NATIONAL DEFENCE ACADEMY / NAVAL ACADEMY

NDA/NA

MATHEMATICS

24 TOPIC WISE SORTED PREVIOUS YEARS SOLVED PAPERS



2015 TO SEPT 2026
(PHASE I AND II)

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PREFACE

The **National Defence Academy & Naval Academy Examination (NDA & NA)**, conducted by the **Union Public Service Commission (UPSC)**, is one of India's most prestigious examinations for young aspirants who wish to serve as officers in the **Indian Army, Navy, and Air Force**. The Mathematics paper plays a **decisive role** in the written examination and demands strong **conceptual understanding, speed, accuracy, and regular practice**.

This book is designed to make NDA Mathematics preparation more **systematic and exam-oriented**. It presents **Previous Year Questions (PYQs) from 2015 to 2026**, each with a clear, step-by-step solution. Practising these questions helps students understand the actual examination pattern, recognise frequently tested concepts, and build effective problem-solving techniques.

The book covers the major areas of **Paper I – Mathematics**: Algebra, Matrices and Determinants, Trigonometry, Analytical Geometry, Differential Calculus, Integral Calculus and Differential Equations, Vector Algebra, Statistics, and Probability. These are organised into **eight chapters**, with units and sub-topics clearly marked for easier learning and revision.

Key Features

- Complete coverage of the NDA Mathematics syllabus
- PYQs from 2015 to 2026
- Topic-wise arrangement with year tags for easy reference
- Detailed solutions that explain every stage of the working
- Emphasis on concepts, speed, accuracy, and exam strategy
- Ideal for revision, practice, and self-assessment

For best results, **revise the concepts first**, **attempt each question independently**, and then use the solutions to analyse your mistakes and improve your speed and accuracy. **Prepare with purpose, practise with confidence**, and move closer to your dream of serving the nation.

Best wishes for your success and a bright future in the Armed Forces.

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2026

NDA MATHEMATICS (PHASE-II)

QUESTIONS WITH EXPLANATIONS

For the following two (02) items that follow:

From the top of a building, the angles of depression of the top and bottom of a tower of height h are α and β respectively. Let H be the height of the building and D be the horizontal distance between the building and the tower.

1. What is $\mathbf{H^2 + D^2}$ equal to?

(a) $\frac{h^2\cos^2\alpha}{\sin^2(\beta-\alpha)}$ (b) $\frac{h^2\cos^2\beta}{\sin^2(\beta-\alpha)}$

(c) $\frac{h^2\cos^2\alpha}{\sin^2(\beta+\alpha)}$ (d) $\frac{h^2\cos^2\beta}{\sin^2(\beta+\alpha)}$

2. If $\alpha = 15^\circ$ and $\beta = 45^\circ$, then which one of the following is correct?

(a) $H = \sqrt{2}D$ (b) $D = \sqrt{2}H$

(c) $2H = \sqrt{3}D$ (d) $H = D$

For the next two (02) items that follow:

The angles A, B and C of a triangle ABC are in the ratio 1:2:7.

3. What is the ratio of the side of greatest length to the side of least length?

(a) 7: 1

(b) $\sqrt{7}$: 1

(c) $(3+\sqrt{5})$: 2

(d) $(3+\sqrt{5})$: 3

4. What is $\sin A \cdot \cos B$ equal to?

(a) 1/4 (b) 1/2

(c) 1 (d) 2

For the next two (02) items that follow:

In a triangle ABC, $\frac{a+b}{13} = \frac{b+c}{11} = \frac{c+a}{12}$.

5. What is $\sin A : \sin B : \sin C$ equal to?

(a) 5: 6: 7 (b) 7: 6: 5

(c) 13: 11: 12 (d) 12: 11: 13

6. What is $\cos A : \cos B : \cos C$ equal to?

(a) 7: 19: 25 (b) 11: 12: 13

(c) 7: 11: 13 (d) 12: 11: 13

For the next two (02) items that follow:

Given that $\tan\left(\frac{A}{2}+\frac{B}{2}\right) = p$ and $\tan\left(\frac{A}{2}-\frac{B}{2}\right) = q$, where $pq \neq \pm 1$.

7. What is $\tan A$ equal to?

(a) $\frac{p-q}{1+pq}$ (b) $\frac{p+q}{1-pq}$

(c) $\frac{p+q}{1+pq}$ (d) $\frac{p-q}{1-pq}$

8. What is $\tan B$ equal to?

(a) $\frac{p-q}{1+pq}$ (b) $\frac{p+q}{1-pq}$

(c) $\frac{p+q}{1+pq}$ (d) $\frac{p-q}{1-pq}$

For the next two (02) items that follow:

Let $4(A+B) = \pi$.

9. What is $(1+\tan A)(1+\tan B)$ equal to?

(a) 1 (b) 2

(c) 4 (d) 6

10. What is $(\cot A - 1)(\cot B - 1)$ equal to?

(a) 1/2 (b) 1

(c) 3/2 (d) 2

11. What is $6\sin(\pi/18) - 8\sin^3(\pi/18)$ equal to?

(a) 1/4 (b) 1/2

(c) 1 (d) 2

12. How many values of θ satisfy the equation $\tan^2\theta + \cot^2\theta = 2$, where $0^\circ < \theta < 360^\circ$?

(a) 1 (b) 2

(c) 4 (d) 8

13. What is $\sin(\tan^{-1} 0.75)$ equal to?

(a) 1 (b) 4/5

(c) 3/5 (d) 1/2

14. What is $\tan^2\left(\frac{1}{2}\cos^{-1}\frac{1}{3}\right)$ equal to?

(a) 1/2 (b) 1

(c) 2 (d) 4

15. What is $\tan(1125^\circ)\cot(405^\circ) + \tan(765^\circ)\cot(675^\circ)$ equal to?

(a) 2 (b) 1

(c) 0 (d) -2

16. If A is a square matrix of order 3 and $|A| = 2$, then what is adj(adj A) equal to?

(a) $8A$

(b) $4A$

(c) $2A$

(d) $2I$, where I is the identity matrix of order 3

17. If A is a square matrix of order 3 such that

$$A(\text{adj } A) = \begin{bmatrix} 64 & 0 & 0 \\ 0 & 64 & 0 \\ 0 & 0 & 64 \end{bmatrix}$$

then what is $|A|$ equal to?

