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CHAPTER

01

The Language of Numbers

Digits · BODMAS · Cyclicity · Surds and Indices

IN THIS CHAPTER

Numbers are the alphabet of every quantitative subject. This chapter shows you which family each number belongs to, how to combine them in the right order, and three time-savers — unit-digit cycles, surds and indices, and a first peek at logarithms — that reappear in many later chapters. After this chapter you will be able to —

- name the set ($\mathbb{N}$, $\mathbb{W}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$) of any number you are shown;
- use BODMAS to evaluate a long expression without confusion;
- predict the last digit of any large power;
- simplify surds and apply the four index laws confidently.

1.1 What is a Number?

A number is a symbol used for three different jobs — to **count** (How many cells in the culture? Two thousand), to **measure** (How concentrated is the buffer? 0.15 M), and to **label** (Take tube number 3). The same ten symbols 0, 1, 2, ..., 9 — called **digits** — do all three jobs, because we group them in powers of ten. This is the **decimal system**, and it works in the lab, in the library and in the exam hall.

Most numbers you meet in life science look harmless: 37 (body temperature in °C), 7.4 (blood pH), 6.022×10^{23} (Avogadro's number). They all use the same ten digits but cover very different 'sizes', from a fraction to an enormous power of ten. Before working with them in formulae we should know which family of numbers each one belongs to.

WHY TEN DIGITS?

All number systems repeat after a base. Ours is base-10 because we have ten fingers. Computers use base-2 (binary); chemists meet base-16 (hexadecimal) in CAS codes. The arithmetic rules in this chapter work in every base, but every example in this book is base-10.

1.2 The Family of Numbers

Mathematicians group numbers into families. Each family is given a single-letter name in *italic-bold* so we can talk about them quickly. The families are nested: every member of a smaller family is automatically a member of every larger family that contains it.

Picture the families as a set of nested boxes (Figure 1.1). The innermost box holds the counting numbers you have used since childhood; each surrounding box adds a few new kinds of number — first zero, then negatives, then fractions and finally the irrationals. By the time we reach the outermost box we have every number that can sit on the real number line.

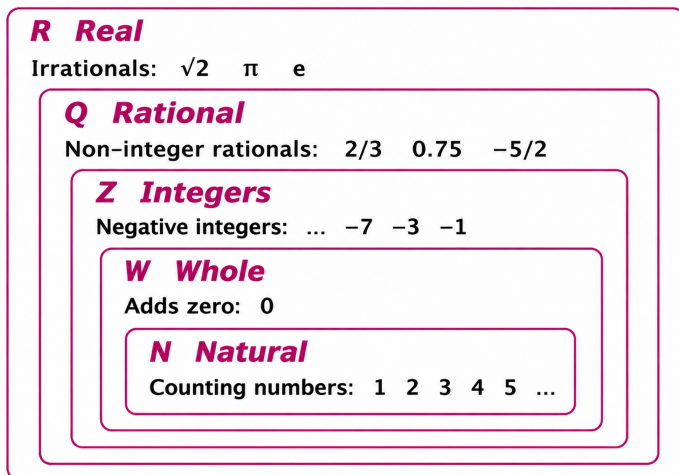


Figure 1.1 Nesting of the five basic number sets — every set contains all sets inside it in this list.

- **Natural numbers ($\mathbb{N}$)** — 1, 2, 3, ... : the everyday counting numbers; never zero, never negative.
- **Whole numbers ($\mathbb{W}$)** — 0, 1, 2, 3, ... : the natural numbers together with zero.
- **Integers ($\mathbb{Z}$)** — ..., -2, -1, 0, 1, 2, ... : the whole numbers together with their negatives. (Z is from Zahlen — German for ‘numbers’.)
- **Rational numbers ($\mathbb{Q}$)** — any number expressible as $\frac{p}{q}$, where p and q are integers and $q \neq 0$; e.g. $\frac{3}{4}$, -7, 0.25. (Q is for quotient.)
- **Irrational numbers** — cannot be expressed as $\frac{p}{q}$, where p and q are integers and $q \neq 0$; their decimal forms neither terminate nor repeat ($\sqrt{2}$, π , e).
- **Real numbers ($\mathbb{R}$)** — rationals together with irrationals; every point on the number line corresponds to a unique real number.

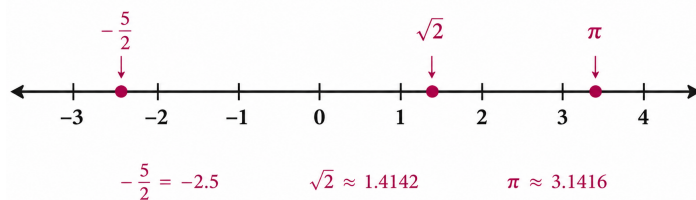


Figure 1.2 The real number line; integers sit at the marked ticks, while $\sqrt{2}$, π and $-5/2$ occupy intermediate points.

