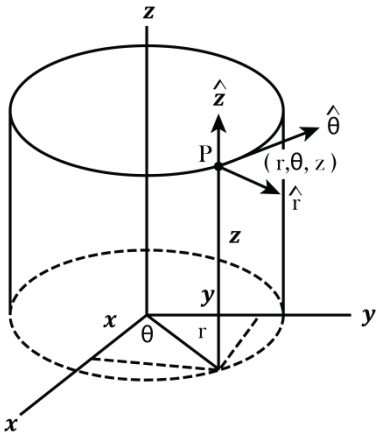
$\hat{x} = sin\,\theta\,cos\,\phi\,\hat{r} + cos\,\theta\,cos\,\phi\,\hat{\theta} - sin\,\phi\,\hat{\phi}$

$(4) \times sin\,\phi\; + (3)cos\,\phi$

$\hat{y} = sin\,\theta\,sin\,\phi\,\hat{r} + sin\,\phi\,cos\,\theta\,\hat{\theta} + cos\,\phi\,\hat{\phi}$

(1) $\times cos\ \theta\ -(2) \times sin\ \theta$

$\hat{z} = cos\ \theta\ \hat{r} - sin\ \theta\ \hat{\theta}$



(B) Cylindrical co-ordinate system

(Image 1)

Let the point P (in above figure) has Cartesian coordinates (x, y, z) and cylindrical coordinates (r, ϕ, z) so the relation between Cartesian and cylindrical co-ordinates.

$x = rcos\ \phi$

$y = rsin\ \phi$

$z = z$

In Cylindrical co-ordinate

$r = \sqrt{x^2 + y^2}$

$\phi = \tan^{-1}\left(\frac{y}{x}\right)$ angle which Q makes with positive x-axis

$z =$ height of that point above the x-y plane.

Unit vector in cylindrical coordinates

The position vector of the point is $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

Putting the value of (x, y, z) in the above relation.

$\vec{r} = rcos\ \phi\ \hat{\imath} + rsin\ \phi\ \hat{\jmath} + z\hat{k}$

The unit vector along r is given by

$\hat{r} = \frac{\partial\vec{r}/\partial\delta}{\left|\frac{\partial r}{\partial\delta}\right|} = cos\ \phi\ \hat{\imath} + sin\ \phi\ \hat{\jmath}$

$\hat{\phi} = -sin\ \phi\ \hat{\imath} + cos\ \phi\ \hat{\jmath}$

$\hat{z} = \hat{k}$

Here $\hat{r}, \hat{\phi}$ and $\hat{z}$ are orthogonal to each other and they follow.

$\hat{r} \times \hat{\phi} = \hat{z}$

$\hat{\phi} \times \hat{z} = \hat{r}$

$\hat{z} \times \hat{r} = \hat{\phi}.$

Line element

Line element between r to r +dr is

$\overrightarrow{dl}_r = dr\ \hat{r}$ (along r curve)

Line element between ϕ to $\phi + d\phi$

$\overrightarrow{dl}_\phi = r\ d\phi\ \hat{\phi}$ (along ϕ curve)

Line element z to z+dz is

$\overrightarrow{dl}_z = dz\ \hat{z}$ (along z curve).

Surface element

Constant r surface: $\overrightarrow{dS}_r = \overrightarrow{dl}_\phi \times \overrightarrow{dl}_z$

$\overrightarrow{dS}_r = r\ d\phi dz\ \hat{r}$

(curved surface of circular cylinder)

Constant ϕ surface: $\overrightarrow{dS}_z = \overrightarrow{dl}_r \times \overrightarrow{dl}_\phi$

$= r\ dr\ d\phi\hat{z}$ (plane surface of a circular cylinder parallel to x-y plane)

Volume Element

The volume element in the cylindrical polar coordinate is given by

$dv = \overrightarrow{dl}_r.(\overrightarrow{dl}_\phi \times \overrightarrow{dl}_z)$

$= r\ dr\ d\phi\ dz.$

Gradient:- It is defined for a scalar.

We cannot take gradient of any vector suppose a function varry with $x\ f\ (x)$ then

$df = \left(\frac{\partial f}{\partial x}\right)_{slope} dx$

This derivative shows how f increases with x. This is the slope of the graph.

Now consider a function $T\ (x, y, z)$ now T is changing with x, y, z. It can't describe by one variable.

Its differential

$dT = \left(\frac{\partial T}{\partial x}\right) dx + \left(\frac{\partial T}{\partial y}\right) dy + \left(\frac{\partial T}{\partial z}\right) dz$

We know that

$\nabla = \left(\frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} + \frac{\partial}{\partial z}\hat{k}\right)$

$\overrightarrow{dr} = dx\hat{\imath} + dy\hat{\jmath} + dz\hat{k}$

So we can write

$dT = (\nabla T).\overrightarrow{dr}$

Gradient of a function.

Gradient of a scalar is a vector quantity.

$dT = \nabla T\ drcos\ \theta$

Magnitude of grad of scalar

If $\theta = 0$

Then change is maximum

If $\theta = 90^\circ$, then change is minimum direction of grad of scalar shows direction of maximum Increases of the function.

E.g. $E = -\nabla V$

Direction of electric field is the direction of maximum decrease of potential (because there is-ve sign) Let us consider a conducting sphere on which electric field is the normal to the surface. It is Equipotential surface so electric field is the direction of maximum decrease in potential.

$\vec{\nabla}T = \frac{\partial T}{\partial x}\hat{\imath} + \frac{\partial T}{\partial y}\hat{\jmath} + \frac{\partial T}{\partial z}\hat{k}$

In Cartesian

$\vec{\nabla}T = \frac{\partial T}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial T}{\partial \theta}\hat{\theta} + \frac{1}{rsin\ \theta}\frac{\partial T}{\partial \phi}\hat{\phi}$

In spherical polar coordinate.

$\vec{\nabla}T = \frac{\partial T}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial T}{\partial \phi}\hat{\phi} + \frac{\partial T}{\partial z}\hat{z}$

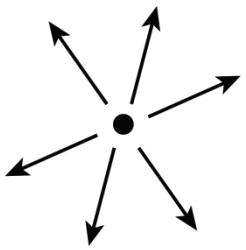
In Cylindrical

Divergence → It is defined for a vector divergence means spread (how much a vector can spread in space). How much a vector diverges from a point in the space.

For source → $+ve$ divergence.

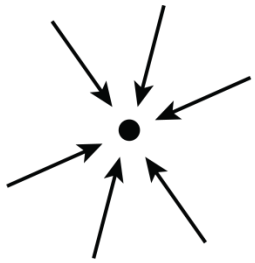
For $sink \rightarrow -ve$ divergence.

Outward direction is +ve



Outward direction is +ve

Inward direction is –ve



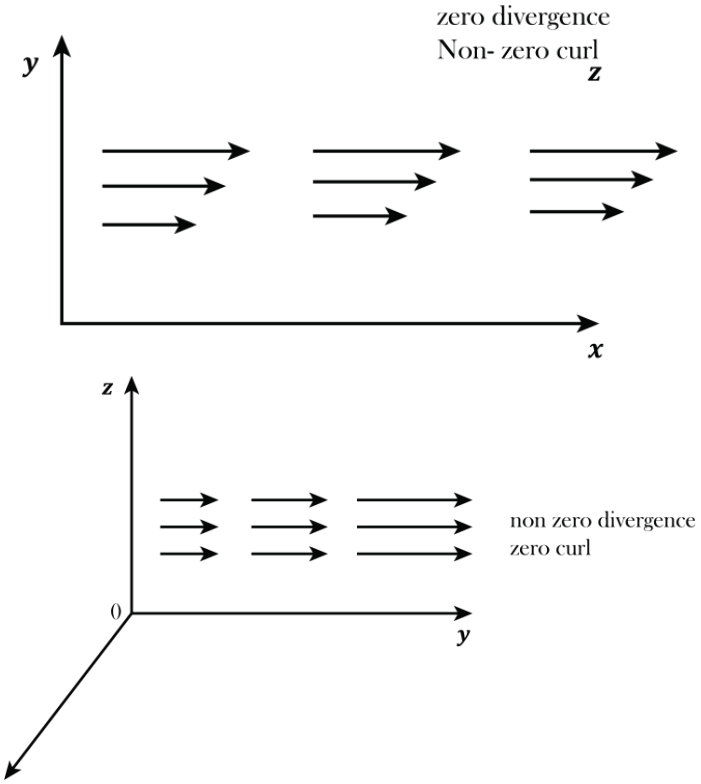
Inward direction is –ve

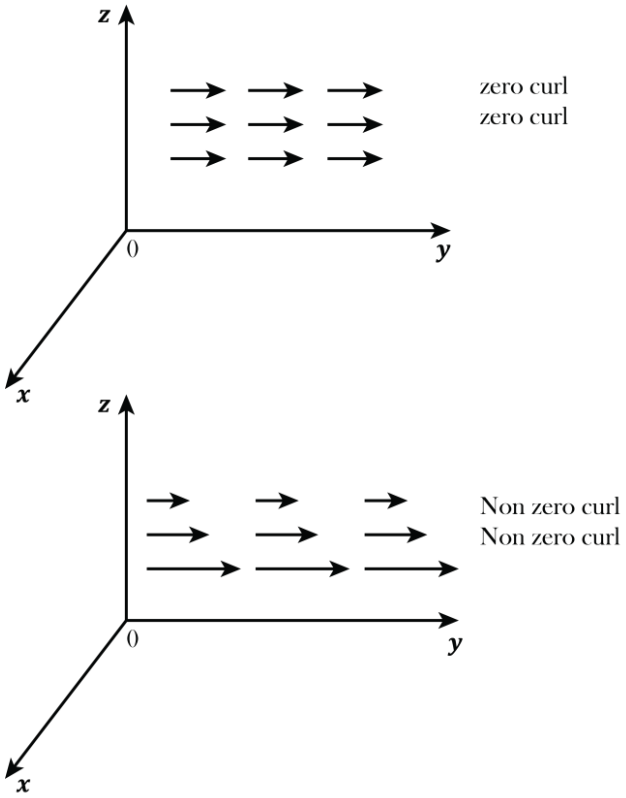
If $\vec{A} = \frac{\partial}{\partial x}A_x + \frac{\partial}{\partial y}A_y + \frac{\partial}{\partial z}A_z$

If the divergence of a vector is zero then it is called solenoidal vector

$\vec{\nabla} \cdot \vec{A} = 0.$

Graphical ____





Curl → It is defined for a vector. Curl of a vector is also a vector. Curl is related to rotation (how much a vector is rotates about a point in the space) It curl of a vector is zero then vector is irrotational.

$\vec{\nabla} \times \vec{A} = \left|\hat{\imath}\hat{\jmath}\hat{k}\, \frac{\partial}{\partial x}\frac{\partial}{\partial y}\frac{\partial}{\partial z}\, A_x A_y A_z\right|$

This is the curl in Cartesian co-ordinate system. If vector in x-direction then it will change in perpendicular direction i.e. A_x change with y & z axis then curl is non zero but if A_x is changes with x-axis then we have zero curl.