(a) 4 (b) 8

(c) 16 (d) 64

18. Let

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

What is the least value of θ for which $A + A^T = I$, where I is the identity matrix of order 2?

(a) $\pi/2$ (b) $\pi/3$

(c) $\pi/4$ (d) $\pi/6$

19. Let A, B and C be the square matrices of order 2 such that $AB = AC$.

Which of the statements given below is/are correct?

I. B and C are not necessarily equal if A is a singular matrix.

II. B and C are equal if A is a non-singular matrix.

Select the answer using the code given below:

(a) I only
(b) II only
(c) Both I and II
(d) Neither I nor II

20. If A is a square matrix of order 3 and $|A| = 4$, then what is $|\text{adj } A|$ equal to?

(a) 64 (b) 16
(c) 8 (d) 4

21. Let A and B be symmetric matrices of the same order.

Which of the following statements is/are correct?

I. $(AB - BA)$ is also a symmetric matrix.

II. $(BA - AB)$ is a skew-symmetric matrix.

Select the answer using the code given below:

(a) I only
(b) II only
(c) Both I and II
(d) Neither I nor II

22. If $\mathbf{A} = \begin{bmatrix} -1 & 2 \\ 3 & -4 \end{bmatrix}$, then what is $|\mathbf{A}^{-1}|$ equal to?

(a) -2 (b) -1
(c) $-1/2$ (d) 2

23. Let S be the set of all real numbers and R be a relation on S defined by $\mathbf{xRy} \Rightarrow |\mathbf{x}| \leq \mathbf{y}$. Then R is

(a) reflexive only
(b) transitive only
(c) both reflexive and transitive
(d) neither reflexive nor transitive

24. 4-digit numbers are formed with 1, 2, 3 and 4. What is the number of the 4-digit numbers in which at least one digit is repeated?

(a) 256 (b) 244
(c) 232 (d) 220

25. Given $S = \{(x, y, z) \in \mathbf{R}^3 : xyz = 210, x < y < z$, where x, y, z are positive integers$\}$ What is the total number of elements in S?

(a) 10
(b) 12
(c) 13
(d) More than 13

26. If $x = 4 + i$, where $i = \sqrt{-1}$, then what is $x^3 + 2x^2 - 63x + 171$ equal to?

(a) -1 (b) 0
(c) 1 (d) 2

27. Let α and β be the roots of the equation

$$\mathbf{x^2 - p(x + 1) - q = 0}$$

What is

$$\frac{\alpha^2 + 2\alpha + 1}{\alpha^2 + 2\alpha + q} + \frac{\beta^2 + 2\beta + 1}{\beta^2 + 2\beta + q}$$

equal to?

(a) -1 (b) 0
(c) 1 (d) 2

28. If the sum of the roots of the equation

$$\frac{1}{2x + p} + \frac{1}{2x + q} = \frac{1}{r}$$

is zero, then what is r equal to?

(a) $p - q$ (b) $p + q$
(c) $(p - q)/2$ (d) $(p + q)/2$

29. Let α and β be the roots of the equation $ax^2 + bx + c = 0$ and $p_n = \alpha^n + \beta^n$, where $n > 1$. What is

$$ap_{n+1} + bp_n + cp_{n-1}$$

equal to?

(a) 0 (b) 1
(c) $a + b + c$ (d) $a - b + c$

30. If p, q and r are in AP, then what is

$$p^3 + 3pr(p + r) + r^3$$

equal to?

(a) q^3 (b) $2q^3$
(c) $3q^3$ (d) $8q^3$

31. If $\frac{1}{b-a} + \frac{1}{b-c} = \frac{2}{b}$, then a, b and c are in

(a) AP
(b) GP
(c) HP
(d) Neither in AP nor in GP nor in HP

32. Let P, Q, R and S be the sum of n terms, 2n terms, 3n terms and 4n terms respectively of an AP such that $Q = 3P$.

Which one of the following is correct?

(a) $5R = 3S$ (b) $6R = 5S$
(c) $8R = 7S$ (d) $9R = 8S$

33. In a GP, the 3rd, 5th and 7th terms are x, $x^2 + 2$ and $x^3 + 10$ respectively, where $x > 1$.

What is the 8th term?

(a) 18 (b) $18\sqrt{3}$
(c) 54 (d) $54\sqrt{3}$

34. If $(1 + i)^n - 16 = 0$, where $i = \sqrt{-1}$ and n is a positive integer, then what is the least value of n?

(a) 2 (b) 4
(c) 8 (d) 16

35. What is $i \times i^4 \times i^9 \times i^{16} \times \ldots \times i^{576}$, where $i = \sqrt{-1}$, equal to?

(a) $-i$ (b) -1
(c) 1 (d) i

36. If $z = x + iy$ be such that $\left|\frac{z+\lambda i}{z-\lambda i}\right| = 1$, where $i = \sqrt{-1}$ and λ is a positive real number, then which of the following statements is/are correct?

I. z lies on the line $y = x$.

II. The amplitude of z is $\frac{\pi}{4}$.

Select the answer using the code given below.