READING THE NESTING SYMBOL

The symbol $\subset$ means ‘is contained in’. So $\mathbb{N} \subset \mathbb{W}$ reads ‘every natural number is also a whole number’ — not that the two sets are the same. The full chain $\mathbb{N} \subset \mathbb{W} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$ simply says: every natural number is whole number, every whole number is an integer, every integer is rational, and every rational is real. Irrational numbers (such as $\sqrt{2}$ and π) live inside $\mathbb{R}$ but outside $\mathbb{Q}$.

★ EXAM SHORTCUT

To decide whether a decimal is rational or irrational, look at its tail. If the decimal terminates (0.75) or eventually repeats (0.333...), it is rational. If it never settles into a pattern (1.41421356...), it is irrational.

DID YOU KNOW

The irrationals are uncountably infinite. Georg Cantor proved in 1874 that almost every real number selected at random is irrational — the rationals form only a vanishingly small fraction of the real line.

Within the integers we recognise a few useful sub-families that the examiner asks about often.

- **Even numbers** — divisible by 2 (... , -4, -2, 0, 2, 4, ...).
- **Odd numbers** — not divisible by 2 (... , -3, -1, 1, 3, 5, ...).
- **Prime numbers** — natural numbers greater than 1 with exactly two divisors, 1 and itself (2, 3, 5, 7, 11, 13, 17, ...). Note that 2 is the only even prime.
- **Composite numbers** — natural numbers greater than 1 with more than two divisors (4, 6, 8, 9, 10, ...). The number 1 is neither prime nor composite.

MEMORISE THESE PRIMES

All primes below 50 — useful for factorisation and divisibility shortcuts: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47.

1.3 Digit, Place Value and Face Value

A **digit** is one of the ten symbols 0, 1, 2, ..., 9. The same digit can have very different 'worth' depending on where it sits in a number.

Consider 4,705:

- The **face value** of 7 is just 7 — the symbol itself.
- The **place value** of 7 is $7 \times 100 = 700$, because 7 sits in the hundreds column.

For integers,

$$\text{Place value} = \text{Face value} \times 10^{(\text{position from right} - 1)} \quad (1.1)$$

The same principle extends to decimal numbers. Digits to the right of the decimal point occupy places worth negative powers of 10: tenths (10^{-1}), hundredths (10^{-2}), thousandths (10^{-3}), and so on. For example, in 0.375, the digit 3 has place value $3 \times 10^{-1} = 0.3$, the digit 7 has place value $7 \times 10^{-2} = 0.07$, and the digit 5 has place value $5 \times 10^{-3} = 0.005$. Thus, moving one place to the right divides the place value by 10, while moving one place to the left multiplies it by 10.

The same idea applies when rounding. To round a decimal to two decimal places we look at the third decimal; if it is 5 or more we round up.

ROUNDING CONVENTION

When the digit immediately after the cut-off is exactly 5 followed by non-zero digits, round up. If it is exactly 5 followed only by zeros, the banker's rule rounds to the nearest even digit; for school arithmetic, the everyday rule simply rounds up.

EXAMPLE 1.1

Rounding off 4.58500001 to the second decimal place gives:

- (a) 4.6 (b) 4.58
(c) 4.59 (d) 4.585

Solution. The digit after the second decimal is 5 and the digits behind it (0 0 0 0 1) are not all zero — so we round upward. The 8 in the second decimal place becomes 9, giving 4.59. **Answer (c).**

1.4 BODMAS — Order of Operations

When several operations appear in a single expression, **BODMAS** tells us which runs first. Within the same priority level ($\times$ and $\div$, or $+$ and $-$) we evaluate from left to right.

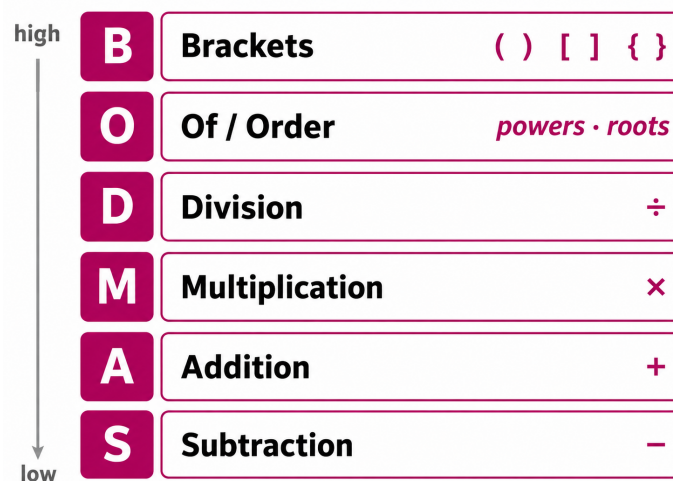


Figure 1.3 The BODMAS hierarchy — operations higher in the stack are applied first.

KEY POINT

Brackets are always evaluated first; a nested inner bracket is solved before its outer one. The word 'of' behaves like multiplication but ranks immediately above $\times$ and $\div$.

EXAMPLE 1.2

Dividing sixty by half and adding half of forty yields:

- (a) 35 (b) 50
(c) 70 (d) 140

Solution. 'Dividing 60 by half' means $60 \div (1/2) = 120$, not $60/2$. Then add half of 40, namely 20. Hence $120 + 20 = 140$.