Table for general formula

System	u	ν	w	f	g	h
				1	1	1
C	$\boldsymbol{x}$	$\boldsymbol{y}$	$\boldsymbol{y}$			
				1	r	r sin θ
S.P	r	θ	∅			
C$\boldsymbol{y}$	S	∅	$\boldsymbol{y}$	1	r	1

Gradient

$\underline{\nabla} T = \frac{1}{F}\frac{\partial T}{\partial u}\hat{u} + \frac{1}{g}\frac{\partial T}{\partial v}\hat{v} + \frac{1}{h}\frac{\partial T}{\partial w}\hat{w}$

$\vec{\nabla}.\vec{A} = \frac{1}{fgh}\left[\frac{\partial}{\partial u}(gh\, A_u) + \frac{\partial}{\partial v}(fh\, A_v) + \frac{\partial}{\partial w}(fg\, A_w)\right]$

Curl

$\vec{\nabla} \times \vec{A} = \frac{1}{fgh}\left|f\hat{u}g\hat{v}h\hat{w}\, \frac{\partial}{\partial u}\frac{\partial}{\partial v}\frac{\partial}{\partial w}\, fA_u gA_v hA_w\right|$

Laplacian

Laplacian is defined for both scalar and vector

$\vec{\nabla}.\vec{\nabla} = \nabla^2$

$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$

$\nabla^2 A = \frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} + \frac{\partial^2 A}{\partial z^2}$

If $\nabla^2 A = 0$ Laplacian equation

Spherical polar

$\nabla^2 A = \frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} A + \frac{1}{r^2 sin\,\theta}\frac{\partial}{\partial \theta} sin\,\theta\ \frac{\partial}{\partial \theta} A + \frac{1}{r^2 \theta}\frac{\partial^2 A}{\partial \phi^2}$

Cylindrical

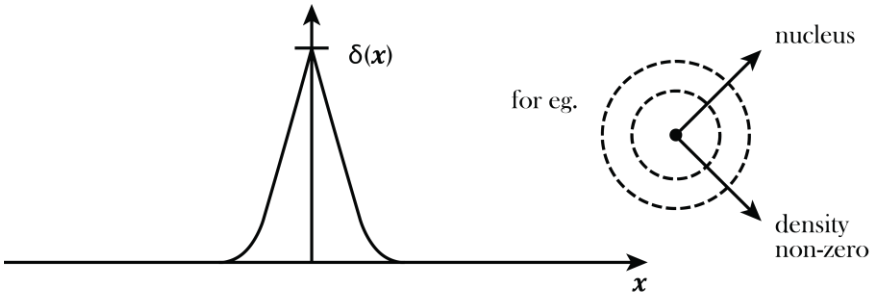
$\nabla^2 A = \frac{1}{r}\frac{\partial}{\partial r}\ r\frac{\partial A}{\partial r} + \frac{1}{r^2}\frac{\partial^2 A}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}$

Dirac-Delta Function:-This is not a ordinary function but generalized function or distribution function. It is has the property that it is ∞ when $x = 0$

i.e. $\delta(x) = \infty (x = 0)$

$= 0\ (x \neq 0)$

Where this function is centralized it is ∞ at other point, it is 0.



Physical ex- density

$\vec{A} = \frac{\hat{r}}{r^2} = \frac{\hat{r}}{r^2}$

$\nabla . \vec{A} = \nabla . \left(\frac{\hat{r}}{r^2}\right) = 4\pi \oint^3 (r)$

$4\pi \int_{-1}^{1} \delta^3(r) d\tau \neq 0 \& 4\pi \int_{1}^{2} \delta^3(r) dz = 0$

This Integral will be non-zero when limits of r enclose it

Properties

For 1 dim

1. $\int_{-\infty}^{\infty} \delta(x) dx = 1$
2. $f(x)\delta(x) = f(0)\delta(x)$
3. $\int_{-\infty}^{\infty} f(x)\delta(x) dx = f(0) \int_{-\infty}^{\infty} \delta(x) dx = f(0)$

Limit must enclose the origin of dirac delta function.

If centre at $x = a$ then

$\delta(x - a) = \infty x = a$

$= 0\ x \neq a$

$\int_{-\infty}^{\infty} \delta(x - a) dx = 1$

E.g. $\delta(x - 2)$ centre at 2

$\int_{3}^{4} \delta(x - 2) dx = 0$

Because 2 is not between 3 & 4 i.e. limits are not enclosing $x = 2$.

2. $f(x)\delta(x - a) = f(a)\delta(x - a)$

3. $\int_{-\infty}^{\infty} f(x)\delta(x - a) dx = f(a) \int_{-\infty}^{\infty} \delta(x - a) dx$

$= f(a)$

Area of Dirac-Delta function is 1.

(height × width = finite)

If $\delta(kx) = \frac{1}{|K|}\delta(x)$

E.g.:- $\delta(-3x) = \frac{1}{|3|}\delta(x)$

Dirac Delta Function is Even Function

$\delta(-x) = \delta(x)$

Derivative of dirac delta function

$\frac{1}{dx}\delta(x) = \delta'(x) = \frac{-1}{x}\delta(x)$

If there is a polynomial is dirac delta function as $\delta[x^2 + 2x + 1]$

First find the roots $x = -1, -1$

If $\delta[(x-a)(x-b)]$ is a dirac delta function who has two centres then.

$\delta[(x-a)(x-b)] = \frac{1}{|a-b|}[\delta(x-a) + \delta(x-b)]$

$\delta(x^2 - a^2) = \frac{1}{2|a|}[\delta(x-a) + \delta(x+a)], a \neq 0$

 # For nth Derivative of Dirac-Delta Function

$\int_{-\infty}^{\infty} f(x)\delta^n(x)dx = (-1)^n f^n(0)$

E.g. $\int_{-\infty}^{\infty} x^2\delta'(x)\,dx = (-1)^1\,2x$

$= 0$

 # For nth derivative of dirac-delta function

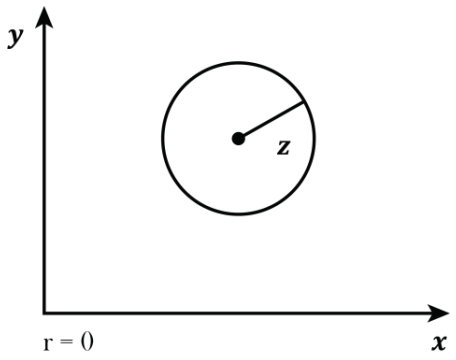
Spherical-polar $\delta^3(r)$

Cartesian $\delta(x)\delta(y)\delta(z)$

Properties

(1) $\int_{all\ space}$ If limit of r enclosing the $r = 0$ then its value is 1 otherwise 0

E.g. for this sphere



$\int \delta^3(r)dr = 0$

Because sphere is not enclosing centre at $r = 0$

(2) $\underline{\nabla}.\left(\frac{\hat{r}}{r^2}\right) = 4\pi\delta^2(r)$

(3) $f(r)\delta^2(r) = f(0)\delta^3(r)$

Note $\underline{\nabla}.\underline{E} = \frac{\delta}{\epsilon_0}$

$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$

In $\vec{\nabla}.\vec{E}$ we use $\vec{\nabla}.\left(\frac{\hat{r}}{r^2}\right) = 4r\delta^3(r)$

& $\delta^3(r) = 1, r = 0$

$= 0, r \neq 0$

Ex. $\int_{-1}^{1} 9x^2\delta(3x+1)dx$

$\frac{1}{3}\int_{-1}^{1}\frac{1}{3}9\left(\frac{-1}{3}\right)^2=\frac{1}{3}$

Ex. $\int_{-1}^{3}(x^2+2)\delta(x^2+5x+4)$

$\int_{-1}^{3}(x^2+2)\delta[(x+4)(x+1)]$

$\delta[f(x)]=\sum\frac{\delta(x-xi)}{|f'(xi)|}$

$\delta(x-a)\delta(b-x)=\delta(b-a)$

SOLVED EXAMPLE

1. For the function $\phi=x^2y+yz$ at the point $(1,2,-1)$ find its rate of change with distance in the direction $\vec{a}=\hat{\imath}+2\hat{\jmath}+3\hat{k}$

(a) $10/\sqrt{14}$ (b) $5/\sqrt{14}$

(c) $20/\sqrt{14}$ (d) 0

Solution:

$\nabla\phi=\left(\hat{\imath}\frac{\partial}{\partial x}+\hat{\jmath}\frac{\partial}{\partial y}+\hat{k}\frac{\partial}{\partial z}\right)(x^2y+yz)$

$=2xy\,\hat{\imath}+(x^2+z)\hat{\jmath}+y\,\hat{k}$

Therefore $\nabla\phi|_{(1,2,-1)}=4\hat{\imath}+2\hat{k}$

Therefore, rate of change with distance in the direction $\vec{a}=\hat{\imath}+2\hat{\jmath}+3\hat{k}$ is

$(\vec{\nabla}\phi)(\hat{a})=\left(4\hat{\imath}+2\hat{k}\right)\frac{\hat{\imath}+2\hat{\jmath}+3\hat{k}}{\sqrt{14}}$

$=\frac{4+6}{\sqrt{14}}$

$=\frac{10}{\sqrt{14}}$

2. Find value of the integral

$\int_{-5}^{5}(x^2+2x+1)\;S\,(x^2+x-6)dx$

(a) 4.9 (b) 3.4

(c) 8.1 (d) 10.6

Solution:

$\int_{-5}^{5}(x^2+2x+1)\;S\,(x^2+x-6)$

$=\int_{-5}^{5}(x^2+2x+1)\times\frac{1}{5}[S(x-3)+S(x+2)]dx$

$=\frac{1}{5}\int_{-5}^{5}(x^2+2x+1)\,S(x-3)dx+\frac{1}{5}\int_{-5}^{5}(x^2+2x+1)\,S(x+2)dx$

$=\frac{1}{5}[9+6+1]+\frac{1}{5}[4-4+1]=\frac{16}{5}+\frac{1}{5}=\frac{17}{5}$

$=3.4$

3. The value of the integral $\int_0^\infty x\,e^{-x^2/4}\,dx$ is

(a) 10 (b) 1

(c) 0 (d) 2

Solution:

$\int_0^\infty x\,e^{-ax^2}\,dx=\frac{-1}{2a}\int_1^0 dt=\frac{1}{2a}$

$\left[e^{-ax^2}=t\Rightarrow dt=-2ax\,e^{-ax^2}\;\Rightarrow x\,e^{-ax^2}dx-\frac{1}{20}dt\right]$

Now $a=\frac{1}{4}$

$\Rightarrow\frac{1}{2a}=\frac{1}{2\times\frac{1}{4}}=2$

$\Rightarrow$ Correct option is (d)

4. If $\vec{a}=\vec{b}+\lambda\vec{c}$ then , which of the following is correct option (a, b, c) are along unit direction)

(a) $\vec{a}\times\vec{c}=\vec{b}\times\vec{c}$ (b) $\vec{a}\times\vec{c}=-\vec{b}\times\vec{c}$

(c) $\vec{a}.\vec{b}=\lambda\,\vec{b}.\vec{c}$ (d) $\vec{a}.\vec{b}=\vec{C}$

Solution:

$\vec{a}\times\vec{c}$

$\Rightarrow(\vec{b}+\lambda\vec{c})\times\vec{c}$

$=\vec{b}\times\vec{c}+\lambda\,\vec{c}\times\vec{c}$

$=\vec{b}\times\vec{c}$ Here $\vec{c}\times\vec{c}=0$

Option a is correct

5. Find the minimum distance from point P with coordinates (1, 2, 1) to the line $\vec{v}=\vec{a}+\lambda\,\vec{b}$ where $\vec{a}=1+j+\hat{k}$ and $\vec{b}=2i-j+3k$

(a) $\sqrt{\frac{13}{14}}$ (b) $\sqrt{\frac{9}{14}}$

(c) $\sqrt{\frac{11}{14}}$ (d) $\sqrt{\frac{1}{2}}$

Solution:

Show that line passes through the point (1, 1, 1) and has direction $(2i-j+3k)$

$\hat{b}=\frac{1}{\sqrt{14}}(2i-j+3k)$

The position vector of P is

$\vec{P}=\hat{\imath}+2j+k$ We find

$\left|(\vec{p}-\vec{a})\times\vec{b}\right|=\frac{1}{\sqrt{14}}(j\times(2i-j+3k))$

$= \frac{1}{\sqrt{14}}(3\hat{\imath} - 2\dot{k})$

$= \sqrt{\frac{13}{14}}$

6. The position vector of a particle at fine t in cartesian coordinate is given by

$r(t) = 2t^2\hat{\imath} + c(3t-2)\hat{\jmath} + (3t^2-1)\hat{k}$

Find the speed of particle at t = 1 n

(a)= $3\hat{\imath} + 6j + 6k$ (b)= $4\,i + 3\hat{\jmath} + 6\dot{k}$

(c)= $3j - 13j + \dot{k}$ (d)= $4i + 4\,j + 6\dot{k}$

Solution:

$v(f) = \frac{d}{dt}\,r\,(t)$

$= \frac{d}{dt}\,(\,2t^2 + (\,3t-2)j + (3t^2-1)k)$

$= 4\,t\,\hat{\imath} + (3\text{t-2})\text{j} + 6\,t\,\hat{k}$

$= 4\hat{t}\imath + 3\hat{\jmath} + 6t\hat{k}$

$= 4\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$

7. The values of a and b for which the force $F = (axy + z^3)\hat{\imath} + x^2\hat{\jmath} + bxz^2\hat{k}$ is conservative are

(a) $a = 2, b = 3$ (b) $a = 1, b = 3$

(c) $a = 2, b = 6$ (d) $a = 3, b = 2$

Solution: (a)

Function given:

$\vec{F} = (axy + z^3)\hat{\imath} + x^2\hat{\jmath} = bxz^2k$

For function to be conservative

$\vec{\nabla} \times \vec{F} = 0$

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (axy + z^2) & x^2 & bxz^2 \end{vmatrix} = \hat{\imath}|0 - 0| +$$

$\hat{\jmath}|3z^2 - bz^2|$

Thus $b = 3a = 2$

8. Consider the three vectors $\vec{v}_1 = 2\hat{\imath} + 3\hat{k}, \vec{v}_2 = \hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ and $\vec{v}_3 = 5\hat{\imath} + \hat{\jmath} + a\hat{k}$ where $\hat{\imath}, \hat{\jmath}$ and $\hat{k}$are the standard unit vectors in a three-dimensional Euclidean space. These vectors will be linearly dependent if the value of a is

(a) $\frac{31}{4}$ (b) $\frac{23}{4}$

(c) $\frac{27}{4}$ (d) 0

Solution: (a)

Given vector will be linearly dependent If and only if determinant is zero.