(a) I only
(b) II only
(c) Both I and II
(d) Neither I nor II

37. What is

$$\begin{vmatrix} \frac{1}{a} & bc & a^3 \\ \frac{1}{a} + \frac{1}{b} & c(a + b) & a^3 + b^3 \\ \frac{1}{a} + \frac{1}{b} + \frac{1}{c} & ab + bc + ca & a^3 + b^3 + c^3 \end{vmatrix}$$

equal to?

(a) 0
(b) 3
(c) $(a + b + c)(ab + bc + ca)$
(d) $a^3 + b^3 + c^3 - 3abc$

(a) 15% (b) 20%
(c) 25% (d) 30%

111. If two fair dice are tossed, then what is the probability that the sum of the numbers on the faces of the dice is neither 5 nor 7?
(a) $11/18$ (b) $2/3$
(c) $13/18$ (d) $5/6$

112. Consider the following statements for three events A, B and C:
I. $P(A \cap B \cap C) \leq P(A) + P(B) + P(C) - 2$
II. $P(A \cup B \cup C) \geq P(A) + P(B) + P(C)$
Which of the statements given above is/are correct?
(a) I only
(b) II only
(c) Both I and II
(d) Neither I nor II

113. A Mathematics problem is given to two students X and Y to solve. The odds in favour of X solving the problem are 6 to 9 and the odds against Y in solving the problem are 6 to 5. What is the probability that the problem will be solved if both X and Y try to solve the problem?
(a) $27/55$ (b) $31/55$
(c) $37/55$ (d) $39/55$

114. The events A and B are independent among three events A, B and D. If $P(A \cap B \cap D) = 0.04$, $P(D|A \cap B) = 0.25$ and $P(B) = 4P(A)$, then what is the value of $P(A \cup B)$?
(a) 0.88 (b) 0.84
(c) 0.28 (d) 0.16

For the next three (03) items that follow:
Let A, B, C and D be mutually exclusive and exhaustive events such that
$\frac{P(A)}{6} = \frac{P(B)}{3} = \frac{P(C)}{4} = \frac{P(D)}{2}$

115. What is $\frac{P(A)+2P(B)}{3P(C)+P(D)}$ equal to?
(a) $5/4$ (b) $6/7$
(c) $2/5$ (d) $17/6$

116. If G is the geometric mean of $P(A)$, $P(B)$, $P(C)$ and $P(D)$, then what is G equal to?
(a) $\frac{5}{2\sqrt{3}}$ (b) $\frac{2}{5\sqrt{3}}$
(c) $\frac{\sqrt{3}}{5}$ (d) $\frac{1}{5\sqrt{3}}$

117. If H is the harmonic mean of $P(A)$, $P(B)$, $P(C)$ and $P(D)$, then what is H equal to?
(a) $14/75$ (b) $15/76$
(c) $16/75$ (d) $16/15$

For the next three (03) items that follow:
Five numbers are randomly picked from the numbers 1,2,3,4,5,6,7,8,9,10 and arranged them in ascending order [$x_1 < x_2 < x_3 < x_4 < x_5$; x_1 being the smallest number and x_5 being the largest number selected].

118. What is $P(x_1 = 4)$ equal to?
(a) $5/84$ (b) $1/14$
(c) $1/7$ (d) $19/84$

119. What is $P(x_3 = 6)$ equal to?
(a) $5/21$ (b) $10/21$
(c) $11/21$ (d) $17/21$

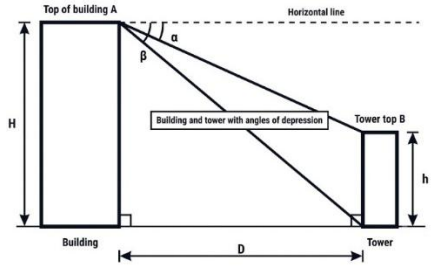
120. What is $P(x_1 = 2, x_2 = 3, x_3 = 8)$ equal to?
(a) $1/12$ (b) $1/60$
(c) $1/252$ (d) 0

ANSWER KEY

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
(a)	(d)	(c)	(a)	(b)	(a)	(b)	(a)	(b)	(d)	(c)	(c)	(c)	(a)	(c)	(c)	(d)	(b)	(c)	(b)
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
(b)	(c)	(b)	(c)	(c)	(c)	(c)	(d)	(a)	(d)	(c)	(a)	(b)	(c)	(c)	(d)	(a)	(d)	(b)	(d)
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
(b)	(a)	(b)	(b)	(c)	(b)	(c)	(a)	(a)	(c)	(a)	(b)	(d)	(c)	(c)	(a)	(b)	(b)	(a)	(c)
61	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
(b)	(a)	(a)	(b)	(b)	(c)	(b)	(c)	(d)	(c)	(d)	(d)	(a)	(b)	(a)	(b)	(a)	(a)	(c)	(a)
81	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**	**91**	**92**	**93**	**94**	**95**	**96**	**97**	**98**	**99**	**100**
(b)	(d)	(b)	(d)	(b)	(c)	(c)	(c)	(c)	(b)	(d)	(c)	(a)	(c)	(c)	(c)	(a)	(a)	(c)	(d)
101	**102**	**103**	**104**	**105**	**106**	**107**	**108**	**109**	**110**	**111**	**112**	**113**	**114**	**115**	**116**	**117**	**118**	**119**	**120**
(b)	(b)	(b)	(b)	(d)	(d)	(b)	(d)	(a)	(b)	(c)	(d)	(c)	(b)	(b)	(b)	(c)	(a)	(a)	(c)

EXPLANATION

1. Explanation: (a)



Let H be the height of the building and D be the horizontal distance between the building and the tower.