Answer (d).

EXAMPLE 1.3

Simplify $16 - 4 \times 3 + 2$ of $6 \div 4$.

- (a) 7 (b) 9
(c) 11 (d) 13

Solution. Apply BODMAS in order. '2 of 6' = 12 ; $12 \div 4 = 3$. Now the expression reads $16 - 4 \times 3 + 3 = 16 - 12 + 3 = 7$.

Answer (a).

★ EXAM SHORTCUT

When the question contains 'of', mentally replace it by 'times, but earlier'. The phrase 'a of b' acts like multiplication and is settled before any nearby $\times$ or $\div$ symbol.

1.5 Unit-Digit Cyclicity

Questions of the form ‘what is the **last digit** of 2 raised to a large power?’ are common because the full power cannot be computed by hand. The unit digit follows a **short cycle**, and knowing the cycle solves the question in seconds.

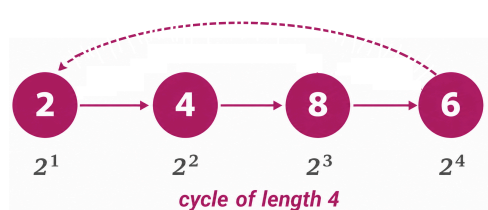


Figure 1.4 Powers of 2 cycle through the unit digits 2, 4, 8, 6.

- 0, 1, 5, 6 — cycle length 1 (powers always end in 0, 1, 5, 6 respectively).
- 4, 9 — cycle length 2 (4: 4, 6, 4, 6, ... 9: 9, 1, 9, 1, ...).
- 2, 3, 7, 8 — cycle length 4.

$$\text{unit digit } (a^n) = \text{unit digit } (a^{n \bmod L}) \quad (1.2)$$

where L = cycle length

EXAMPLE 1.4

What is the digit at the unit's place of 7^{2025} ?

- (a) 1 (b) 3
(c) 7 (d) 9

Solution. The cycle of 7 has length 4: 7, 9, 3, 1, 7, ... Since $2025 \bmod 4 = 1$, the unit digit matches that of 7 raised to the first power, which is 7. **Answer (c).**

NOTE

If $n \bmod L$ equals 0, use L itself (not 0) as the working exponent. For example, the unit digit of 2 raised to the 8th power is found from 2 raised to the 4th = 16, giving 6.

★ EXAM SHORTCUT

Ignore everything except the unit digit of the base. The unit digit of 2974^{301} is identical to that of 4^{301} — because every other digit of the base disappears under multiplication of only the last column.

1.6 Surds and Indices

A **surd** is a root that cannot be reduced to a rational number (for example $\sqrt{2}$ or $\sqrt[3]{5}$). An **index**, or exponent, is

the superscript that tells how many times a base multiplies itself ($2^3 = 2 \times 2 \times 2 = 8$).

$$a^m \cdot a^n = a^{m+n}$$

$$a^m / a^n = a^{m-n}$$

$$(a^m)^n = a^{mn} \quad (1.3)$$

$$a^0 = 1 \quad (a \neq 0)$$

$$a^{-n} = 1 / a^n$$

$$a^{1/n} = \sqrt[n]{a} \quad (1.4)$$

COMMON SURD IDENTITIES

$\sqrt{50} = \sqrt{(25 \cdot 2)} = 5\sqrt{2}$. $\sqrt{(a^2b)} = a\sqrt{b}$. To rationalise the denominator, multiply by the appropriate surd; e.g. $1/\sqrt{3} = \sqrt{3}/3$.

EXAMPLE 1.5

The geometric mean of the squares of two positive integers is 10. The smallest possible sum of these two integers is:

- (a) 7 (b) 8
(c) 10 (d) 11

Solution. $GM(a^2, b^2) = \sqrt{(a^2 b^2)} = ab = 10$. Integer pairs with $ab = 10$ are (1, 10) and (2, 5), with sums 11 and 7 respectively. The smallest is 7. **Answer (a).**

1.7 A First Look at Logarithms

If $a^x = N$ we write $x = \log_a N$ and read it ‘the logarithm of N to base a ’. Logarithms convert multiplication into addition — a habit life sciences already exploit in pH, in sound-intensity decibels, and in absorbance spectroscopy.

$$\log(ab) = \log a + \log b$$

$$\log(a/b) = \log a - \log b$$

$$\log(a^n) = n \cdot \log a \quad (1.5)$$

SCIENTIFIC NOTATION

A number written as $a \times 10^n$, with $1 \leq a < 10$, is in scientific notation. Examples: 6.022×10^{23} (Avogadro), 1.66×10^{-27} kg (atomic mass unit). Multiplying these is easy: add the exponents ($10^m \cdot 10^n = 10^{m+n}$).

DID YOU KNOW

The familiar definition $pH = -\log_{10} [H^+]$ is just a logarithm renamed: pure water with $[H^+] = 10^{-7}$ mol/L therefore has pH 7. A jump of one pH unit means a ten-fold change in hydrogen-ion concentration.