Thus,

$$\begin{vmatrix} 2 & 1 & 5 \\ 0 & 2 & 1 \\ 3 & 2 & a \end{vmatrix} = 0$$

$2(2a-2) - (-3) + 5(-6) = 0$

$a = \frac{31}{4}$

9. Let A be a non-singular 3x3 matrix, the columns of which are denoted by the vectors $\vec{a}, \vec{b}\ and\ \vec{c}$ respectively. Similarly$\vec{u}, \vec{v}$ and $\vec{w}$ denote the vectors that form thecorresponding columns of $(A^T)^{-1}$.Which of the following is true?

(a) $\vec{u}.\vec{a} = 0, \vec{u}.\vec{b} = 0, \vec{u}.\vec{c} = 1$

(b) $\vec{u}.\vec{a} = 0, \vec{u}.\vec{b} = 1, \vec{u}.\vec{c} = 0$

(c) $\vec{u}.\vec{a} = 1, \vec{u}.\vec{b} = 0, \vec{u}.\vec{c} = 0$

(d) $\vec{u}.\vec{c} = 0, \vec{u}.\vec{b} = 0, \vec{u}.\vec{c} = 0$

Solution: (c)

Take $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \Rightarrow (A^T)^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/3 \end{bmatrix}$

Column of both the matrices are vector

For A=>vectors $\vec{a}, \vec{b}\ and\ \vec{c}$

$(A^T)^{-1} => vectors\ \vec{u}, \vec{v}$ and $\vec{w}$

We see that

$\vec{u}.\vec{a} = 1.1 + 0.0 + 0.0 = 1$

$\vec{u}.\vec{b} = 1.0 + 0.2 + 0.0 = 0$

$\vec{u}.\vec{c} = 1.0 + 00 + 0.3 = 0$

10. Let $\vec{r}$ denote the position vector of any point in three-dimensional space, and $r = |\vec{r}|$.Then

(a) $\vec{\nabla}.\vec{r} = 0\ and\ \vec{\nabla} \times \vec{r} = \vec{r}/r$

(b) $\vec{\nabla}.\vec{r} = 0\ and\ \nabla^2\,r = 0$

(c) $\vec{\nabla}.\vec{r} = 3\ and\ \nabla^2\vec{r} = \vec{r}/r^2$

(d) $\vec{\nabla}.\vec{r} = 3\ and\ \vec{\nabla} \times \vec{r} = 0$

Solution: (d)

As

$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$\vec{\nabla}.\vec{r} = \left(\frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial x}\hat{\jmath} + \frac{\partial}{\partial x}\hat{k}\right).\left(x\hat{\imath} + y\hat{\jmath} + z\hat{k}\right) =$

$(1 + 1 + 1) = 3$

$\vec{\nabla} \times \vec{r}\left(\frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial x}\hat{\jmath} + \frac{\partial}{\partial x}\hat{k}\right) \times \left(x\hat{\imath} + y\hat{\jmath} + z\hat{k}\right) = 0$

$\vec{\nabla}.\vec{r} = 3\ and\ \vec{\nabla} \times \vec{r} = 0$

PREVIOUS YEAR QUESTIONS

[IIT JAM 2020]

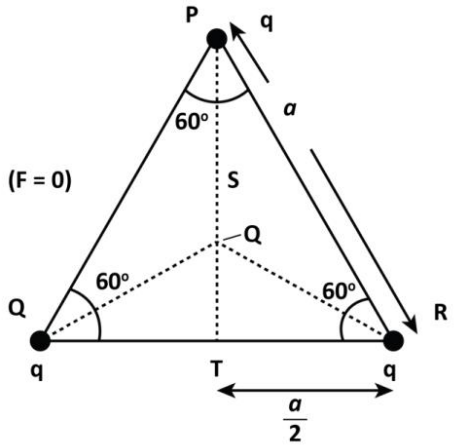
1. Three-point charges q are placed at the corners of an equilateral triangle. Another point charge $-Q$ is placed at the centroid of the triangle. If the force on each of the charges q vanishes, then the ratio Q/ q is

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$

(c) $\frac{1}{3\sqrt{3}}$ (d) $\frac{1}{3}$

Solution: (b)

Force on q should be zero $(F = 0)$



$PT = \sqrt{a^2 - \left(\frac{a}{2}\right)^2} \Rightarrow \sqrt{a^2 - \frac{a^2}{4}} \Rightarrow \sqrt{\frac{3a^2}{4}} \Rightarrow \frac{\sqrt{3}a}{2}$

$PS \Rightarrow \frac{2}{3}$ of PT

$PS = \frac{2}{3} \times \frac{\sqrt{3}a}{2} \Rightarrow \frac{a}{\sqrt{3}}$

Angle of equilateral triple is $60°$. So

Force on q due to all 3 charpes $q, q \ \& - Q$.

$\frac{kq^2 \cos 30°}{a^2} + \frac{kq^2 \cos 30°}{a^2} \Rightarrow k\frac{qQ}{\left(\frac{Q}{\sqrt{3}}\right)^2}$

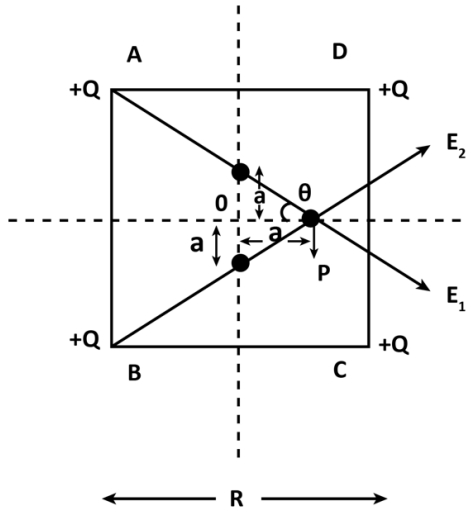
[IIT JAM 2016]

2. Four equal charge of $+Q$, each are kept at the vertices of a square of side R. A particle of mass m and charge $+Q$ is placed in the plane of the square at a short distance $a(\ll R)$ from the centre. If the motion of the particle is confined to the plane, it will undergo small oscillations with an angular frequency

(a) $\sqrt{\frac{Q^2}{2\pi\varepsilon_0 R^3 m}}$ (b) $\sqrt{\frac{Q^2}{\pi\varepsilon_0 R^3 m}}$

(c) $\sqrt{\frac{\sqrt{2}Q^2}{\pi\varepsilon_0 R^3 m}}$ (d) $\sqrt{\frac{Q^2}{4\pi\varepsilon_0 R^3 m}}$

Solution: (c)



Consider particle is at point P which is at a distance (a) from the centre O.

$E_1 \& E_2$ be the electric field due to charges present at A & B resp.

$E_1 = E_2$ because distance & charges are equal due to which we calculate electric field at point P.

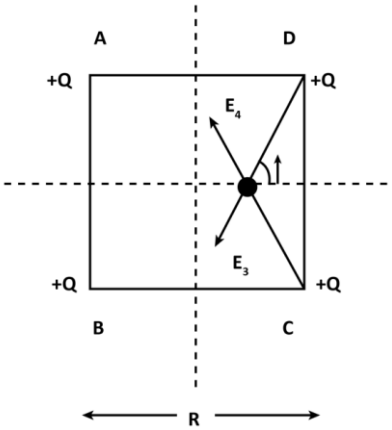
$E_1 = E_2 = \frac{k\,Q}{\left[\left(\frac{R}{2}\right)^2 + \left(\frac{R}{2} + \frac{a}{2}\right)^2\right]}$

Resultant of electric field's present at point P due to charges present at A & B are.

$E_{2,y} = 2\,E_1 \cos\theta$

$E_{12,,y} = \frac{2k\theta}{\left[\left(a + \frac{R}{2}\right)^2 + \left(\frac{R}{2}\right)^2\right]^{\frac{3}{2}}}\left[a + \frac{R}{2}\right) \frac{2k\theta}{\left[\frac{R^2}{2}\right]^{\frac{3}{2}}}\left(a + \frac{R}{2}\right)$

$E_{12,y} \Rightarrow \frac{4\sqrt{2}k\theta}{R^3}\left[a + \frac{R}{2}\right]$ ..... (1)



Similarly, $E_3 = E_4 = \frac{k\theta}{\left[\left(\frac{R}{2} - a\right)^2 + \left(\frac{R}{2}\right)^2\right]}$

Resultant $E_{3\,4,y} = 2E_3 \cos d = \frac{2k\theta}{\left[\left(\frac{R}{2} - a\right)^2 + \frac{R^2}{4}\right]^{\frac{3}{2}}}\left(\frac{R-a}{2}\right)$

$E_{34,y} = \frac{4\sqrt{2}k\theta}{R^3}\left[\frac{R}{2} - a\right]$ ... (2)

Now, add equation (1) & (2)

Resultant $E \Rightarrow \frac{4\sqrt{2}k\theta}{R^3}\left[\left(\frac{R}{2} - a\right) - \left(\frac{R}{2} + a\right)\right] \Rightarrow -\frac{8\sqrt{2}k\theta}{R^3}$

$E \Rightarrow \frac{-8\sqrt{2}}{R^3}\frac{\theta a}{4\pi\epsilon}$

$E \Rightarrow \frac{-2\sqrt{2}\theta}{\pi\epsilon_0 R^3}$

Force on that charge present at point P is $F \Rightarrow QEF \Rightarrow \frac{-2\sqrt{2}Qa}{\pi\epsilon_0 R^3}$

$w \Rightarrow \sqrt{\frac{2\sqrt{2}Q^2}{\pi\epsilon_0 m\, R^3}}$

[IIT JAM 2016]

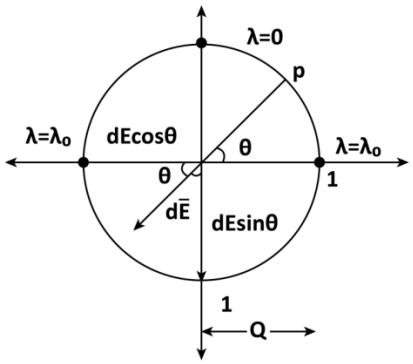
3. The charge per unit length of a circular wire of radius a in the $xy-$plane, with its centre at the origin, is $\lambda = \lambda_0 cos\,\theta$, where λ_0 is a constant and the angle θ is measured from the positive $x-$axis. The electric field at the center of the circle is

(a) $\vec{E} = -\frac{\lambda_0}{4\epsilon_0 a}\hat{\imath}$ (b) $\vec{E} = \frac{\lambda_0}{4\epsilon_0 a}\hat{\imath}$

(c) $\vec{E} = -\frac{\lambda_0}{4\epsilon_0 a}\hat{\jmath}$ (d) $\vec{E} = \frac{\lambda_0}{4\epsilon_0 a}\hat{k}$

Solution: (a)

Given Angle measured From $x - axis$.



Correct option is from a & b.

Electric field due to a charged element at P is.

$dE = -dE\cos\theta\,\hat{\imath} - dE\sin\theta\,\hat{\jmath}$

So, the total electric field at at the centre is.