Using $\tan\theta = \frac{\text{vertical height}}{\text{horizontal distance}}$,

$\tan\beta = \frac{H}{D}$ and $\tan\alpha = \frac{H-h}{D}$.

Subtracting,

$$\tan\beta - \tan\alpha = \frac{h}{D}.$$

Using $\tan\beta - \tan\alpha = \frac{\sin(\beta-\alpha)}{\cos\alpha\cos\beta}$, we get

$$D = \frac{h\cos\alpha\cos\beta}{\sin(\beta-\alpha)}.$$

Also, $H = D\tan\beta$. Therefore,

$$H^2 + D^2 = D^2(\tan^2\beta + 1).$$

Using the identity $1 + \tan^2\beta = \sec^2\beta$,

$$H^2 + D^2 = D^2\sec^2\beta = \frac{D^2}{\cos^2\beta}.$$

Substituting the value of D,

$$H^2 + D^2 = \frac{h^2\cos^2\alpha\cos^2\beta}{\sin^2(\beta-\alpha)\cos^2\beta} = \frac{h^2\cos^2\alpha}{\sin^2(\beta-\alpha)}.$$

Hence, the correct answer is Option (a): $\frac{h^2\cos^2\alpha}{\sin^2(\beta-\alpha)}$.

2. Explanation: (d)

Using the relation obtained in Solution 1,

$$\tan\beta = \frac{H}{D}.$$

Given $\beta = 45°$, and using $\tan 45° = 1$,

$$\frac{H}{D} = 1.$$

Therefore,

$$H = D.$$

Hence, the correct answer is Option (d): $H = D$.

3. Explanation: (c)

The angles A, B, and C of $\triangle ABC$ are in the ratio 1: 2: 7.

Let $A = x$, $B = 2x$, $C = 7x$.

Using the angle-sum property of a triangle,

$$A + B + C = 180°,$$

we get

$$x + 2x + 7x = 180°$$
$$\Rightarrow 10x = 180° \Rightarrow x = 18°.$$

Therefore,

$A = 18°$, $B = 36°$, $C = 126°$.

In a triangle, the greatest side lies opposite the greatest angle, and the least side lies opposite the least angle.

Therefore, the greatest and least sides are opposite $126°$ and $18°$, respectively.

Using the Sine Rule,

$$\frac{\text{greatest side}}{\text{least side}} = \frac{\sin 126°}{\sin 18°}.$$

Using $\sin(180° - \theta) = \sin\theta$,

$$\sin 126° = \sin 54°.$$

Also,

$\sin 54° = \frac{1+\sqrt{5}}{4}$ and $\sin 18° = \frac{\sqrt{5}-1}{4}$.

Therefore,

$$\frac{\sin 126°}{\sin 18°} = \frac{\frac{1+\sqrt{5}}{4}}{\frac{\sqrt{5}-1}{4}} = \frac{1+\sqrt{5}}{\sqrt{5}-1}.$$

Rationalising the denominator,

$$\frac{\sin 126°}{\sin 18°} = \frac{1+\sqrt{5}}{\sqrt{5}-1}\times\frac{\sqrt{5}+1}{\sqrt{5}+1} = \frac{(1+\sqrt{5})^2}{5-1} = \frac{1+2\sqrt{5}+5}{4} = \frac{3+\sqrt{5}}{2}.$$

Thus, the required ratio is

$$(3+\sqrt{5}):2.$$

Hence, the correct answer is Option (c): $(3+\sqrt{5}):2$.

4. Explanation: (a)

Using the result obtained in Question 3,

$A = 18°$ and $B = 36°$.

Therefore,

$$\sin A\cos B = \sin 18°\cos 36°.$$

Using

$\sin 18° = \frac{\sqrt{5}-1}{4}$ and $\cos 36° = \frac{\sqrt{5}+1}{4}$,

we get

$$\sin A\cos B = \frac{\sqrt{5}-1}{4}\times\frac{\sqrt{5}+1}{4}.$$

Using the identity $(a-b)(a+b) = a^2 - b^2$,

$$\sin A\cos B = \frac{(\sqrt{5})^2 - 1^2}{16} = \frac{5-1}{16} = \frac{1.}{4}$$

Hence, the correct answer is Option (a): $\frac{1}{4}$.

5. Explanation: (b)

Given: $\frac{a+b}{13} = \frac{b+c}{11} = \frac{c+a}{12} = k$,

we get

$a + b = 13k$, $b + c = 11k$, $c + a = 12k$.

Using $2a = (a+b) + (a+c) - (b+c)$,

$$2a = 13k + 12k - 11k = 14k \Rightarrow a = 7k.$$

Similarly, $2b = (a+b) + (b+c) - (a+c)$,

$$2b = 13k + 11k - 12k = 12k \Rightarrow b = 6k,$$

And $2c = (b+c) + (a+c) - (a+b)$,

$$2c = 11k + 12k - 13k = 10k \Rightarrow c = 5k.$$

Therefore,

$$a: b: c = 7: 6: 5.$$

Using the Sine Rule, the sines of the angles are proportional to their opposite sides:

$$\sin A : \sin B : \sin C = a: b: c = 7: 6: 5.$$

Hence, the correct answer is Option (b): 7: 6: 5.

6. Explanation: (a)

Using the result obtained in Question 5,

$$a: b: c = 7: 6: 5.$$

Let $a = 7k$, $b = 6k$, $c = 5k$.

