PREFACE

IFAS, one of the finest institutes in India, has cumulated the IIT JAM Practice Physics Book. This book is useful for IIT JAM, TIFR, JEST, JNU, DU, BHU, HCU, CUCET, Autonomous, Deemed, Private, State and Central University Entrance Exams.

There are more than 2000 Questions in the book, which comprises questions from different topics mentioned in the latest syllabus of IIT JAM physics. All questions are sorted topic-wise with answer keys & detailed solutions. It will help the reader to practice questions from a particular topic at one place. This book relates the present study to its wider context, which would serve as a pedestal upon which the remaining part of the study will be built. This Book will give exposure to questions that are very important for building concepts and quick exercises.

This book will give you a competitive edge and the much-needed skill to ace the race of All M.Sc. Physics entrance exams.

A C K N O W L E D G M E N T

We are eternally grateful to the entire students at Institute for Advanced Studies (IFAS). We would like to thank Mr. Radheshyam Choudhary, Founding CEO, IFAS Edutech Pvt. Ltd. For their constant motivation, keen insight, and ongoing support in bringing our idea into reality. It is because of their efforts and encouragement that we have a legacy to pass on to the IFAS family.

Having an idea and turning it into a book is not easy. The experience is extremely challenging and rewarding. We especially want to thank people that helped make this happen. Complete thanks to Mr. Jeevitesh Kumar Rajput, Ms. Abhilasha Sharma & Ms. Anjali Ghanghass.

Next, we would like to extend our heartfelt gratitude to the entire IFAS team for the hard work. With your workaholic attitude we were able to further continue our teaching, training, and especially learning the many facets of the process of building these creative questions in books.

We take pride to express our gratitude for everyone's restless support, and producing it would be impossible without the outstanding people of IFAS publication. It was our immense pleasure to work with many other passionate people of IFAS during the production of this book. Special thanks to Vikendra Metha who crafted our ideas for the wonderful design of the cover page and Govind Kumar for formatting and type setting.

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1 CHAPTER

MATHEMATICAL PHYSICS

Unit 1.1 Calculus of Single and Multiple Variables

1. Evaluate $\lim_{x\to\infty} x \sin\frac{1}{x}$.
   (a) ∞ (b) 1
   (c) 0 (d) 2

2. Evaluate $\lim_{x\to -1} \frac{(x+2)(3x-1)}{x^2-3x-2}$
   (a) -1 (b) 1
   (c) 0 (d) 2

3. Evaluate $\lim_{x\to 0} \frac{e^{\frac{1}{x}}}{e^{\frac{1}{x}}+1}$
   (a) 0 (b) 1
   (c) ∞ (d) does not exists.

4. The function $f(x) = \begin{cases} \frac{\sin 2x}{x} & when\ x \neq 0 \\ 0 & when\ x = 0 \end{cases}$
   (a) is continuous at origin
   (b) has removable discontinuity at origin
   (c) is discontinuous at all points
   (d) none of the above

5. Evaluate $\lim_{x\to 0} \frac{xe^x - \log(1+x)}{x^2}$
   (a) $\frac{1}{2}$ (b) $\frac{3}{2}$
   (c) 1 (d) $\frac{5}{2}$

6. The function $f(x) = (x-3)^5(x+1)^4$
   (a) has maxima at $x = 1$ (b) has minima at $x = \frac{7}{9}$
   (c) both (a) and (b) (d) none of the above

7. If f, g be two function defined on some neighborhood of point (a, b) such that
   $\lim f(x, y) = l$
   $\lim g(x, y) = m$
   when $(x, y) \to (a, b)$, then
   Which of the following is incorrect?
   (a) $\lim(f+g) = \lim f + \lim g = l + m$
   (b) $\lim(f-g) = \lim f + \lim g = l + m$
   (c) $\lim(f \cdot g) = \lim f \cdot \lim g = l \cdot m$
   (d) all are correct.

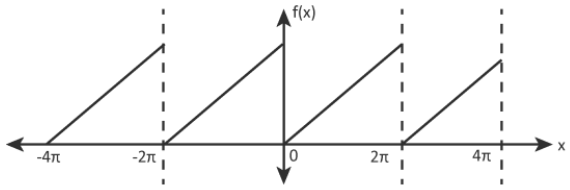
8. The maxima of the function $2x^3 - 3x^2 - 36x + 10$ lies at
   (a) -1 (b) -2
   (c) -3 (d) -4

9. The function $f(x) = \frac{2}{3}x^2 - 6x^2 + 20x - 5$
   (a) Possess only minimum value
   (b) Possess only maximum value
   (c) Does not possess a maxima or minima
   (d) Have both maxima and minima

10. The function $(x - [x])$, where $[x]$ = largest integer $\leq x$.
    (a) is continuous at all points
    (b) is continuous at all points except integral value 1, 2, 3...
    (c) is discontinuous
    (d) none of the above

Unit 1.2: Fourier Series

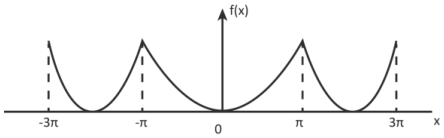
1. The Fourier series for the function shown below is:-