$\vec{E} = -i \int \frac{\lambda\, dl}{4\pi\epsilon_0 a^2}\cos\theta - \hat{\jmath}\int \frac{\lambda\, dl}{4\pi\epsilon_0 a^2}\sin\theta$

Given :- $\lambda = \lambda_0\cos\theta$

$\vec{E} = -\hat{\imath}\int_0^{2\pi}\frac{\lambda_0 cos^2\,\theta}{4\pi\epsilon_0 a^2}(a\, d\theta) -$

$\hat{\jmath}\int\frac{\lambda_0\cos\theta(a\, d\,\theta)\sin\theta}{4\pi\epsilon_0 a^2}[dl = a\, d\theta]$

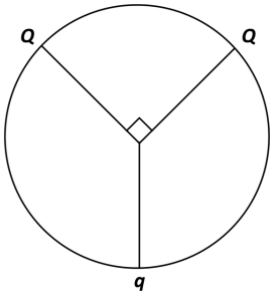
$\vec{E} \Rightarrow \frac{-\hat{\imath}\lambda_0}{4\pi\epsilon_0 a}\int_0^{2\pi} cos^2\theta\, d\,\theta - \hat{\jmath}\int\frac{\lambda_0}{4\pi\epsilon_0 a}\cos\theta\sin\theta\, d\theta$

$\vec{E} \Rightarrow -\hat{\imath}\left(\frac{\lambda_0}{4\pi\epsilon_0 a}\right)\left(\frac{2\pi}{2}\right) = -\frac{\lambda_0}{4\epsilon_0 a}\hat{\imath}$

$\vec{E} = \frac{-\lambda_0\hat{\imath}}{4\epsilon_0 a}$

[IIT JAM]

4. Three charges are located on the circumference of a circle of radius $'R'$ as shown in the figure below. The two charges Q subtend an angle 90˚ at the centre of the circle. The charge 'q' is symmetrically placed with respect to the charges Q. If the electric field at the centre of the circle is zero, what is the magnitude of Q?

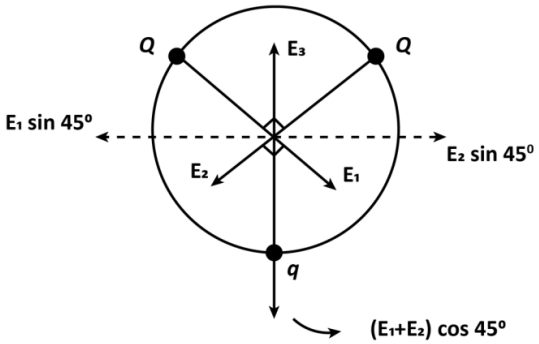


(a)$\frac{q}{\sqrt{2}}$ (b) $\sqrt{2}q$

(c) $2q$ (d) $4q$

Solution: (a)

Electric field due to the two equal charges Q will be



$E_1 = E_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$

Electric field due to the charge q will be $E_3 = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$

Resultant of E_1 and E_2 is $= \sqrt{E_1^2 + E_2^2} = \sqrt{2}E_1$,

$E_1 = E_2 = \frac{\sqrt{2}Q}{4\pi\epsilon_0 R^2} = \frac{q}{4\pi\epsilon_0 R^4}$

$\Rightarrow Q = \frac{q}{\sqrt{2}}$

[IIT JAM]

5. Four equal point charges are kept fixed at the four vertices of a square. How many neutral points (i.e. points where the electric field vanishes) will be found inside the square?

(a) 1 (b) 4

(c) 5 (d) 7

Solution: (c)

Inside the square, there is only five points where the field vanishes due to symmetry one at the centre and other 4 also present due to the 4 charges present at the vertices.

PRACTICE QUESTION

1. For a constant vector field$\vec{a}$, the value of $\nabla\ (\vec{a}.\overrightarrow{r})$ will be
   (a) $4\vec{a}$ (b) 0
   (c) $\vec{a}$ (d) $2\vec{a}$

2. Consider a conservative field$\vec{A}$. What will be the work done by the conservative field along a closed curve $x^2 + y^2 = 1$
   (a) 0 (b) -2
   (c) 2 (d) ½

3. For a vector field $\vec{A} = \frac{-y\,\hat{\imath} + x\,\hat{\jmath}}{x^2 + y^2}$, what will be the work done by $\vec{A}$ over a square having sides 1 unit and centre at (1,1)
   (a) 13 (b) 9
   (c) 0 (d) -25

4. For a vector field $\vec{F} = \frac{-y\,\hat{\imath} + x\,\hat{\jmath}}{x^2 + y^2}$, what will be the work done by $\vec{A}$ over a curve $x^2 + y^2 = 12$
   (a) 0 (b) π
   (c) 2 π (d) - π

5. The vector field $\vec{A} = -(x^2\hat{\imath} + y\,\hat{\jmath})$ is:
   (a) Solenoidal (b) Irrotational
   (c) both a and b (d) none of these

ANSWER KEY				
1	**2**	**3**	**4**	**5**
(c)	(a)	(c)	(c)	(b)

Chapter 2

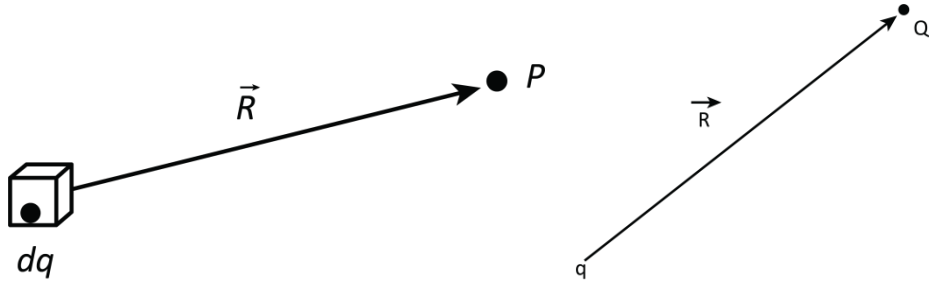
Electric Force & Field

Introduction

The study of electric charge at rest is known as electrostatics. In the present chapter, we shall study two fundamental laws namely Coulomb's law and and Electric field. Based on Coulomb's law, the concept of Electric field, electric potential will be introduced and applied in cases involving point, line, surface and volume charge distribution.

Coulomb's Law and Superposition Principle

The electric force on a test charge Q due to a single point charge q, which is at rest and a distance R apart is given by Coulomb's law $\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{R^2}\hat{R}$.



The constant ε_0 is called the permittivity of free space.

In mks units, $\varepsilon_0 = 8.85 \times 10^{-12}\frac{C^2}{N.m^2}$

$\vec{R}$ is the separation vector from $\vec{r'}$ (the location of q) to $\vec{r}$ (the location of Q): $\vec{R} = \vec{r} - \vec{r'}$: R is its magnitude, and $\hat{R}$ is its direction. The force points along the line from q to Q; it is repulsive if q and Q have the same sign, and attractive if their signs are opposite.

Limitations of Coulomb's Law:

(1) Coulomb's law is applicable in case of point charge at rest. The charged bodies, whose sizes are much smaller than distance between them, are referred as point charges.

(2) This law holds goodover the wide range from 10^{-13}meter to many kilometres. For distances less than 10^{-13}meter, it breaks down due to predominance of nuclear force.

Electric Field

We have seen that like charges repel each other while unlike charges attract each other. To explain how the charges exert a force on each other, even though there is no physical contact between them, the concept of electric field or electrostatic field has been introduced by Faraday. According to this concept, an electric charge produces an electric field in its surrounding space. When another charge is brought in this region of space, it experiences an electric force.

Thus, an electric field is said to exist in the region if another charge experiences an electric force in that region. The strength of electric field at any point in the region is described in terms of Quantity called electric intensity or intensity of electric field. It is denoted by $\vec{E}$ *The intensity of an electric field at any point in an electric field is defined as the force acting on a unit positive charge placed at that point.*

To determine the electric intensity at a point in an electric field, imagine a small positive test charge $q_0$is placed at that point. If $\vec{F}$ is force acting on the test charge, the electric intensity at that point is given by,

$$\vec{E} = \frac{\vec{F}}{q_0}$$

Electric Intensity is a vector quantity and its direction is the direction of force experienced by positive test charge. Here, a test charge q_0 should be so small that it should not change the original electric field.

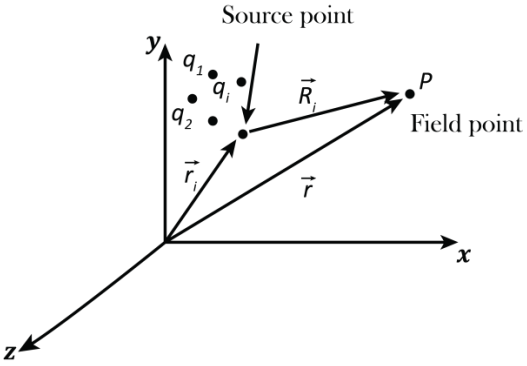
Therefore, above equation in correct form is expressed as,

$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0}$$

If we have many point charges $q_1, q_2, \ldots$ at distances $R_1, R_2, R_3 \ldots$ from test charge Q, then according to the principle of superposition the total force on Q is

$$\vec{F} = \vec{F}_1 + \overrightarrow{F_2} + \cdots = \frac{Q}{4\pi\varepsilon_0}\left(\frac{q_1}{R_1^2}\hat{R}_1 + \frac{q_2}{R_2^2}\hat{R}_2 + \cdots\right)$$

$$\Rightarrow \vec{F} = Q\vec{E}$$



Where $\vec{E}(P) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{R_i^2}\hat{R}_i$

$\vec{E}$ is called the electric field of the source charges. Physically $\vec{E}(P)$ is the force per unit charge that would be exerted on a test charge placed at P.

If charge is distributed continuously over some region, then ***xyz***

$\vec{E}(r) = \frac{1}{4\pi\varepsilon_0}\int_{line}\frac{1}{R^2}\hat{R}dl'$.

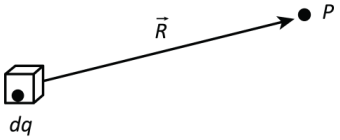
The electric field of a line charge is $(dq = \lambda dl')$

$\vec{E}(r) = \frac{1}{4\pi\varepsilon_0}\int_{line}\frac{\lambda(r')}{R^2}\hat{R}dl'$where λ is charge per unit length.

For surface charge $(dq = \sigma da')$

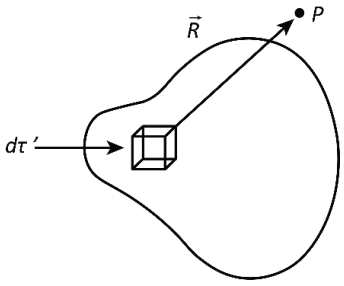
$\vec{E}(r) = \frac{1}{4\pi\varepsilon_0}\int_{surface}\frac{\sigma(r')}{R^2}\hat{R}da'$where σ is charge per unit area.

For a volume charge $(dq = \rho d\tau')$0



(a) Continuous distribution

$\vec{E}(r) = \frac{1}{4\pi\varepsilon_0}\int_{volume}\frac{\rho(r')}{R^2}\hat{R}d\tau'$where ρ is charge per unit volume***λ***



(d) Volume charge, *P*

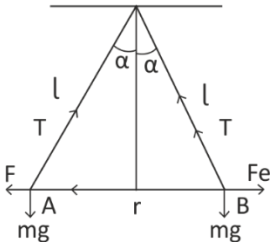
SOLVED EXAMPLE

1. Two-point charges of equal mass m, and charge Q are suspended at a common point by two threads of negligible mass and length l. The system attains equilibrium at angle α such that α is really small. The inclination angle is

(a) $\sqrt[3]{\frac{Q^2}{16\pi\varepsilon_0 mgl^2}}$ (b) $\sqrt[3]{\frac{Q^2}{4\pi\varepsilon_0 mgl^2}}$

(c) $\sqrt[3]{\frac{Q^2}{8\pi\varepsilon_0 mgl^2}}$ (d) $\sqrt[3]{\frac{Q^2}{16\pi\varepsilon_0 mgl^2}}$

Solution:



$Fe \rightarrow$ coloumb force

$T \rightarrow$ Tension in each thread

$mg \rightarrow$Weight of each charge

At A and B

$$T\sin\alpha = Fe$$
$$T\cos\alpha = mg$$
$$\therefore \frac{\sin\alpha}{\cos\alpha} = \frac{Fe}{mg} = \frac{1}{mg}\frac{Q^2}{4\pi\varepsilon_0 r^2}$$

$r = 2l\sin\alpha$ (from trigonometry)

$$Q^2\cos\alpha = 16\pi\varepsilon_0\, mgl^2\sin^3\alpha$$
$$Q^2 = 16\pi\varepsilon_0\, mgl^2\sin^3\alpha tan\,\alpha$$