Using the Cosine Rule,

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
$$= \frac{(6k)^2 + (5k)^2 - (7k)^2}{2(6k)(5k)}$$
$$= \frac{36 + 25 - 49}{60} = \frac{12}{60} = \frac{1.}{5}$$

Similarly,

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$
$$= \frac{(7k)^2 + (5k)^2 - (6k)^2}{2(7k)(5k)}$$
$$= \frac{49 + 25 - 36}{70} = \frac{38}{70} = \frac{19}{35},$$

and

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$
$$= \frac{(7k)^2 + (6k)^2 - (5k)^2}{2(7k)(6k)}$$
$$= \frac{49 + 36 - 25}{84} = \frac{60}{84} = \frac{5.}{7}$$

Therefore,

$$\cos A : \cos B : \cos C = \frac{1}{5} : \frac{19}{35} : \frac{5.}{7}$$

Multiplying each term by 35,

$$\cos A : \cos B : \cos C = 7 : 19 : 25.$$

Hence, the correct answer is Option (a): $7 : 19 : 25$.

7. Explanation: (b)

Given

$$\tan\left(\frac{A + B}{2}\right) = p \text{ and } \tan\left(\frac{A - B}{2}\right) = q.$$

We can write A as

$$A = \frac{A + B}{2} + \frac{A - B}{2},$$

because

$$\frac{A + B}{2} + \frac{A - B}{2} = \frac{A + B + A - B}{2}$$
$$= \frac{2A}{2} = A.$$

Therefore,

$$\tan A = \tan\left[\frac{A + B}{2} + \frac{A - B}{2}\right].$$

Using the tangent addition formula

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y},$$

we get

$$\tan A = \frac{\tan\left(\frac{A + B}{2}\right) + \tan\left(\frac{A - B}{2}\right)}{1 - \tan\left(\frac{A + B}{2}\right)\tan\left(\frac{A - B}{2}\right)}.$$

Substituting

$$\tan\left(\frac{A + B}{2}\right) = p \text{ and } \tan\left(\frac{A - B}{2}\right) = q,$$

we obtain

$$\tan A = \frac{p + q}{1 - pq}.$$

The given condition $pq \neq 1$ ensures that the denominator $1 - pq$ is not zero.

Hence, the correct answer is Option (b): $\frac{p+q}{1-pq}$.

8. Explanation: (a)

Given: $\tan\left(\frac{A+B}{2}\right) = p$ and $\tan\left(\frac{A-B}{2}\right) = q$.

We can write B as

$$B = \frac{A + B}{2} - \frac{A - B}{2},$$

because

$$\frac{A + B}{2} - \frac{A - B}{2} = \frac{A + B - A + B}{2}$$
$$= \frac{2B}{2} = B.$$

Therefore,

$$\tan B = \tan\left[\frac{A + B}{2} - \frac{A - B}{2}\right].$$

Using the tangent subtraction formula

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y},$$

we get

$$\tan B = \frac{\tan\left(\frac{A + B}{2}\right) - \tan\left(\frac{A - B}{2}\right)}{1 + \tan\left(\frac{A + B}{2}\right)\tan\left(\frac{A - B}{2}\right)}.$$

Substituting the given values,

$$\tan B = \frac{p - q}{1 + pq}.$$

Hence, the correct answer is Option (a): $\frac{p-q}{1+pq}$.

9. Explanation: (b)

Given

$$4(A + B) = \pi \Longrightarrow A + B = \frac{\pi}{4}.$$

Using the tangent addition formula,

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}.$$

Since $\tan\frac{\pi}{4} = 1$,

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = 1,$$

so

$$\tan A + \tan B = 1 - \tan A \tan B.$$

Using the distributive law,

$$(1 + \tan A)(1 + \tan B) = 1 + \tan A + \tan B + \tan A \tan B$$
$$= 1 + (1 - \tan A \tan B) + \tan A \tan B$$
$$= 2.$$

Hence, the correct answer is Option (b): 2.

10. Explanation: (d)

Given: $4(A + B) = \pi$, we get

$$A + B = \frac{\pi}{4}.$$

Using the result obtained in Question 9,

$$\tan A + \tan B = 1 - \tan A \tan B.$$

Now, using $\cot\theta = \frac{1}{\tan\theta}$,

$$(\cot A - 1)(\cot B - 1) = \left(\frac{1}{\tan A} - 1\right)\left(\frac{1}{\tan B} - 1\right).$$

Taking the LCM inside each bracket,

$$= \left(\frac{1 - \tan A}{\tan A}\right)\left(\frac{1 - \tan B}{\tan B}\right)$$
$$= \frac{(1 - \tan A)(1 - \tan B)}{\tan A \tan B}.$$

Using the distributive law: $(a - b)(c - d) = ac - ad - bc + bd$, with $a = c = 1$, $b = \tan A$, and $d = \tan B$,

$$(1 - \tan A)(1 - \tan B) = 1 - \tan B - \tan A + \tan A \tan B.$$

Therefore,

$$(\cot A - 1)(\cot B - 1) = \frac{1 - \tan A - \tan B + \tan A \tan B}{\tan A \tan B}.$$

Using the result from Question 9: $\tan A + \tan B = 1 - \tan A \tan B$, we get

$$(\cot A - 1)(\cot B - 1) = \frac{1 - (1 - \tan A \tan B) + \tan A \tan B}{\tan A \tan B}$$
$$= \frac{1 - 1 + \tan A \tan B + \tan A \tan B}{\tan A \tan B}$$
$$= \frac{2 \tan A \tan B}{\tan A \tan B}$$
$$= 2.$$

Hence, the correct answer is Option (d): 2.