★ EXAM SHORTCUT

$\log_{10} 2 \approx 0.301$ and $\log_{10} 3 \approx 0.477$. With just these two values you can estimate $\log$ of any number whose prime factors are 2, 3 and 5 (since $\log 5 = \log 10 - \log 2 = 0.699$). Worth memorising.

1.8 Sequences, Sums and a Quick Algebra Toolkit

Many counting and arithmetic shortcuts in later chapters depend on the same handful of results. We close Chapter 1 with the three sum formulas every student should know by heart, the rules for arithmetic and geometric sequences, the modulus of a real number, and the quadratic formula. None of them is hard; they just save a great deal of time in the exam hall.

- **Sum of the first n natural numbers** — the result Gauss famously discovered as a schoolboy by pairing $1 + n$, $2 + (n - 1)$, ... each pair summing to $n + 1$.

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2} \quad (1.6)$$

- **Sum of the squares** of the first n natural numbers.

$$1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6} \quad (1.7)$$

- **Sum of the cubes** — equals the square of the simple sum.

$$1^3 + 2^3 + \dots + n^3 = \left[\frac{n(n+1)}{2} \right]^2 \quad (1.8)$$

An **arithmetic progression (AP)** is a sequence whose successive terms differ by a constant d . The first term is a , the n -th term is $a_n = a + (n - 1)d$, and the sum of the first n terms is $S_n = \frac{n}{2}(2a + (n - 1)d) = \frac{n}{2}(a + l)$, where l is the last term.

A **geometric progression (GP)** is one in which each term is r times the previous. The n -th term is $a_n = a \cdot r^{n-1}$, and the sum of the first n terms is $S_n = \frac{a(1-r^n)}{1-r}$ for $r \neq 1$. If $|r| < 1$ the infinite GP converges to $\frac{a}{1-r}$ — the foundation of every geometric-decay argument used later in the book.

MODULUS OF A REAL NUMBER

$|x|$ is the size of x without its sign — so $|3| = |-3| = 3$. Two useful identities: $|x \cdot y| = |x| \cdot |y|$ and the triangle inequality $|x + y| \leq |x| + |y|$.

QUADRATIC FORMULA

The roots of $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

The discriminant $D = b^2 - 4ac$ decides the type: $D > 0$ gives two distinct real roots, $D = 0$ gives one repeated root, and $D < 0$ gives a pair of complex conjugates.

★ EXAM SHORTCUT

Whenever an exam question asks for ‘the sum of the first n ...’, ‘the n -th term of ...’ or ‘the dots in row n of a pyramid’, reach straight for one of (1.6)–(1.8) above. Recognising the pattern is half the work.

Quick Recap

- Number sets nest: every natural is whole, every whole is integer, every integer is rational, and every rational is real — i.e. $\mathbb{N} \subset \mathbb{W} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$. Irrational numbers live inside $\mathbb{R}$ but outside $\mathbb{Q}$.
- Place value = face value $\times 10$ raised to (position $- 1$). For decimal numbers, moving one place to the right of decimal divides the place value by 10, while moving one place to the left multiplies it by 10.
- BODMAS order: Brackets $\rightarrow$ Of/Order $\rightarrow \div \rightarrow \times \rightarrow + \rightarrow -$. Within the same priority, work left to right.
- Unit-digit cycles — length 1 (0, 1, 5, 6); length 2 (4, 9); length 4 (2, 3, 7, 8).
- Index laws: $a^m \cdot a^n = a^{m+n}$; $(a^m)^n = a^{mn}$; $a^{-n} = 1/a^n$.
- Logarithm: $\log(ab) = \log a + \log b$. Familiar example: $\text{pH} = -\log_{10} [\text{H}^+]$.
- Sum identities: $1+2+\dots+n = n(n+1)/2$; $1^2+2^2+\dots+n^2 = n(n+1)(2n+1)/6$; $1^3+2^3+\dots+n^3 = [n(n+1)/2]^2$.
- AP n -th term $a_n = a + (n-1)d$; GP n -th term $a_n = a \cdot r^{n-1}$; infinite GP ($|r| < 1$) sums to $a/(1-r)$.
- $|x|$ = magnitude of x ; quadratic roots = $(-b \pm \sqrt{b^2 - 4ac})/(2a)$.

Practice

Attempt all 15 questions within 15 minutes.

- The digit at the unit's place of 3 raised to the 100th power is
(a) 1 (b) 3
(c) 7 (d) 9
- Which of the following is irrational?
(a) $22/7$ (b) $\sqrt{16}$
(c) $\sqrt{7}$ (d) 0.333...