   (a) $\frac{\pi}{2} - 2\left[\sin x + \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x + \cdots\right]$
   (b) $\frac{\pi}{4} - 2\left[\sin x + \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x + \cdots\right]$
   (c) $\pi - 2\left[\sin x + \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x + \cdots\right]$
   (d) $-2\left[\sin x + \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x + \cdots\right]$

2. The function $f(x) = \left(\frac{\pi - x}{2}\right)^2$, $0 < x < 2\pi$ can be expanded as:-
   (a) $\frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{\cos nx}{n^2}$ (b) $\frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{\sin nx}{n^2}$
   (c) $\frac{\pi^2}{6} + \sum_{n=1}^{\infty} \frac{\cos nx}{n^2}$ (d) $\frac{\pi^2}{6} + \sum_{n=1}^{\infty} \frac{\sin nx}{n^2}$

3. For the given function



Which of the following relations satisfies?

(a) $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}\ldots\ldots=\frac{\pi^2}{12}$

(b) $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}\ldots\ldots=\frac{\pi^2}{6}$

(c) $\frac{1}{1^2}+\frac{1}{2^2}-\frac{1}{3^2}+\frac{1}{4^2}\ldots\ldots=\frac{\pi^2}{12}$

(d) $\frac{1}{1^2}+\frac{1}{2^2}-\frac{1}{3^2}+\frac{1}{4^2}\ldots\ldots=\frac{\pi}{6}$

4. Evaluate the $d.c.$ compound in the expansion of output of Half Wave Rectifier

$$f(t)=\begin{cases}0 & , \quad -\pi<\omega t<0\\ E_0\sin\omega t & , \quad 0<\omega t<\pi\end{cases}$$

(a) $\frac{E_0}{2}$ (b) $\frac{E_0}{\sqrt{2}}$

(c) $\frac{E_0}{\pi}$ (d) 0

5. Determine half range cosine series of $f(x)=x(\pi-x)\ ;\ 0<x<\pi$

(a) $\frac{\pi^2}{6}-\left[\frac{\cos 2x}{1^2}+\frac{\cos 4x}{2^2}+\frac{\cos 6x}{3^2}\ldots\right]$

(b) $\frac{\pi^2}{6}-\left[\frac{\cos 4x}{1^2}+\frac{\cos 6x}{2^2}+\frac{\cos 8x}{3^2}\ldots\right]$

(c) $\frac{\pi^2}{12}-\left[\frac{\cos 2x}{1^2}+\frac{\cos 4^x}{2^2}+\frac{\cos 6x}{3^2}\ldots\right]$

(d) $\frac{\pi}{12}-\left[\frac{\cos 4x}{1^2}+\frac{\cos 6x}{2^2}+\frac{\cos 8x}{3^2}\ldots\right]$

6. Determine half range sine series of $f(x)=x(\pi-x)\ ;\ 0<x<\pi$

(a) $\frac{8}{\pi}\left[\frac{\sin x}{1^3}+\frac{\sin 3x}{3^3}+\frac{\sin 5x}{5^3}+\ldots\right]$

(b) $\frac{8}{\pi^2}\left[\frac{\sin x}{1^3}+\frac{\sin 3x}{3^3}+\frac{\sin 5x}{5^3}+\ldots\right]$

(c) $\frac{8}{\pi}\left[\frac{\sin x}{1^2}-\frac{\sin 3x}{3^2}+\frac{\sin 5x}{5^2}+\ldots\right]$

(d) $\frac{8}{\pi^2}\left[\frac{\sin x}{1^2}+\frac{\sin 3x}{3^2}+\frac{\sin 5x}{5^2}+\ldots\right]$

7. Find cosine series expansion of $f(x)=1+x, 0\le x\le 2$

(a) $4+\frac{8}{\pi^2}\left[\frac{1}{1^2}\cos\frac{\pi x}{2}+\frac{1}{3^2}\cos\frac{3\pi x}{2}+\ldots\right]$

(b) $4-\frac{8}{\pi^2}\left[\frac{1}{1^2}\cos\frac{\pi x}{2}+\frac{1}{3^2}\cos\frac{3\pi x}{2}+\ldots\right]$

(c) $2+\frac{4}{\pi^2}\left[\frac{1}{1^2}\cos\frac{\pi x}{2}+\frac{1}{3^2}\cos\frac{3\pi x}{2}+\ldots\right]$

(d) $2-\frac{4}{\pi^2}\left[\frac{1}{1^2}\cos\frac{\pi x}{2}+\frac{1}{3^2}\cos\frac{3\pi x}{2}+\ldots\right]$

8. Find fourier sine series for the function $f(x)=e^{ax}$ for $0<x<\pi$

(a) $\sum\frac{2n}{(a^2+n^2)\pi}[1-(-1)^n e^{a\pi}]\sin nx$

(b) $\sum\frac{2}{(a^2+n^2)\pi}[1-(-1)^n e^{a\pi}]\sin nx$

(c) $\sum\frac{2n}{(a^2+n^2)\pi}[1+(-1)^n e^{a\pi}]\sin nx$

(d) $\sum\frac{2}{(a^2+n^2)\pi}[1+(-1)^n e^{a\pi}]\sin nx$

9. Which of the following is not the Dirichlet's conditions for a fourier series of function $f(x)$ for integral $(-\pi,\pi)$?