11. Explanation: (c)

Given the expression: $6\sin\left(\frac{\pi}{18}\right) - 8\sin^3\left(\frac{\pi}{18}\right)$

Factoring out 2, we get:

$$= 2\left[3\sin\left(\frac{\pi}{18}\right) - 4\sin^3\left(\frac{\pi}{18}\right)\right]$$

Using the triple-angle formula for sine: $\sin(3\theta) = 3\sin\theta - 4\sin^3\theta$, with $\theta = \frac{\pi}{18}$, the expression inside the brackets becomes:

$$3\sin\left(\frac{\pi}{18}\right) - 4\sin^3\left(\frac{\pi}{18}\right) = \sin\left(3 \times \frac{\pi}{18}\right) = \sin\left(\frac{\pi}{6}\right)$$

Since $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$, substituting this back into our expression:

$$= 2 \times \frac{1}{2} = 1$$

Hence, the correct answer is Option (c): 1.

12. Explanation: (c)

Given: $\tan^2\theta + \cot^2\theta = 2$.

Using : $\cot^2\theta = \frac{1}{\tan^2\theta}$,

let

$$x = \tan^2\theta.$$

Then

$$x + \frac{1}{x} = 2.$$

Multiplying throughout by x,

$$x^2 + 1 = 2x.$$

Therefore,

$$x^2 - 2x + 1 = 0.$$

Using the identity $a^2 - 2ab + b^2 = (a-b)^2$,

$$(x-1)^2 = 0 \Longrightarrow x = 1.$$

Since $x = \tan^2\theta$,

$$\tan^2\theta = 1 \Longrightarrow \tan\theta = \pm 1.$$

For $0° < \theta < 360°$,

$\tan\theta = 1$ at $\theta = 45°, 225°$,

and

$\tan\theta = -1$ at $\theta = 135°, 315°$.

Thus, there are 4 possible values of θ.

Hence, the correct answer is Option (c): 4.

13. Explanation: (c)

Given the expression: $\sin(\tan^{-1}0.75)$

Convert the decimal 0.75 into a fraction:

$$0.75 = \frac{3}{4}$$

Let $\theta = \tan^{-1}\left(\frac{3}{4}\right)$, which means:

$$\tan\theta = \frac{3}{4}$$

In a right-angled triangle, $\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{3}{4}$.

Using Pythagoras' theorem to find the hypotenuse:

$$\text{Hypotenuse} = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$$

Now, find $\sin\theta$:

$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{3}{5}$$

Hence, the correct answer is Option (c): $\frac{3}{5}$.

14. Explanation: (a)

Given the expression:

$$\tan^2\left(\frac{1}{2}\cos^{-1}\frac{1}{3}\right)$$

Let $\theta = \cos^{-1}\left(\frac{1}{3}\right)$, so that:

$$\cos\theta = \frac{1}{3}$$

The expression then becomes:

$$\tan^2\left(\frac{\theta}{2}\right)$$

Using the half-angle formula for tangent in terms of cosine,

$$\tan^2\left(\frac{\theta}{2}\right) = \frac{1-\cos\theta}{1+\cos\theta}$$

Substitute $\cos\theta = \frac{1}{3}$ into the formula:

$$\tan^2\left(\frac{\theta}{2}\right) = \frac{1-\frac{1}{3}}{1+\frac{1}{3}} = \frac{\frac{2}{3}}{\frac{4}{3}} = \frac{2}{4} = \frac{1}{2}$$

Hence, the correct answer is Option (a): $\frac{1}{2}$.

15. Explanation: (c)

Using the periodicity formulas: $\tan(\theta + 180°n) = \tan\theta$ and $\cot(\theta + 180°n) = \cot\theta$,

we get

$$\tan 1125° = \tan(6 \times 180° + 45°) = \tan 45° = 1,$$

$$\cot 405° = \cot(2 \times 180° + 45°) = \cot 45° = 1,$$

$$\tan 765° = \tan(4 \times 180° + 45°) = \tan 45° = 1,$$

and

$$\cot 675° = \cot(3 \times 180° + 135°) = \cot 135° = -1.$$

Therefore,

$$\tan 1125°\cot 405° + \tan 765°\cot 675° = (1)(1) + (1)(-1) = 1 - 1 = 0.$$

Hence, the correct answer is Option (c): 0.

16. Explanation: (c)

Given that A is a square matrix of order $n = 3$ and $|A| = 2$.

We know the standard property for the adjoint of a matrix:

$$\text{adj}(\text{adj}\,A) = |A|^{n-2}A$$

Substituting the values $n = 3$ and $|A| = 2$:

$$\text{adj}(\text{adj}\,A) = (2)^{3-2}A = (2)^1A = 2A$$

Hence, the correct answer is Option (c): 2A.

17. Explanation: (d)

For any square matrix A, using the matrix-adjoint property,

$$A(\text{adj}\,A) = |A|I,$$

where I is the identity matrix of the same order.

Given

$$A(\text{adj}\,A) = \begin{bmatrix} 64 & 0 & 0 \\ 0 & 64 & 0 \\ 0 & 0 & 64 \end{bmatrix} = 64I.$$

Comparing

$$|A|I = 64I,$$

we get

$$|A| = 64.$$

Hence, the correct answer is Option (d): 64.

18. Explanation: (b)

Given: $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$.

Using the definition of transpose, the rows and columns are interchanged:

$$A^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}.$$

Therefore,

$$A + A^T = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} + \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} 2\cos\theta & 0 \\ 0 & 2\cos\theta \end{bmatrix}.$$

Given that $A + A^T = I$, where $I = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, comparing the corresponding elements gives

$$2\cos\theta = 1.$$

Thus,

$$\cos\theta = \frac{1.}{2}$$

The least positive value of θ for which $\cos\theta = \frac{1}{2}$ is

$$\theta = \frac{\pi}{3}.$$

Hence, the correct answer is Option (b): $\frac{\pi}{3}$.