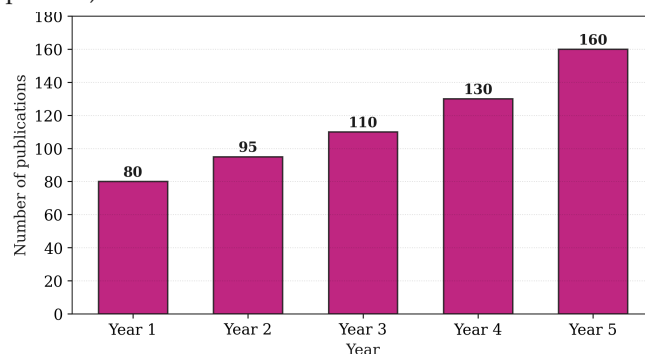
3. Simplify $60 \div 5$ of $2 + 4 \times 3 - 7$.
 (a) 8 (b) 11
 (c) 12 (d) 17
4. If 2 raised to x equals $1/32$, then x equals
 (a) 5 (b) -5
 (c) $1/5$ (d) $-1/5$
5. The place value of 8 in 38 904 is
 (a) 8 (b) 800
 (c) 8 000 (d) 80 000
6. How many zeros does the product $5 \times 10 \times 15 \times 20 \times 25 \times 30$ end with?
 (a) 3 (b) 4
 (c) 5 (d) 6
7. The value of $(\sqrt{3} + 1)(\sqrt{3} - 1)$ is
 (a) 1 (b) 2
 (c) 3 (d) 4
8. $\log_{10} 2 + \log_{10} 5$ equals
 (a) 0 (b) 1
 (c) 5 (d) $\log_{10} 7$
9. The smallest positive integer that leaves remainder 1 when divided by 2, 3, 4 and 5 is
 (a) 31 (b) 41
 (c) 61 (d) 121
10. If a raised to $(1/3)$ equals 2, then a^2 equals
 (a) 8 (b) 16
 (c) 32 (d) 64
11. Rounded to two decimal places, 0.49500001 becomes
 (a) 0.49 (b) 0.50
 (c) 0.495 (d) 0.500
12. The number of positive factors of 60 is
 (a) 8 (b) 10
 (c) 12 (d) 16
13. The value of $\sqrt{50}$ lies between
 (a) 6 and 7 (b) 7 and 8
 (c) 8 and 9 (d) 9 and 10
14. If p and q are two distinct primes both greater than 5, the unit digit of $p \cdot q$ can NEVER be
 (a) 1 (b) 3
 (c) 5 (d) 9
15. Simplify $(a^4 b^{-2}) \div (a^{-1} b^3)$.
 (a) $a^3 b^5$ (b) $a^5 b^5$
 (c) $a^5 b^{-5}$ (d) $a^3 b^{-5}$

Solutions

1. The cycle of unit digits of powers of 3 is (3, 9, 7, 1), of length 4. Since $100 \equiv 0 \pmod{4}$, we use the 4th term of the cycle, which is 1. **Answer (a).**
2. $22/7$ is a ratio of integers; $\sqrt{16} = 4$; $0.333... = 1/3$ are all rational. Only $\sqrt{7}$ is irrational because 7 is not a perfect square. **Answer (c).**
3. Apply BODMAS. '5 of 2' is treated as multiplication and evaluated first inside the division step: $60 \div (5 \times 2) = 60 \div 10 = 6$. Now $4 \times 3 = 12$. Combining, $6 + 12 - 7 = 11$. **Answer (b).**
4. $1/32 = 1 / 2^5 = 2^{-5}$, so $x = -5$. **Answer (b).**
5. In 38 904 the digit 8 sits in the thousands position. Place value = $8 \times 10^3 = 8\,000$. **Answer (c).**
6. Trailing zeros equal the number of (2, 5) factor pairs. Counting 5s in the factors: $5(1) + 10(1) + 15(1) + 20(1) + 25(2) + 30(1) = 7$. Counting 2s: $5(0) + 10(1) + 15(0) + 20(2) + 25(0) + 30(1) = 4$. Minimum is 4. **Answer (b).**
7. Difference of squares: $(\sqrt{3})^2 - (1)^2 = 3 - 1 = 2$. **Answer (b).**
8. $\log a + \log b = \log(ab)$; hence $\log_{10} 2 + \log_{10} 5 = \log_{10} 10 = 1$. **Answer (b).**
9. Let N be the required integer. Then $N - 1$ must be a common multiple of 2, 3, 4 and 5; the smallest such multiple is $\text{LCM}(2, 3, 4, 5) = 60$. So $N = 60 + 1 = 61$. **Answer (c).**
10. Cube both sides of the given relation: $a = 2^3 = 8$. Therefore $a^2 = 8^2 = 64$. **Answer (d).**
11. The third decimal is 5 and the digits behind it (0 0 0 0 1) are not all zero, so we round upward. $0.49 \rightarrow 0.50$. **Answer (b).**
12. $60 = 2^2 \cdot 3^1 \cdot 5^1$. Total number of divisors = $(2 + 1)(1 + 1)(1 + 1) = 12$. **Answer (c).**
13. $7^2 = 49$ and $8^2 = 64$. Since $49 < 50 < 64$, we have $7 < \sqrt{50} < 8$. **Answer (b).**
14. Every prime greater than 5 ends in 1, 3, 7 or 9. Products of two such unit digits give unit digits drawn from {1, 3, 7, 9} only. The digit 5 is therefore impossible. **Answer (c).**
15. Subtract exponents separately. For the a -terms: $4 - (-1) = 5$, giving a^5 . For the b -terms: $-2 - 3 = -5$, giving b^{-5} . **Answer (c).**