Now, when $\alpha \rightarrow 0$ then $\sin\alpha \rightarrow a$

$$\tan\alpha = \alpha$$
$$Q^2 = 16\pi\, mgl^2\alpha^3$$
$$\alpha = \sqrt[3]{\frac{Q^2}{16\pi\, mgl^2}}$$

2. The electrostatic potential V(x, y) in free space in a region where the charge density ρ is zero is given V(x,y) $=10e^{2x} - f(x) + y^2$. Given that the x-component of the electric field Ex, and V are zero at the origin, f (x) is

(a) $10e^{2x} - x^2$ (b) $10e^{2x} - x^2 + 5$

(c) $5e^{2x} - x^2$ (d) $5e^{2x} - x^2 + 10$

Solution:

$$V(x,y) = 10\, e^{2x} - f(x) + y^2$$

Since $\rho = 0$ $\nabla^2 V = 0$

$\rightarrow 40e^{2x} - f"(x) + 2 = 0$

Since $E_x = 0$ $E = -\nabla V$

$=-[20e^{2x} - f'(x)] = 0$ at $x = 0$

So we can write

$$-[20 - f'(0)] = 0$$
$$f'(0) = 20$$

V at $(0,0)$ is 0

$$[10 - f(0)] = 0$$
$$f(0) = 10$$

Solve equation

$$40e^{2x} - f"(x) - 2 = 0$$
$$f"(x) = 40e^{2x} - 2$$
$$f'(x) = 20e^{2x} - 2x + c_1$$
$$f'(0) = 20 + c_1$$
$$20 = 20 + c_1$$
$$c_1 = 0$$

Again $f'(x) = 20e^{2x} - 2x + 0$

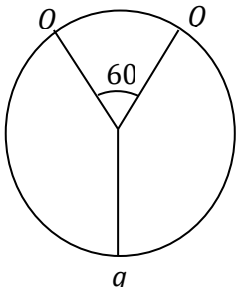
$$f(x) = 10e^{2x} - x^2 + c_2$$
$$f(0) = 10$$
$$10 = 10 - 0 + c_2$$
$$c_2 = 0$$
$$f(x) = 10e^{2x} - x^2$$

3. Three charges are located on the circumference of a circle of radius R . The two charges Q subtend an angle 60^0 at the center of the circle. The charge q is symmetrically placed with respect to the charges Q. If the electric field at the center of the circle is zero, what is the magnitude of Q?

(a) $\frac{q}{\sqrt{3}}$ (b) $\sqrt{3}q$

(c) $\sqrt{2}$q (d) $\frac{q}{\sqrt{2}}$

Solution:



Force at center is 0

This means that net force $due\ to$ to both Q should be equal and opposite due to charge q

Net force due to both $Q, F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 60}$

Where $F_1 = \frac{KQ}{R^2}$ $F_2 = \frac{KQ}{R^2}$

Net force $F = \sqrt{\left(\frac{KQ}{R^2}\right)^2 + \left(\frac{KQ}{R^2}\right)^2 + 2\left(\frac{KQ}{R^2}\right)^2 \times \frac{1}{2}}$

$$F = \sqrt{\left(\frac{KQ}{R^2}\right)^2 + \left(\frac{KQ}{R^2}\right)^2 + \left(\frac{KQ}{R^2}\right)^2}$$

$$F = \sqrt{3\left(\frac{KQ}{R^2}\right)^2} = \sqrt{3}\frac{KQ}{R^2}$$

Now this force must be equal to force due to charge q

$$\frac{Kq}{R^2} = \sqrt{3}\frac{KQ}{R^2}$$

$$q = \sqrt{3}Q$$

$$Q = \frac{q}{\sqrt{3}}$$

4. A charge distribution is given by $\rho=2x^2y^2$ C/m^3• Determine the total charge enclosed by a cube of side 2 m centered at the origin and whose edges are parallel to the axes.

(a) 9/16 C (b) 16/9 C

(c) 10 C (d) 8 C

Solution:

$$\rho = 2x^2y^2$$

Total charge $\int \rho dV$

$dV \rightarrow$ volume element $= dxdydz$

$$\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} 2x^2y^2 \; dxdydz$$

Because cube is at Centre $x\ varies$ From -1 to 1 similarly for y, z

$$Q = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} 2x^2y^2 \; dxdydz$$

$$\iint 2x^2y^2 \, |z|_{-1}^{1} \, dy\, dx$$

$$\iint 2x^2y^2 \, (1+1) dy\, dx$$

$$= \int_{-1}^{1}\int_{-1}^{1} 4x^2y^2 \; dy\, dx$$

$$= \int y^2 \left|\frac{4x^3}{3}\right|_{-1}^{1} dy$$

$$\int y^2 \frac{8}{3} dy = \left|\frac{8y^3}{9}\right|_{-1}^{1} = \frac{16}{9}$$

$= \frac{16}{9}$ C charge

5. Find the force by the lower hemisphere applies on upper hemisphere of a sphere of total charge Q and radius R

(a) $\frac{Q^2}{64\pi\varepsilon_0 R^2}$ (b) $\frac{3Q^2}{128\pi\varepsilon_0 R^2}$

(c) $\frac{3Q^2}{64\pi\varepsilon_0 R^2}$ (d) $\frac{Q^2}{16\pi\varepsilon_0 R^2}$

Solution:

Electric field inside solid sphere

E= $\frac{Qr}{4\pi\epsilon_0 R^3}$

Force per unit volume

$$= f = \rho E = \frac{Q}{\frac{4\pi}{3}R^3}\frac{Qr}{4\pi\epsilon_0 R^3} = \frac{3}{\epsilon_0}\left(\frac{Q}{4\pi R^3}\right)^2 r$$

Force in z direction on $d\tau$ is

$$dF_2 = f_2 d\tau = \frac{3}{\epsilon_0}\left(\frac{Q}{4\pi R^3}\right)^2 r\cos\theta$$

$$(r^2 \sin\theta \; dr d\theta d\phi)$$

The total force on upper hemisphere is

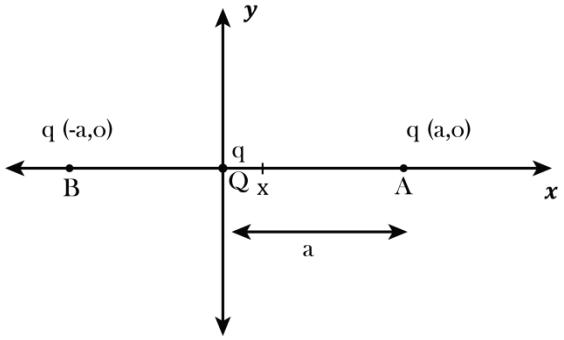
$$F_2 = \int f d\tau$$

$$= \frac{3}{\epsilon_0}\left(\frac{Q}{4\pi R^3}\right)^2 \int_0^R r^3 \, dr \int_0^{\pi/2} \cos\theta \sin\theta \; d\theta \int_0^{2\pi} d\phi$$

$$= \frac{3}{\epsilon_0}\left(\frac{Q}{4\pi R^3}\right)^2 \frac{R^4}{4}\left(-\frac{cos2\theta}{2}\Big|_0^{\pi/2}\right)\frac{2\pi}{2}$$

$$= \frac{3\,Q^2}{6\,4\pi\epsilon_0 R^2}$$

6. If a third charge +q place at Q. What is the force on the charge → zero. If small charge is displaced a small distance x toward the charge placed at A. Find the freq. of oscillation of the charge placed at Q.



Oscillating charges is in stable equilibrium. If –ve is placed between +ve charges then there will be unstable equilibrium.

In simple harmonic motion

$$F \propto +x$$

$$F = kx$$

$$w = \sqrt{\frac{k}{m}} \ldots \text{(i)}$$

If the mass of +q charge is m then resultant force on D

$$\vec{F}_D = \frac{1}{4\pi \in_0}\frac{q^2}{(a-x)}(-\hat{x}) + \frac{1}{4\pi \in_0}\frac{q^2}{(a+x)^2}(\hat{x})$$

$$\vec{F}_0 = \frac{-q^2}{4\pi \in_0}\left[\frac{1}{(a-x)^2} - \frac{1}{(a+x)^2}\right]$$

$$\vec{F}_D = \frac{-q}{4\pi \in_0}\left[\frac{(a+x)^2-(a-x)^2}{a^4}\right] x \ll a$$

$\vec{F}_D = \frac{-q^2 x}{\pi \epsilon_0 a^3}$ .. (ii)

Comparing (i) & (ii)

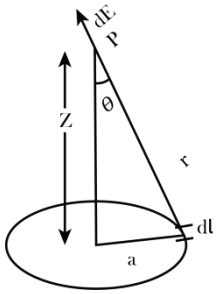
$$kx = \frac{q^2}{\pi \in_0 a^3}$$

$$w = \sqrt{\frac{k}{m}} = \sqrt{\frac{q^2}{\pi \in a^3 m}}$$

$$w = \sqrt{\frac{q^2}{\pi \in_0 a^3 m}}$$

$$f = \frac{w}{2\pi} = \frac{q}{2\pi}\sqrt{\frac{1}{\pi \in_0 a^3 m}}$$

7. Electric field of a circular ring of radius a having uniform line density λ at a dist z above the centre?



$$\overrightarrow{dE} = \frac{kdq}{r^2}\cos\theta$$

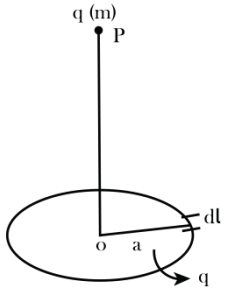
$$\overrightarrow{dE} = \frac{kQ}{2\pi a}\frac{dl}{(a^2+z^2)}\cos\theta\,, \cos\theta = \frac{z}{(r^2+z^2)^{\frac{1}{2}}}$$

$$\overrightarrow{dE} = \frac{KQ}{2\pi a}\frac{z}{(a^2+z^2)^{\frac{3}{2}}}\int dl$$

$$E = \frac{Qz}{4\pi \in_0 (z^2+a^2)^{\frac{3}{2}}}$$

Now if the distance z is small & put a charge q at the small distance z from the centre of ring If total charge = q

Then force on it



$$F = \frac{Kq^2 z}{(z^2+a^2)^{\frac{3}{2}}}$$

$$F = \frac{kq^2 z}{a^3}$$

$$F \propto z$$

$$F = kz$$

$$k = \frac{q^2}{4\pi \in_0 a^3}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$T = 2\pi\sqrt{\frac{m4\pi \in_0 a^3}{q^2}}$$

$$w = \sqrt{\frac{k}{m}} = \frac{q}{2a}\sqrt{\frac{1}{\pi \in_0 am}}$$

8. An electric field in a region is given by $\vec{E}(x,y,z) = ax\hat{\imath} + cz\hat{\jmath} + 6by\hat{k}$. For which of a, b, c does this represent an electrostatic field?

(a) 13, 1, 12 (b) 17, 6, 1

(c) 13, 1, 6 (d) 45, 6, 1

Solution:- We have $\vec{E} = ax\hat{\imath} + cz\hat{\jmath} + 6by\hat{k}$

For electrostatic field

$\vec{\nabla} \times \vec{E}$ should be zero.

$\therefore \vec{\nabla} \times \vec{E} = 0$

$$\left|\hat{\imath}\hat{\jmath}\hat{k}\frac{\partial}{\partial x}\frac{\partial}{\partial y}\frac{\partial}{\partial z}axcz6by\right| = 0 \Rightarrow 6b = c$$

Only option (c) satisfy the condition.

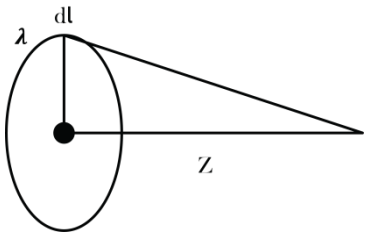
Correct option is (c)

9. A circular loop of radius R, carries a uniform line charge density λ. The electric field. Calculated at a distance z directly above the center of the loop, is maximum if z is equal to?

(a) $\frac{R}{\sqrt{3}}$ (b) $\frac{R}{\sqrt{2}}$

(c) $\frac{R}{2}$ (d) $2R$

Solution:-



$$\phi = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\lambda dl}{\sqrt{R^2+z^2}} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda 2\pi R}{\sqrt{R^2+z^2}}$$

$$\therefore E = \frac{\partial\phi}{\partial z} = \frac{\lambda}{2\varepsilon_0}\frac{Rz}{(R^2+z^2)^{3/2}}$$

For $\vec{E}\frac{\partial E}{\partial z}=0$

$$\therefore \frac{1}{(R^2+z^2)^{\frac{3}{2}}}-\frac{3}{2}\frac{2z^2}{(R^2+z^2)^{\frac{5}{2}}}=0$$

$$\Rightarrow (R^2+z^2)-3z^2=0 \Rightarrow z=\frac{R}{\sqrt{2}}$$

Correct option is (b) $\boldsymbol{\lambda}$

10. A thin conducting ring of radius r has an electric charge $+Q$ what would be the increase in the tension of the wire, if a point charge +q is placed at the centre of the ring?