19. Explanation: (c)

Given: $AB = AC$.

Taking all terms to one side and using the distributive law $AX - AY = A(X - Y)$,

$$AB - AC = 0$$
$$\Rightarrow A(B - C) = 0.$$

Statement I: B and C are not necessarily equal if A is singular.

If A is singular, then A^{-1} does not exist, so A cannot be cancelled from $AB = AC$.

For example, if A is the zero matrix, then

$$AB = 0 = AC$$

for any matrices B and C, even when $B \neq C$. Therefore, Statement I is correct.

Statement II: B and C are equal if A is non-singular.

If A is non-singular, then A^{-1} exists. Premultiplying $AB = AC$ by A^{-1},

$$A^{-1}AB = A^{-1}AC.$$

Using $A^{-1}A = I$,

$$IB = IC \Longrightarrow B = C.$$

Therefore, Statement II is also correct.

Hence, the correct answer is Option (c): Both I and II.

20. Explanation: (b)

Given that A is a square matrix of order $n = 3$ and $|A| = 4$.

We know the standard property for the determinant of the adjoint of a matrix:

$$|\operatorname{adj} A| = |A|^{n-1}$$

Substituting the values $n = 3$ and $|A| = 4$:

$$|\operatorname{adj} A| = (4)^{3-1} = (4)^2 = 16$$

Hence, the correct answer is Option (b): 16.

21. Explanation: (b)

Let A and B be symmetric matrices of the same order, which means $A^T = A$ and $B^T = B$.

Statement I: $(AB - BA)$ is also a symmetric matrix.

To check if $(AB - BA)$ is symmetric, we take its transpose:

$$(AB - BA)^T = (AB)^T - (BA)^T$$
$$= B^TA^T - A^TB^T$$

Since A and B are symmetric ($A^T = A$ and $B^T = B$):

$$(AB - BA)^T = BA - AB$$
$$= -(AB - BA)$$

This shows that $(AB - BA)$ is a skew-symmetric matrix, not a symmetric matrix.

Thus, Statement I is incorrect.

Statement II: $(BA - AB)$ is a skew-symmetric matrix.

Reasoning: To check if $(BA - AB)$ is skew-symmetric, we take its transpose:

$$(BA - AB)^T = (BA)^T - (AB)^T$$
$$= A^TB^T - B^TA^T$$

Using the symmetry of A and B ($A^T = A$ and $B^T = B$):

$$(BA - AB)^T = AB - BA$$
$$= -(BA - AB)$$

Since the transpose of the matrix is equal to its negative, $(BA - AB)$ is indeed a skew-symmetric matrix.

Thus, Statement II is correct.

Since only Statement II is correct:

Hence, the correct answer is Option (b): II only.

22. Explanation: (c)

Given

$$A = \begin{bmatrix}-1 & 2\\ 3 & -4\end{bmatrix}.$$

Using the determinant formula: $\begin{vmatrix}a & b\\ c & d\end{vmatrix} = ad - bc$,

$$|A| = (-1)(-4) - (2)(3) = 4 - 6$$
$$= -2.$$

Using the determinant property of an inverse matrix,

$$|A^{-1}| = \frac{1}{|A|} = \frac{1}{-2} = -\frac{1.}{2}$$

Hence, the correct answer is Option (c): $-\frac{1}{2}$.

23. Explanation: (b)

The relation R on $\mathbb{R}$ is defined by

$$xRy \Leftrightarrow |x| \leq y.$$

For reflexivity, xRx must hold for every $x \in \mathbb{R}$, which requires

$$|x| \leq x.$$

For $x = -1$,

$$|-1| = 1 \not\leq -1.$$

Therefore, R is not reflexive.

For transitivity, suppose xRy and yRz. Then

$$|x| \leq y \quad \text{and} \quad |y| \leq z.$$

Since $|x| \geq 0$ and $|x| \leq y$, we get $y \geq 0$, so $|y| = y$. Therefore,

$$|x| \leq y = |y| \leq z,$$

which gives xRz.

Hence, R is transitive.

Hence, the correct answer is Option (b): transitive only.

24. Explanation: (c)

Using the Multiplication Principle, each of the four places can be filled with any of the digits 1,2,3,4.

Therefore, the total number of four-digit numbers is

$$4 \times 4 \times 4 \times 4 = 4^4 = 256.$$

The number of four-digit numbers having no repeated digit is

$$4! = 4 \times 3 \times 2 \times 1 = 24.$$

Using the Complement Rule,

Numbers with at least one repeated digit

$$= \text{Total numbers}$$
$$- \text{Numbers with no repetition}$$
$$= 256 - 24$$
$$= 232$$

Hence, the correct answer is Option (c): 232.

25. Explanation: (c)

Given

$$xyz = 210 = 2 \times 3 \times 5 \times 7, \quad x < y < z.$$

Since 210 is a product of four distinct primes, consider two cases.

Case 1:

If $x = 1$, then $yz = 210$. The possible pairs (y, z) are

$$(2,105), (3,70), (5,42), (6,35),$$
$$(7,30), (10,21), (14,15).$$

Thus, there are 7 triples.

Case 2:

If $x > 1$, all three factors must contain at least one prime factor.

Therefore, one factor contains two primes and each of the other two factors contains one prime. Using the combination formula,

$$\binom{4}{2} = 6.$$

Thus, the total number of elements in S is

$$7 + 6 = 13.$$

Hence, the correct answer is Option (c): 13.