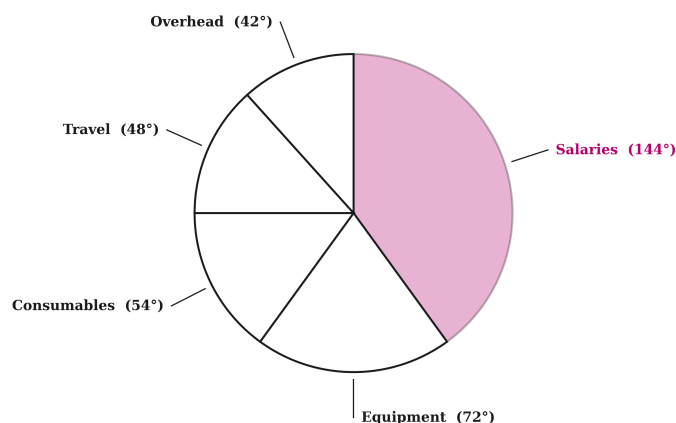
MOCK TEST 1

- A pharmacist is preparing 750 mL of an antiseptic solution by mixing two stocks of strengths 70 % and 90 % by volume. The final mixture must be exactly 76 %. How many mL of the 70 % stock are required?
(a) 450 mL (b) 500 mL
(c) 525 mL (d) 600 mL
- In a screen of 400 plant cells, 65 % express gene A, 50 % express gene B, and 30 % express both. How many cells express exactly one of A or B?
(a) 140 (b) 180
(c) 200 (d) 220
- ₹ 50 000 is invested at 8 % per annum compounded annually for 2 years. By how much rupees does the CI exceed the SI on the same principal at the same rate over the same period?
(a) ₹ 200 (b) ₹ 240
(c) ₹ 320 (d) ₹ 400
- Two motorcycles ride between a campus and a hospital 60 km apart. The first leaves the campus at 9 a.m. at 40 km/h; the second leaves the hospital at the same 9 a.m. at 80 km/h. When and where do they meet?
(a) 9:30 a.m., 20 km from campus (b) 9:45 a.m., 25 km from campus
(c) 10 a.m., 30 km from campus (d) 9:30 a.m., 40 km from campus
- Technicians P, Q and R can do a protocol alone in 12, 15 and 20 hours. They start together but Q leaves after exactly 4 hours. How many more hours after Q leaves will P and R take?
(a) 1.0 h (b) 1.5 h
(c) 2.0 h (d) 2.5 h
- In a clinical trial, 80 % of patients with a disease test positive and 5 % of patients without it also test positive. Prevalence is 20 %. Given a positive test, what is the probability the patient has the disease?
(a) 0.62 (b) 0.72
(c) 0.80 (d) 0.92
- A laboratory orders 30 pieces of glassware: pipettes ₹ 100, burettes ₹ 400, conical flasks ₹ 50 each. Total cost ₹ 8 200, and pipettes = flasks + 4. How many burettes were ordered?
(a) 12 (b) 15
(c) 18 (d) 21
- A rectangular plot has perimeter 50 m and area 144 m². Find the lengths of its longer and shorter sides using the sum-and-product identity.
(a) 18 m and 8 m (b) 16 m and 9 m
(c) 14 m and 11 m (d) 12 m and 13 m
- A lab has 8 distinct reagents; 3 reactive ones must occupy 3 red cabinets and the rest 5 blue cabinets. In how many ways can the eight reagents be allocated to the cabinets?
(a) 120 (b) 720
(c) 6 720 (d) 40 320
- From 6 PhD students and 4 postdocs a panel of 4 is to be chosen with at least one postdoc. In how many distinct ways can such a panel be selected?
(a) 165 (b) 180
(c) 195 (d) 210
- The mean marks of 50 students in class A is 64 and that of 40 students in class B is 75. If the two classes are merged, what is the combined mean (to one decimal place)?
(a) 67.5 (b) 68.9
(c) 69.5 (d) 70.0
- The bar chart shown depicts a university's research publications over five years. Compute the percentage growth from the first to the fifth year (round to the nearest whole percent).



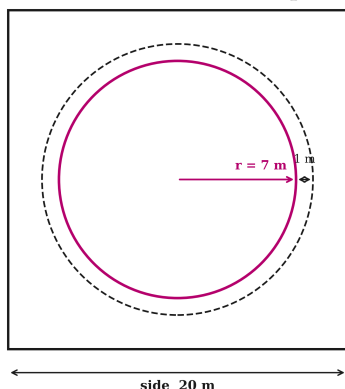
- 60 %
- 80 %
- 100 %
- 120 %

- The pie chart shown depicts the allocation of a ₹ 5 crore research budget across five categories. Using the central angle of the 'Salaries' sector, find the amount allocated to salaries.