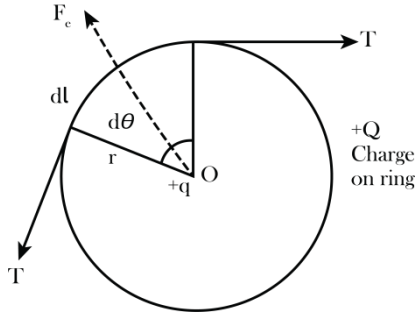
The situation is shown in figure. Charge on a small element dl of the ring $dQ=\frac{Q}{2\pi r}dl$, outward electric force on this element ?$F_e=\frac{1}{4\pi\varepsilon_0}\left(\frac{Qdl}{2\pi r}\right)\left(\frac{q}{r^2}\right)$.

Let the tension be increased by T, to balance this force F_e. The increase in tension is given by

$$=2Tsin\left(\frac{d\theta}{2}\right)\theta$$

$$=2T\left(\frac{d\theta}{2}\right)$$

$$=T\left(\frac{dl}{r}\right)(\because dl=rd\theta)$$

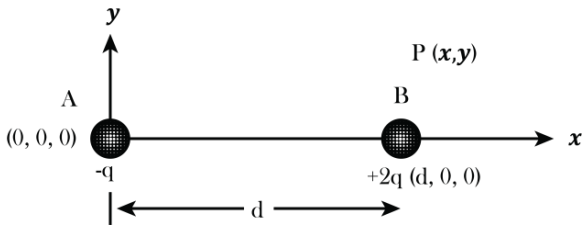


Hence, $T\frac{dl}{r}=-\frac{2}{4\pi\varepsilon_0}\left(\frac{Qdl}{2\pi r}\right)\left(\frac{q}{r^2}\right)$

Therefore, $T=\frac{Qd}{8\pi^2\varepsilon_0 r^2}$.

11. Two points charges $+2q$ and $-q$ are placed distance d apart. Find the locus of all points at which the resultant electric field is parallel to the line joining the charges?

In. Let the two point charges $-q$ and $+2q$ are placed at point A and B respectively which are d distance apart as shown in figure.



Net electronic field at point $P(x,y,0)$ due to this charge distribution is

$$\vec{E}=-\frac{q}{4\pi\varepsilon_0}\left[\frac{x\hat{\imath}+y\hat{\jmath}}{(x^2+y^2)^{\frac{3}{2}}}+2\frac{(x-d)\hat{\imath}+y\hat{\jmath}}{[(x-d)^2+y^2]^{\frac{3}{2}}}\right]$$

If $\vec{E}$ is parallel to line AB

$$\vec{E}\cdot\hat{\jmath}=0$$

$$\frac{y}{(x^2+y^2)^{3/2}}-\frac{2y}{[(x-d)^2+y^2]^{\frac{3}{2}}}=0$$

$$(x-d)^2+y^2=2(x^2+y^2)$$

$$x^2+y^2+2xd+d^2=0$$

$$(x+d)^2+y^2=0$$

$$y=\pm(x+d)$$

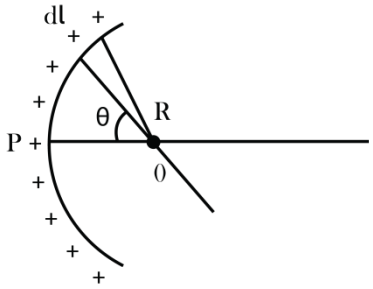
At every point on this straight line $\vec{E}$ is parallel to line AB.

12. A thin half ring of radius $R=20$ cm is uniformly charged with a total charge $q=0.70nC$. Find the field at the centre of curve.

Line charge density?

Solution: $\lambda=\frac{q}{\pi R}$

Therefore, the field at centre due to elementary length θ



$$dE=\frac{1}{4\pi\varepsilon_0}\frac{\lambda dl}{R^2}$$

It has two components, $dEcos\,\theta$ and $dE\,sin\,sin\,\theta$ but to symmetry $dEsin\,\theta$ will be cancelled out.

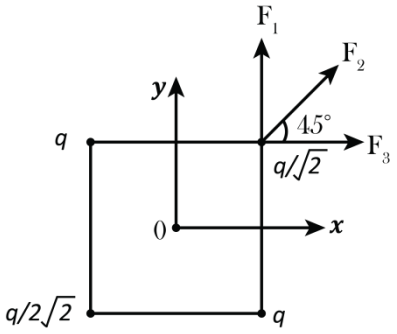
$$\therefore E=\frac{1}{4\pi\varepsilon_0}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{\lambda R d\theta}{R^2}cos\,\theta\,d\theta$$

$$=\frac{q}{4\pi^2\varepsilon_0 R^2}[sin\,\theta\,]_{-\pi/2}^{\pi/2}$$

$$=\frac{q}{4\pi^2\varepsilon_0 R^2}2=\frac{q}{2\pi^2\varepsilon_0 R^2}$$

$$=\frac{0.70\times10^{-9}}{2\times(3.14)^2\times8.85\times(0.2)^2}=0.10\frac{kV}{m}$$

13. Let us choose coordinate system as shown in figure. Three forces acting on $q/\sqrt{2}$ due to other three charges are also shown in the figure?



Therefore, $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$

$$= \frac{q^2/\sqrt{2}}{4\pi\varepsilon_0 a^2}\hat{\imath} + \frac{\frac{q^2}{4}}{4\pi\varepsilon_0\left(a\sqrt{2}\right)^2}(cos45^\circ\hat{\imath} + sin45^\circ\hat{\jmath}) + \frac{q^2/\sqrt{2}}{4\pi\varepsilon_0 a^2}\hat{\jmath}$$

$$= \frac{q^2}{4\sqrt{2}\pi\varepsilon_0 a^2}\left[\hat{\imath} + \frac{1}{8}(\hat{\imath}+\hat{\jmath}) + \hat{\jmath}\right]$$

$$= \frac{9q^2}{32\sqrt{2}\pi\varepsilon_0 a^2}[\hat{\imath}+\hat{\jmath}] \Rightarrow \left|\vec{F}\right| = \frac{9q^2}{32\pi\varepsilon_0 a^2}$$

14. Three charges $q, -q,$ and 2q are placed at $(a,a,a)(a,-a,a)$ and $(a,-a,-a)$ respectively. Calculate net force on $2q$?

Solution. $2q$ will experience force due to two other charges.

Let $q_1 = q, q_2 = -q, q_3 = 2q$

$$\vec{r}_1 = a\left(\hat{\imath}+\hat{\jmath}+\hat{k}\right), \vec{r}_3 = a\left(\hat{\imath}-\hat{\jmath}-\hat{k}\right)$$

Force on $2q$

$$\vec{F} = \vec{F}_{31} + \vec{F}_{32}$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_3 q_1}{r_{31}^3}\vec{r}_{31} + \frac{q_3 q_2}{r_{32}^3}\vec{r}_{32}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{2q^2}{|\vec{r}_3 - \vec{r}_1|^3}(\vec{r}_3 - \vec{r}_1) - \frac{2q^2}{|\vec{r}_3 - \vec{r}_2|}(\vec{r}_3 - \vec{r}_2)\right]$$

$$= \frac{q^2}{2\pi\varepsilon_0}\left[\frac{a\left(-2\hat{\jmath}-2\hat{k}\right)}{16\sqrt{2}a^3} - \frac{a\left(-2\hat{k}\right)}{8a^3}\right]$$

$$= \frac{q^2}{16\pi\varepsilon_0 a^2}\left[-\frac{1}{\sqrt{2}}\left(\hat{\jmath}+\hat{k}-2\sqrt{2}\hat{k}\right)\right]$$

$$= \frac{-q^2}{16\sqrt{2}\pi\varepsilon_0 a^2}\left[\hat{\jmath} + \left(1-2\sqrt{2}\right)\hat{k}\right]$$

$$= \frac{q^2}{16\sqrt{2}\pi\varepsilon_0 a^2}\left[\left(2\sqrt{2}-1\right)\hat{k} - \hat{\jmath}\right]$$

15. A charge distribution has the charge density given by $\rho = Q\{\delta(x-x_0) - \delta(x+x_0)\}$. For this charge distribution the electric field at $(2x_0, 0,0)$?

(a) $\frac{2Q\hat{x}}{9\pi\epsilon_0 x_0^2}$ (b) $\frac{Q\hat{x}}{4\pi\epsilon_0 x_0^3}$

(c) $\frac{Q\hat{x}}{4\pi\epsilon_0 x_0^2}$ (d) $\frac{Q\hat{x}}{16\pi\epsilon_0 x_0^2}$

Solution: We have one dimension charge density

$$\rho = Q\{\delta(x-x_0) - \delta(x+x_0)\}$$

Therefore, the total charge,

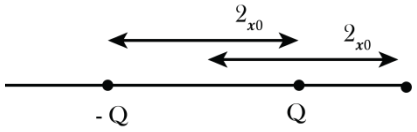
$$Q = \int \rho dx = Q\int \delta(x-x_0)dx - Q\int \delta(x+x_0)dx$$

$$= Q(at\ x = x_0) + (-Q)(at\ x = -x_0)$$

Therefore, the field at $(2x_0, 0,0)$ point is s

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left\{\frac{Q}{x_0^2} - \frac{Q}{(3x_0)^2}\right\}$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{9-1}{9x_0^2}\right] - \frac{2Q}{9\pi\varepsilon_0 x_0^2}\hat{x}$$



Correct option is (a)

16. Consider a particle of electric charge 'e' and mass 'm' moving under the influence of constant horizontal electric field E and constant vertical gravitational field described by acceleration due to gravity 'g'. If particle starts from rest, what will be its trajectory?

(a) Parabolic (b) elliptic

(c) straight line (d) circular

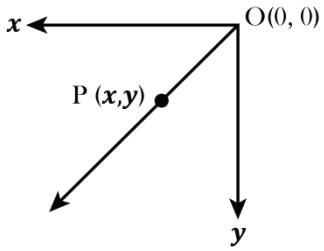
Solution:- Let at $t = 0$, electron origine $(0,0)$ at $t = t$ the particle is at (x, y).

Acceleration along x-axis is $\frac{eE}{m}$ and acceleration along y-axis is g.

Therefore, the position of at a time t is $x = \frac{1}{2}\frac{eE}{m}t^2$ and $y = \frac{1}{2}gt^2$

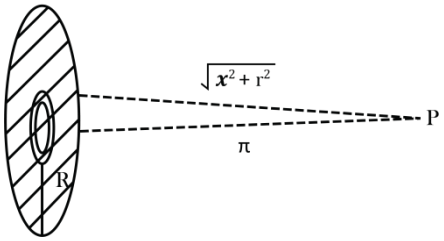
$\therefore \frac{y}{x} = \left(\frac{gm}{eE}\right)$

$$y = \left(\frac{gm}{eE}\right)x$$



It is the equation of straight line.

⇒ Calculate electric field due to a uniformly charged ring.



$$\pi R^2 \to Q$$
$$1 \to \frac{Q}{\pi R^2}$$
$$dq = \frac{Q}{\pi R^2} 2\pi r\, dr$$
$$dE = \frac{kdq}{(x^2+r^2)} \cos\theta$$
$$dE = \frac{kQ}{\pi R^2} \frac{2\pi r\, dr\, x}{(x^2+r^2)^{3/2}}$$
$$dE = K\,\sigma 2\pi x \int_0^R \frac{r\, dr}{(x^2+r^2)^{3/2}}$$

Put $r = x\tan\theta$

$$dr = x\sec^2\theta d\theta$$
$$r\, dr = x^2\theta\tan\theta d\theta$$
$$(x^2+r^2)^{3/2} = (x^2+x^2\tan\theta)^{3/2}$$
$$= x^3\sec^3\theta$$
$$dE = k\sigma 2\pi x \int \frac{x^2\sec^2\theta\tan\theta d\theta}{x^3\sec^3\theta}$$
$$dE = k\,\sigma 2\pi \int \frac{\tan\theta}{\sec\theta} d\theta$$
$$\int dE = k\,\sigma 2\pi \int \sin\theta\, d\theta$$

$$E = -k\sigma 2\pi \left[\frac{x}{\sqrt{x^2+r^2}}\right]_0^R = -k\sigma 2\pi \left[\frac{1}{\sqrt{x^2+R^2}} - \frac{1}{x}\right]$$
$$E = \left[\frac{1}{x} - \frac{1}{\sqrt{R^2+x^2}}\right]$$
$$E = K\sigma 2\pi \left[1 - \frac{x}{\sqrt{R^2+x^2}}\right]$$

17. If E is field of a finite object having total charge Q then what is value of β $E = \frac{k\,Q\,x^\beta}{(x^2+a^2)^{\frac{5}{2}}}$?