26. Explanation: (c)

Given: $x = 4 + i, \quad i^2 = -1.$

Using the identity $(a + b)^2 = a^2 + 2ab + b^2$,

$$x^2 = (4 + i)^2 = 16 + 8i + i^2 = 15 + 8i.$$

Therefore,

$$x^3 = x \cdot x^2 = (4 + i)(15 + 8i).$$

Using the distributive law,

$$x^3 = 60 + 32i + 15i + 8i^2 = 60 + 47i - 8 = 52 + 47i.$$

Substituting these values,

$$x^3 + 2x^2 - 63x + 171 = (52 + 47i) + 2(15 + 8i) - 63(4 + i) + 171$$
$$= 52 + 47i + 30 + 16i - 252 - 63i + 171$$
$$= (52 + 30 - 252 + 171) + (47 + 16 - 63)i$$
$$= 1 + 0i = 1.$$

Hence, the correct answer is Option (c): 1.

27. Explanation: (c)

The given equation is

$$x^2 - p(x + 1) - q = 0,$$

or

$$x^2 - px - (p + q) = 0.$$

Since α and β are its roots, using Vieta's formulas,

$$\alpha + \beta = p, \quad \alpha\beta = -(p + q).$$

Therefore,

$$q = -\alpha\beta - p = -\alpha\beta - \alpha - \beta.$$

Now,

$$\alpha^2 + 2\alpha + q = \alpha^2 + 2\alpha - \alpha\beta - \alpha - \beta = \alpha^2 + \alpha - \alpha\beta - \beta = (\alpha + 1)(\alpha - \beta),$$

where factorization by grouping is used.

Similarly,

$$\beta^2 + 2\beta + q = (\beta + 1)(\beta - \alpha).$$

Therefore,

$$\frac{\alpha^2 + 2\alpha + 1}{\alpha^2 + 2\alpha + q} + \frac{\beta^2 + 2\beta + 1}{\beta^2 + 2\beta + q} = \frac{(\alpha + 1)^2}{(\alpha + 1)(\alpha - \beta)} + \frac{(\beta + 1)^2}{(\beta + 1)(\beta - \alpha)} = \frac{\alpha + 1}{\alpha - \beta} - \frac{\beta + 1}{\alpha - \beta} = \frac{\alpha - \beta}{\alpha - \beta} = 1$$

Hence, the correct answer is Option (c): 1.

28. Explanation: (d)

The given equation is

$$\frac{1}{2x + p} + \frac{1}{2x + q} = \frac{1}{r}.$$

Taking the LCM,

$$\frac{(2x + q) + (2x + p)}{(2x + p)(2x + q)} = \frac{1}{r},$$
$$\frac{4x + p + q}{(2x + p)(2x + q)} = \frac{1}{r}.$$

By cross-multiplication,

$$r(4x + p + q) = (2x + p)(2x + q).$$

Using the distributive law to expand,

$$4rx + r(p + q) = 4x^2 + 2(p + q)x + pq.$$

Therefore,

$$4x^2 + [2(p + q) - 4r]x + [pq - r(p + q)] = 0.$$

Using Vieta's formula, the sum of the roots of $ax^2 + bx + c = 0$ is $-\frac{b}{a}$.

Hence,

$$\text{Sum of roots} = -\frac{2(p + q) - 4r}{4}.$$

Since the sum of the roots is zero,

$$-\frac{2(p + q) - 4r}{4} = 0,$$
$$\Rightarrow 2(p + q) - 4r = 0,$$
$$\Rightarrow r = \frac{p + q}{2}.$$

Hence, the correct answer is Option (d): $(\frac{p+q}{2})$.

29. Explanation: (a)

Let α and β be the roots of

$$ax^2 + bx + c = 0,$$

and let

$$p_n = \alpha^n + \beta^n, \quad n > 1.$$

Since α is a root,

$$a\alpha^2 + b\alpha + c = 0.$$

Multiplying by α^{n-1},

$$a\alpha^{n+1} + b\alpha^n + c\alpha^{n-1} = 0.$$

Similarly, since $\beta$is also a root,

$$a\beta^{n+1} + b\beta^n + c\beta^{n-1} = 0.$$

Adding these equations,

$$a(\alpha^{n+1} + \beta^{n+1}) + b(\alpha^n + \beta^n) + c(\alpha^{n-1} + \beta^{n-1}) = 0$$

Using, $p_n = \alpha^n + \beta^n$,

$$ap_{n+1} + bp_n + cp_{n-1} = 0.$$

Hence, the correct answer is Option (a): 0.

30. Explanation: (d)

Since p, q, r are in an arithmetic progression, using the middle-term property of an AP,

$$2q = p + r.$$

Using the algebraic identity

$$a^3 + b^3 + 3ab(a + b) = (a + b)^3,$$

we get

$$p^3 + 3pr(p + r) + r^3 = (p + r)^3.$$

Since $p + r = 2q$,

$$(p + r)^3 = (2q)^3 = 8q^3.$$

Hence, the correct answer is Option (d): $8q^3$.

31. Explanation: (c)

Given:

$$\frac{1}{b - a} + \frac{1}{b - c} = \frac{2}{b}.$$

Taking the LCM on the left-hand side,

$$\frac{(b - c) + (b - a)}{(b - a)(b - c)} = \frac{2}{b},$$
$$\Rightarrow \frac{2b - a - c}{(b - a)(b - c)} = \frac{2}{b}.$$

By cross-multiplication,

$$b(2b - a - c) = 2(b - a)(b - c).$$

Using the distributive law to expand both sides,

$$2b^2 - ab - bc = 2b^2 - 2ab - 2bc + 2ac.$$

Therefore,

$$ab + bc = 2ac,$$
$$\Rightarrow b(a + c) = 2ac.$$

Dividing by abc,

$$\frac{1}{a} + \frac{1}{c} = \frac{2}{b}.$$

Thus, $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in AP.

By the definition of harmonic progression, if the reciprocals of the