- ₹ 1.8 crore
- ₹ 2.0 crore
- ₹ 2.2 crore
- ₹ 2.5 crore

14. The enrolment in four departments is: Bio 80 M, 120 F; Chem 100 M, 90 F; Phys 110 M, 70 F; Math 60 M, 130 F. In which department is the female-to-total ratio highest?
- (a) Biology (b) Chemistry
(c) Physics (d) Mathematics
15. A $20 \times 20 \text{ m}^2$ square garden has a circular flower bed of radius 7 m at its centre with a 1-m-wide circular pathway around it (see figure). Find the area of the garden NOT covered by either the flower bed or the pathway.



- (a) 180 m^2 (b) 199 m^2
(c) 220 m^2 (d) 246 m^2
16. A solid metallic sphere of radius 6 cm is melted and recast (no loss) into N identical solid cylinders, each of radius 1 cm and height 4 cm. Find N .

- (a) 48 (b) 64
(c) 72 (d) 90

17. A triangular plot has sides 13 m, 14 m and 15 m. A circular fountain of the largest possible radius is placed inside so that it touches all three sides. Find the radius.
- (a) 2 m (b) 3 m
(c) 4 m (d) 5 m

18. A bacterial-count sequence at successive 30-min intervals is 12, 18, 30, 54, 102, ... The first differences grow geometrically. Predict the next term.
- (a) 184 (b) 198
(c) 210 (d) 224

19. In a cipher used by a research group GLYCINE is encoded as HMZDJOF. Using the same rule, find the encoded form of LYSINE.
- (a) MZTJOF (b) KXRHMD
(c) MZSJOF (d) NZTKPG

20. A typical bacterium is roughly spherical, diameter $\approx 1 \mu\text{m}$; a typical human somatic cell is roughly spherical, diameter $\approx 50 \mu\text{m}$. What is the approximate ratio of the bacterium's S/V ratio to that of the human cell?
- (a) 5 (b) 25
(c) 50 (d) 100

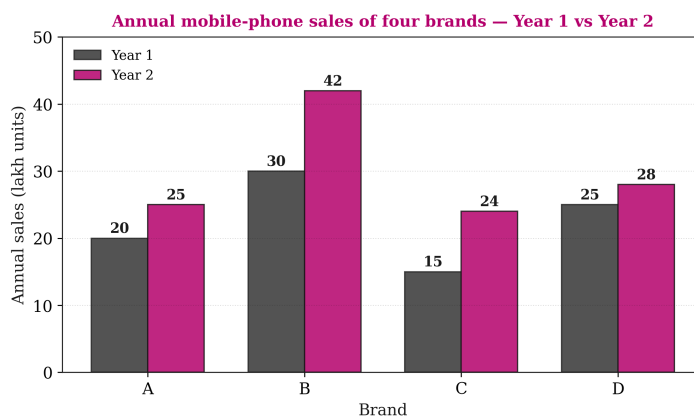
ANSWERS & SOLUTIONS

1. (c) Alligation: 70 %-stock : 90 %-stock = $(90 - 76):(76 - 70) = 7 : 3$, so 70 %-stock = $(7/10) \times 750 = 525 \text{ mL}$. **Shortcut:** Alligation ratio $(90-76):(76-70)$.
2. (d) A only = 35 %, B only = 20 %, exactly one = $55 \% \times 400 = 220$. **Shortcut:** $P(A) + P(B) - 2P(A \cap B) = 0.55$.
3. (c) SI = 8 000; CI = $50\,000 \times 0.1664 = 8\,320$; difference = ₹ 320. **Shortcut:** $CI - SI (2 \text{ yr}) = P \times (R/100)^2 = 320$.
4. (a) Closing speed = 120 km/h; meet in 0.5 h $\Rightarrow$ 9:30 a.m.; campus-side distance = 20 km. **Shortcut:** Distance from one end = its speed $\times$ meeting time.
5. (b) Combined rate of three = $1/5$ per hour; $4/5$ done in 4 h. Remaining $1/5$ at P + R rate $2/15$ takes 1.5 h. **Shortcut:** (remaining fraction) / (new combined rate).
6. (c) $P(+) = 0.20 \times 0.80 + 0.80 \times 0.05 = 0.20$; $P(D | +) = 0.16/0.20 = 0.80$. **Shortcut:** Posterior = (prior $\times$ sens.) / total positive.
7. (c) Substitute $p = f + 4$. The system reduces to $f = 4$, $p = 8$, $b = 18$. **Shortcut:** Substitute the simplest relation first.
8. (b) $L + B = 25$, $LB = 144$. Roots of $t^2 - 25t + 144 = 0$ are $(25 \pm 7)/2 = 16, 9$. **Shortcut:** Sides = roots of $t^2 - (\text{sum})t + (\text{product})$.
9. (b) $3! \times 5! = 6 \times 120 = 720$. **Shortcut:** Independent arrangements multiply.
10. (c) $C(10, 4) - C(6, 4) = 210 - 15 = 195$. **Shortcut:** Complement: total - none.