Solution: $E = \frac{kQ\,x^\beta}{(x^2+a^2)^{\frac{5}{2}}}$ If $x \gg a$

$$x^2 + a^2 = x^2$$
$$E = \frac{k\,Q\,x^\beta}{x^5}$$

If $\beta = 3$ then $E = \frac{KQ}{x^2}$

This can be possible only if $\beta = 3$

18. $\vec{E} = k\,(xy\,\hat{x} + 2yz\,\hat{y} + 3xz\hat{z})$
$\vec{E} = k(y^2\hat{x} + (2xyz + z^2)\hat{y} + 2yz\,\hat{z})$

Find weather these two electric fields are possible electrostatic fields.

Which one of the following is impossible electrostatic field.

(i) $\vec{E} = k(xy\,\hat{x} + 2y\,z\,\hat{y} + 3xz\hat{z})$

$$\vec{\nabla} \times \vec{E} = k \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & 2yz & 3xz \end{vmatrix}$$
$$= k[\hat{x}(0-2y) - \hat{y}(3z-a) + \hat{z}(0-x)]$$
$$= k[-2y\hat{x} - 3z\hat{y} - x\hat{z}]$$
$$\nabla \times \vec{E} \neq 0$$

(ii) $\vec{E} = k\,[y^2\hat{x} + (2xy + z^2)\hat{y} + 2yz\,\hat{z}]$

$$\nabla \times \vec{E} = k \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 & 2xy + z^2 & 2yz \end{vmatrix}$$
$$= k\,[\hat{x}(2x - 2x) = \hat{y}(0-0) + \hat{z}(2y - 2y)]$$
$$= k[0\hat{x} + 0\hat{y} + 0\hat{z}] = 0$$

(i) Impossible electric field

(ii) Possible electric field

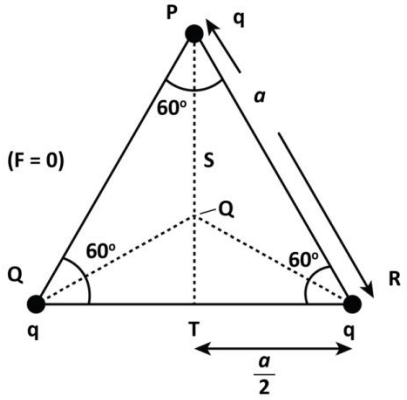
19. Three-point charges q are placed at the corners of an equilateral triangle. Another point charge $-Q$ is placed at the centroid of the triangle. If the force on each of the charges q vanishes, then the ratio Q/ q is

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$

(c) $\frac{1}{3\sqrt{3}}$ (d) $\frac{1}{3}$

Solution:- (b)

Force on q should be zero $(F = 0)$



$$PT = \sqrt{a^2 - \left(\frac{a}{2}\right)^2} \Rightarrow \sqrt{a^2 - \frac{a^2}{4}} \Rightarrow \sqrt{\frac{3a^2}{4}} \Rightarrow \frac{\sqrt{3}a}{2}$$

$PS \Rightarrow \frac{2}{3}$ of PT

$$PS = \frac{2}{3} \times \frac{\sqrt{3}a}{2} \Rightarrow \frac{a}{\sqrt{3}}$$

Angle of equilateral triple is $60°$. So

Force on q due to all 3 charpes q, q & $-Q$.

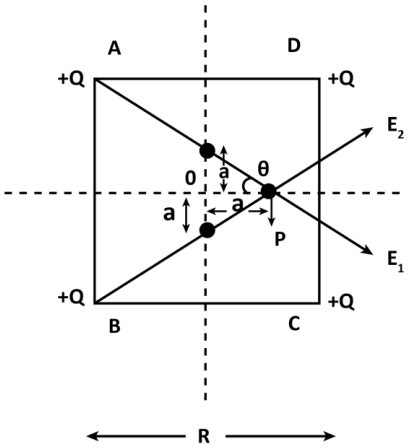
$$\frac{kq^2\cos 30°}{a^2} + \frac{kq^2\cos 30°}{a^2} \Rightarrow k\frac{qQ}{\left(\frac{Q}{\sqrt{3}}\right)^2}$$

20. Four equal charge of $+Q$, each are kept at the vertices of a square of side R. A particle of mass m and charge $+Q$ is placed in the plane of the square at a short distance $a(\ll R)$ from the centre. If the motion of the particle is confined to the plane, it will undergo small oscillations with an angular frequency

(a) $\sqrt{\frac{Q^2}{2\pi\varepsilon_0 R^3 m}}$ (b) $\sqrt{\frac{Q^2}{\pi\varepsilon_0 R^3 m}}$

(c) $\sqrt{\frac{\sqrt{2}Q^2}{\pi\varepsilon_0 R^3 m}}$ (d) $\sqrt{\frac{Q^2}{4\pi\varepsilon_0 R^3 m}}$

Solution:- (c)



Consider particle is at point P which is at a distance (a) from the centre O.

E_1& E_2 be the electric field due to charges present at A & B resp.

$E_1 = E_2$ because distance & charges are equal due to which we calculate electric field at point P.

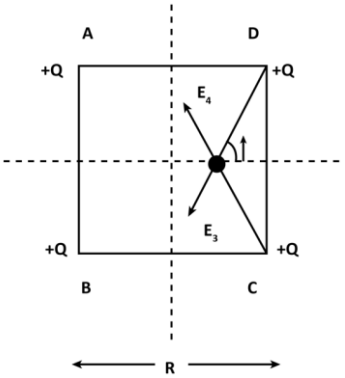
$$E_1 = E_2 = \frac{k\,Q}{\left[\left(\frac{R}{2}\right)^2 + \left(\frac{R}{2}+\frac{a}{2}\right)^2\right]}$$

Resultant of electric field's present at point P due to charges present at A & B are.

$$E_{2,y} = 2\,E_1 \cos\theta$$

$$E_{12,,y} = \frac{2k\theta}{\left[\left(a+\frac{R}{2}\right)^2 + \left(\frac{R}{2}\right)^2\right]^{\frac{3}{2}}}\left[a+\frac{R}{2}\right]\frac{2k\theta}{\left[\frac{R^2}{2}\right]^{\frac{3}{2}}}\left(a+\frac{R}{2}\right)$$

$E_{12,y} \Rightarrow \frac{4\sqrt{2}k\theta}{R^3}\left[a+\frac{R}{2}\right]$ ..... (1)



Similarly, $E_3 = E_4 = \frac{k\theta}{\left[\left(\frac{R}{2}-a\right)^2 + \left(\frac{R}{2}\right)^2\right]}$

Resultant $E_{3\,4,y} = 2E_3 \cos d = \frac{2k\theta}{\left[\left(\frac{R}{2}-a\right)^2 + \frac{R^2}{4}\right]^{\frac{3}{2}}}\left(\frac{R-a}{2}\right)$

$E_{34,y} = \frac{4\sqrt{2}k\theta}{R^3}\left[\frac{R}{2}-a\right]$ ... (2)

Now, add equation (1) & (2)

Resultant $E \Rightarrow \frac{4\sqrt{2}k\theta}{R^3}\left[\left(\frac{R}{2}-a\right)-\left(\frac{R}{2}+a\right)\right] \Rightarrow -\frac{8\sqrt{2}k\theta}{R^3}$

$$E \Rightarrow \frac{-8\sqrt{2}}{R^3}\frac{\theta a}{4\pi \in}$$

$$E \Rightarrow \frac{-2\sqrt{2}\theta}{\pi \in_0 R^3}$$

Force on that charge present at point P is $F \Rightarrow QEF \Rightarrow \frac{-2\sqrt{2}Qa}{\pi\in_0 R^3}$

$$w \Rightarrow \sqrt{\frac{2\sqrt{2}Q^2}{\pi \in_0 m\, R^3}}$$

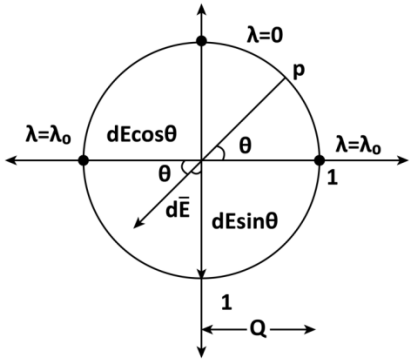
21. The charge per unit length of a circular wire of radius a in the $xy-$plane, with its centre at the origin, is $\lambda = \lambda_0 cos\,\theta$, where λ_0 is a constant and the angle θ is measured from the positive $x-$axis. The electric field at the center of the circle is

(a) $\vec{E} = -\frac{\lambda_0}{4\in_0 a}\hat{\imath}$ (b) $\vec{E} = \frac{\lambda_0}{4\in_0 a}\hat{\imath}$

(c) $\vec{E} = -\frac{\lambda_0}{4\in_0 a}\hat{\jmath}$ (d) $\vec{E} = \frac{\lambda_0}{4\in_0 a}\hat{k}$

Solution:- (a)

Given Angle measured From $x-axis$.



Correct option is from a & b.

Electric field due to a charged element at P is.

$$dE = -dE cos\,\theta\,\hat{\imath} - dE\sin\theta\,\hat{\jmath}$$

So, the total electric field at at the centre is.

$$\vec{E} = -i\int \frac{\lambda\, dl}{4\pi \in_0 a^2}\cos\theta - \hat{\jmath}\int \frac{\lambda\, dl}{4\pi \in_0 a^2}\sin\theta$$

Given:- $\lambda = \lambda_0 \cos\theta$

$$\vec{E} = -\hat{\imath}\int_0^{2\pi} \frac{\lambda_0 cos^2\,\theta}{4\pi\,\epsilon_0\,a^2}(a\,d\theta)$$

$$-\hat{\jmath}\int \frac{\lambda_0 \cos\theta\,(a\,d\,\theta)\sin\theta}{4\pi\,\epsilon_0\,a^2}[dl = a\,d\theta]$$

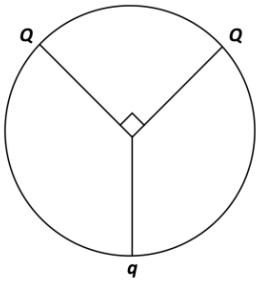
$$\vec{E} \Rightarrow \frac{-\hat{\imath}\lambda_0}{4\pi\,\epsilon_0\,a}\int_0^{2\pi} cos^2\theta\,d\,\theta$$

$$-\hat{\jmath}\int \frac{\lambda_0}{4\pi\,\epsilon_0\,a}\cos\theta\sin\theta\,d\theta$$

$$\vec{E} \Rightarrow -\hat{\imath}\ \left(\frac{\lambda_0}{4\pi\,\epsilon_0\,a}\right)\left(\frac{2\pi}{2}\right) = -\frac{\lambda_0}{4\epsilon_0 a}\hat{\imath}$$

$$\vec{E} = \frac{-\lambda_0\hat{\imath}}{4\,\epsilon_0\,a}$$

22. Three charges are located on the circumference of a circle of radius $'R'$ as shown in the figure below. The two charges Q subtend an angle 90˚ at the centre of the circle. The charge 'q' is symmetrically placed with respect to the charges Q. If the electric field at the centre of the circle is zero, what is the magnitude of Q?

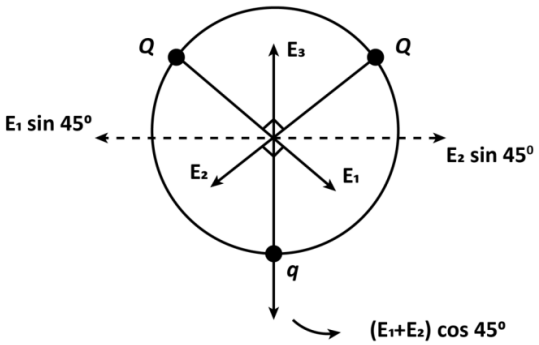


(a) $\frac{q}{\sqrt{2}}$ (b) $\sqrt{2}q$

(c) $2q$ (d) $4q$

Solution: (a)

Electric field due to the two equal charges Q will be



$$E_1 = E_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$$

Electric field due to the charge q will be $E_3 = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$

Resultant of E_1 and E_2 is $= \sqrt{E_1^2 + E_2^2} = \sqrt{2}E_1$,

$E_1 = E_2 = \frac{\sqrt{2}Q}{4\pi\epsilon_0 R^2} = \frac{q}{4\pi\epsilon_0 R^4}$

$\Rightarrow Q = \frac{q}{\sqrt{2}}$

23. Four equal point charges are kept fixed at the four vertices of a square. How many neutral points (i.e. points where the electric field vanishes) will be found inside the square?

(a) 1 (b) 4

(c) 5 (d) 7

Solution:- (c)

Inside the square, there is only five points where the field vanishes due to symmetry one at the centre and other 4 also present due to the 4 charges present at the vertices.

PREVIOUS YEAR QUESTIONS

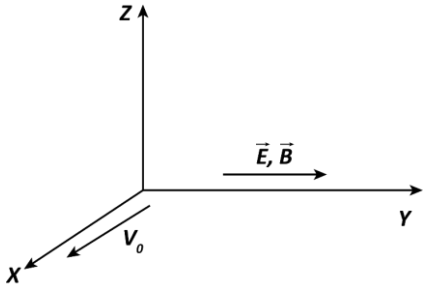
[JAM 2015]

1. A positively charged particle, with a charge q, enters a region in which there is a uniform electric field $\vec{E}$ and a uniform magnetic field $\vec{B}$, both directed parallel to the positive y -axis. At $= 0$, the particle is at the origin and has a speed v_0 directed along the positive x - axis. The orbit of the particle, projected on the x-z plane, is a circle. Let T be the time taken to complete one revolution of this circle. The y -coordinate of the particle at $t = T$ is given by

(a) $\frac{\pi^2\,mE}{2\,qB^2}$ (b) $\frac{2\pi^2\,mE}{qB^2}$

(c) $\frac{\pi^2\,mE}{qB^2} + \frac{v_0\pi m}{qB}$ (d) $\frac{2\pi m v_0}{qB}$

Solution: (b)



From newton's law of motion displacement in y direction, $y = u_y t + \frac{1}{2}a_y t^2$

$$y = \frac{1}{2}\times\frac{qE}{m}(T)^2$$

[T is the time period of revolution T=$\frac{2\pi\ \ m}{qB}$]

$$= \frac{2\pi^2 mE}{qB^2}$$

[JAM 2014]

2. A particle of mass m carrying charge q is moving in a circle in a magnetic field B. According to Bohr's model, the energy of the particle in the nth level is

(a) $\frac{1}{n^2}\left(\frac{hqB}{\pi m}\right)$ (b) $n\left(\frac{hq\,B}{\pi\, m}\right)$

(c) $n\left(\frac{hq\,B}{2\,\pi\, m}\right)$ (d) $n\left(\frac{hq\,B}{4\pi\, m}\right)$

Solution:- (d)

According to Bohr model, Energy of particle in n [th] orbit $E_n = \frac{mv^2}{2}$ ..... (1)

Angular momentum of the particle L $= mv_n r_n = n\hbar$ .....(2)

Radius of an orbit of a particle moving in a magnetic field, $r_n = \frac{mv_n}{qB}$ .....(3)

By putting the value of $r_n from\ equation\ (3)\ to\ (1) \Rightarrow mv_n = \frac{n\hbar qB}{mv_n}$

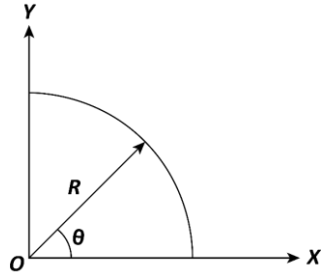
$mv_n^2 = \frac{n\hbar qB}{m}$

$Energy\ E_n = \frac{1}{2}mv_n^2 = \frac{nhqB}{4\pi m}$

$= n\left(\frac{qBh}{4\pi\, m}\right)$

[JAM 2012]

3. A segment of a circular wire of radius R, extending from $= 0\ to\ \pi/2$, carries a constant linear charge density λ . The electric field at origin O is



(a) $\frac{\lambda}{4\pi\varepsilon_0 R}(-\hat{x} - \hat{y})$

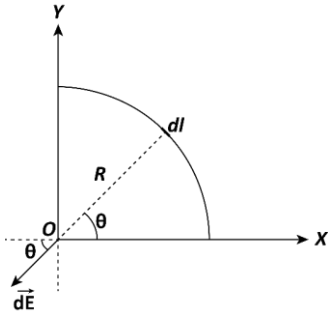
(b) $\frac{\lambda}{4\pi\varepsilon_0 R}\left(-\frac{1}{\sqrt{2}}\hat{x} - \frac{1}{\sqrt{2}}\hat{y}\right)$

(c) $\frac{\lambda}{4\pi\varepsilon_0 R}\left(-\frac{1}{2}\hat{x} - \frac{1}{2}\hat{y}\right)$

(d) 0

Solution:- (a)

As seen in figure $\vec{E} = -E_x\hat{x} - E_y\hat{y}$



Where $E_x = \int_{line}^{\cdot} dE\cos\theta\ and\ E_y = \int_{line}^{\cdot} dE\sin\theta$

And applying coulomb law, $dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dl}{R^2}$

$E_x = \int_{line}^{\cdot}\frac{1}{4\pi\varepsilon_0}\frac{\lambda dl}{R^2}\cos\theta = \frac{\lambda}{4\pi\varepsilon_0}\int_0^{\pi/2}\cos\theta\ \frac{Rd\,\theta}{R^2}$

$E_x = \frac{\lambda}{4\pi\varepsilon_0 R}[\sin\theta]_0^{\pi/2} = \frac{\lambda}{4\pi\varepsilon_0 R}$

In the same way calculating E_y,

$E_y = \int_{line}^{\cdot}\frac{1}{4\pi\varepsilon_0}\frac{\lambda dl}{R^2}\sin\theta = \frac{\lambda}{4\pi\varepsilon_0}\int_0^{\pi/2}\sin\theta\ \frac{Rd\,\theta}{R^2}$

$\Longrightarrow E_y = \frac{\lambda}{4\pi\varepsilon_0 R}[-\cos\theta]_0^{\pi/2} = \frac{\lambda}{4\pi\varepsilon_0 R}$

Therefore $\vec{E} = -E_x\hat{\imath} - E_y\hat{\jmath}$

$$E_x = \frac{\lambda}{4\pi\varepsilon_0 R}(-\hat{\imath} - \hat{\jmath})$$

[JAM 2005]

4. Two point charges $+q_1$ and $+q_2$ are fixed with a finite distance d between them. It is desired to put a third charge q_3 in between these two charges on the line joining them so that the charge q_3 is in equilibrium. This is:

(a) possible only if q_3 is positive

(b) possible only if q_3 is negative

(c) possible irrespective of the sign of q_3

(d) not possible at all

Solution:-(c)

Equilibrium position not depend on the sign of the chargethat we put in between the two point charges. It depends on the magnitude of charges in between we placed q_3.

PRACTICE PROBLEM

1. In terms of mass (M), length (L), time (T) and charge (C) dimension of permittivity is:

(a) $ML^3T^{-2}C^{-2}$ (b) $M^{-1}L^{-3}T^2C^2$

(c) $M^{-1}L^{-2}T^2C^2$ (d) $ML^2T^{-2}C^{-2}$

2. A charge q is placed at the center of the line joining two equal charges Q. The system of these charges will be in equilibrium of q is equal to

(a) $\frac{Q}{2}$ (b) $-\frac{Q}{2}$

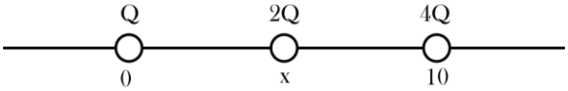
(c) $\frac{Q}{4}$ (d) $-\frac{Q}{4}$

3. The ratio of repulsive Coulomb force and the attractive gravitational force between two protons is

(a) 1.24×10^{36} (b) 2.31×10^{39}

(c) 3.21×10^{26} (d) 1.91×10^{32}

4. Three positively charged particles lie on a straight line at positions $0, x$ and 10 as indicated in the figure below. Their charges are $Q, 20$ and $4Q$ cm respectively.



If the charges at $x = 0$ and $x = 10$ are fixed and the charge at x is movable, the system will be in equilibrium when $x =$

(a) 8 (b) 2

(c) 20/3 (d) 10/3

5. Two points charges $+q_1$ abd $+q_2$ are fixed with a finite a distance 'd' between them. It is desired to put a third charge q_1 in between these two charges on the line joining them so that the charges q_1 is in equilibrium. That is:

(a) Possible only if q_3 is positive

(b) Possible only if q_3 is negative

(c) Possible irrespective of the sign of q_3.

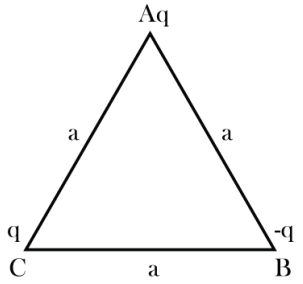
(d) Not possible at all.

6. The ratio F_c/F_G of the electrostatic Coulomb force F_C to the gravitational force F_G between the proton and the electron in the first Bohr orbit (or radius r_B) of a hydrogen atom is closest to the following value.

(a) 2×10^{39} (b) 2×10^{49}

(c) 2×10^{42} (d) $-2 \times 10^{35}/r_B$

7. Three point charges are placed at the corners of an equilateral triangle of side 'a' as shown in the figure. Magnitude of force on charge at A is:



(a) $\frac{\sqrt{3}Kq^2}{a^2}$ (b) $\frac{Ka^2}{a^2}$

(c) $\frac{2Kq^2}{a^2}$ (d) $\frac{Kq^2}{2a^2}$

8. In the previous question if a point charge q is placed at the centroid of the triangle force on it will be:

(a) $\frac{2Ka^2}{a^2}$ (b) $\frac{3Kq^2}{a^2}$

(c) $\frac{6Kq^2}{a^2}$ (d) $\frac{3\sqrt{3}Kq^2}{a^2}$

9. Consider two points charges q and λq located at the points, $x = a$ and $x = \mu a$, respectively. Assuming that the sum of the two charges is constant, what is the value of λ for which the magnitude of the electrostatic force is maximum?

(a) μ (b) 1

(c) $\frac{1}{\mu}$ (d) $1 + \mu$

10. Five negative electric charges of magnitude e are placed symmetrically on a circle of radius R. The magnitude of the electric field at the center of the circle is

(a) $\frac{e}{4\pi\varepsilon_0 R^2}$ (b) 0

(c) $\frac{e}{4\pi\varepsilon_0 R^2}\sin\frac{2\pi}{5}$ (d) $\frac{5e}{4\pi\varepsilon_0 R^2}$

11. When five equal point charges are placed at the corners of regular hexagon, electric field the centre is E_0. If another charge is removed such that charges from two adjacent corners are missing then electric field at centre will be

(a) $2E_0$ (b) $\frac{E_0}{2}$

(c) $\sqrt{3}E_0$ (d) $2\sqrt{3}E_0$

12. Two free point charges +4Q and +Q are placed at a distance r. A third charge q is so placed that all the three are in equilibrium.

(a) q is placed at a distance $\frac{1}{3}r$ from 4Q

(b) q is placed at a distance $\frac{1}{3}r$ from Q

(c) $q = \frac{4Q}{9}$

(d) $q = -\frac{4Q}{9}$

13. Two equal charges are 2d' distance apart. How far form the mid-point should another charge be placed on perpendicular bisector so that it experiences maximum force?

(a) d (b) d/2

(c) $d/\sqrt{2}$ (d) $\sqrt{2d}$

14. Four charges Q_1, Q_2, Q_3 and Q_4 of same magnitude are fixed along the x-axis at $x = -2a, -a, +a$ and $+2a$ respectively. A positive charge q is placed on the positive axis at a distance $b > 0$. Four options of the signs of these charges are given in List-I. The direction of the forces on the charges q is given in List-II. Match List-I with List-II and select the correct answer using the code given below the lists.