11. (b) $(50 \times 64 + 40 \times 75)/90 = 6\,200/90 \approx 68.9$. **Shortcut:** Weighted average.
12. (c) $Y_1 = 80$, $Y_5 = 160$; growth = 100 %. **Shortcut:** Final/initial = 2.
13. (b) Salaries fraction = $144^\circ/360^\circ = 2/5$; salaries = $0.4 \times 5 = ₹ 2$ crore. **Shortcut:** Sector angle / $360^\circ \times \text{total}$.
14. (d) F/Total: Bio 0.60, Chem 0.474, Phys 0.389, Math 0.684. Highest in Mathematics. **Shortcut:** Compute $F/(M+F)$ for each.
15. (b) Outer-circle area = $\pi \times 8^2 \approx 201.14 \text{ m}^2$. Uncovered = $400 - 201.14 \approx 199 \text{ m}^2$. **Shortcut:** $400 - \pi \times R_{\text{outer}}^2$.
16. (c) Sphere $V = 288\pi$; cylinder $V = 4\pi$; $N = 72$. **Shortcut:** Volume conservation.
17. (c) $s = 21$, Heron's area = 84; $r = \text{Area}/s = 4 \text{ m}$. **Shortcut:** Inradius shortcut $r = \text{Area}/s$.
18. (b) Differences double: 6, 12, 24, 48 $\rightarrow$ next 96. Next term = $102 + 96 = 198$. **Shortcut:** Differences double; add latest difference doubled.
19. (a) Each letter shifts by +1: LYSINE $\rightarrow$ MZTJOF. **Shortcut:** Identify the +1 shift, then apply.
20. (c) $S/V = 3/R$ for a sphere; ratio = $R_{\text{human}} / R_{\text{bact}} = 25/0.5 = 50$. **Shortcut:** $S/V \propto 1/R$; just take the inverse-radius ratio.

MOCK TEST 2

1. In a student union election with two candidates, candidate A received 56 % of the valid votes cast and defeated B by exactly 720 valid votes. Assuming every ballot cast was valid, find the total number of votes polled in the election.
(a) 5 000 (b) 6 000
(c) 7 200 (d) 8 000
2. A fruit vendor sells 24 mangoes for ₹ 1 080 and thereby incurs a loss of 10 % on his cost. To gain a profit of 20 % instead, at what price (per mango, in rupees) must he sell each mango?
(a) ₹ 54 (b) ₹ 60
(c) ₹ 66 (d) ₹ 72
3. A research grant of ₹ 3 60 000 is to be distributed among three scientists P, Q and R in the strict ratio 5 : 7 : 8 of their seniority. A supplementary bonus is then paid to Q such that Q's final share equals R's share. Find the bonus paid to Q.
(a) ₹ 12 000 (b) ₹ 15 000
(c) ₹ 18 000 (d) ₹ 24 000
4. The average mark of a class of 30 students is exactly 60. If the highest scorer's mark is excluded, the average of the remaining 29 students drops to 58. What was the mark scored by the highest scorer of the class?
(a) 102 (b) 118
(c) 124 (d) 132
5. A 60-litre container is full of pure milk. 12 litres of milk is withdrawn and replaced by 12 litres of water; the operation is then repeated once more. How many litres of milk remain in the container after the second replacement?
(a) 36.0 L (b) 38.4 L
(c) 40.0 L (d) 42.0 L
6. A motorboat covers 24 km downstream in exactly 2 hours and the same 24 km upstream in exactly 4 hours. Find the speed of the boat in still water and the speed of the stream (in km/h), respectively.
(a) 9 and 3 (b) 8 and 4
(c) 10 and 2 (d) 12 and 6
7. A water tank can be completely filled by inlet tap P in 8 hours and completely emptied by outlet tap Q in 12 hours. If both taps are opened together when the tank is empty, how long will the tank take to fill?
(a) 20 h (b) 24 h
(c) 28 h (d) 36 h
8. A single card is drawn at random from a standard well-shuffled 52-card deck. What is the probability that the card drawn is either a red card or a king (or both)?
(a) $\frac{1}{2}$ (b) $\frac{7}{13}$
(c) $\frac{6}{13}$ (d) $\frac{15}{26}$
9. In how many distinct ways can the letters of the word DAUGHTER be rearranged such that all three of its vowels appear together as one contiguous block in the word?
(a) 720 (b) 2 160
(c) 4 320 (d) 8 640
10. A research team of 5 members is to be selected from 7 senior scientists and 5 junior scientists, subject to the condition that the team must contain at least 3 senior scientists. In how many distinct ways can such a team be chosen?
(a) 525 (b) 546
(c) 580 (d) 612
11. In a triangle the lengths of the three sides are in the ratio 3 : 5 : 7. The largest angle of the triangle is opposite to the largest side. Using the law of cosines, find the measure of this largest angle in degrees.
(a) 90° (b) 105°
(c) 120° (d) 135°
12. The grouped bar chart shown below compares the annual sales (in lakh units) of four mobile-phone brands A, B, C and D across two consecutive years. Identify the brand that showed the highest percentage growth from Year 1 to Year 2.



- (a) A (b) B
(c) C (d) D
13. A school sports survey records the participants in four sports across two age groups: Football (12-15: 80; 16-18: 50), Cricket (70; 60), Hockey (45; 35) and Tennis (25; 30). In which sport does the older age group (16-18) constitute the greatest fraction of the total participants?
(a) Football (b) Cricket
(c) Hockey (d) Tennis