(a) $f(x)$ is single bounded.

(b) $f(x)$ is bounded.

(c) has only a finite number of discontinuity.

(d) Infinite number of discontinuities.

10. Find the constant term in the expansion of function

$$f(x)=\begin{cases}1 & ; \quad 0<x<\frac{\pi}{2}\\ 0 & ; \quad \frac{\pi}{2}<x<\pi\end{cases}$$

(a) 0 (b) 1

(c) $\frac{3}{2}$ (d) $\frac{1}{2}$

Unit 1.3: Vector Algebra

1. Evaluate $\hat{\imath}\times(\vec{a}\times\hat{\imath})+\hat{\jmath}\times(\vec{a}\times\hat{\jmath})+\hat{k}\times(\vec{a}\times\hat{k})$

(a) $\vec{a}$ (b) $2\vec{a}$

(c) $3\vec{a}$ (d) 0

2. The condition for $\vec{a},\vec{b},\vec{c}$ to be coplanar:-

(a) $\vec{a}\cdot(\vec{b}\times\vec{c})=0$ (b) $(\vec{a}\times\vec{b})\times\vec{c}=0$

(c) $(\vec{a}\cdot\vec{c})\times\vec{b}=0$ (d) $\vec{a}+\vec{b}+\vec{c}=0$

3. The value of $\vec{a}\times(\vec{b}\times\vec{c})$ is

(a) $(\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}$ (b) $(\vec{a}\cdot\vec{b})\vec{c}-(\vec{c}\cdot\vec{b})\vec{a}$

(c) $(\vec{a}\cdot\vec{b})\vec{c}-(\vec{a}\cdot\vec{c})\vec{b}$ (d) $(\vec{c}\cdot\vec{b})\vec{a}-(\vec{a}\cdot\vec{b})\vec{c}$

4. The value of λ, if

$$\vec{a}=\hat{\imath}+\hat{\jmath}+\hat{k}$$
$$\vec{b}=2\hat{\imath}-4\hat{k}$$
$$\vec{c}=\hat{\imath}+\lambda\hat{\jmath}+3\hat{k}$$

are coplanar:-

(a) $\frac{3}{5}$ (b) $\frac{5}{3}$

(c) $\frac{2}{3}$ (d) $\frac{1}{3}$

5. If $\vec{a},\vec{b},\vec{c}$ are the vectors representing adjacent sides of a parallelepiped, then it's volume is given by:-

(a) $\vec{a}\cdot(\vec{b}\times\vec{c})$ (b) $[\vec{a}\cdot(\vec{b}\times\vec{c})]$

(c) $[\vec{a}\cdot(\vec{b}\cdot\vec{c})]^2$ (d) $(\vec{a})^3\cdot(\vec{b}\times\vec{c})$

6. Volume of tetrahedron is represented by:-

(a) $\frac{1}{2}[\vec{a}\vec{b}\cdot\vec{c}]$ (b) $\frac{1}{2}[\vec{a}\vec{b}\vec{c}]^2$

(c) $\frac{1}{6}[\vec{a}\vec{b}\vec{c}]$ (d) $\frac{1}{6}[\vec{a}\vec{b}\vec{c}]^3$

7. The volume of tetrahedron if adjacent sides are represented by

$\vec{a} = 3\hat{\imath} - \hat{\jmath}$
$\vec{b} = \hat{\jmath} + 2\hat{k}$
$\vec{c} = \hat{\imath} + 5\hat{\jmath} + 4\hat{k}$ is:-

(a) 20 (b) $\frac{20}{6}$
(c) $\frac{3}{10}$ (d) $\frac{1}{20}$

8. Find the value of m, if given vectors are linearly dependent

$$\vec{a} = 7\hat{\imath} + m\hat{\jmath} + \hat{k}$$
$$\vec{b} = 3\hat{\imath} + 2\hat{\jmath} + \hat{k}$$
$$\vec{c} = 5\hat{\imath} + 3\hat{\jmath} + \hat{k}$$

(a) 1 (b) 2
(c) 3 (d) 4

9. The term $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d})$ is equals to
(a) $(\vec{a} \cdot \vec{c})(\vec{b} \cdot \vec{d}) - (\vec{a} \cdot \vec{d})(\vec{b} \cdot \vec{c})$
(b) $(\vec{a} \cdot \vec{d})(\vec{b} \cdot \vec{c}) - (\vec{a} \cdot \vec{c})(\vec{b} \cdot \vec{d})$
(c) 0
(d) 1

10. The value of

$$\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b})$$

Is:-

(a) 1 (b) $\vec{a}(\vec{b} \times \vec{c})$
(c) $\vec{a} + \vec{b} + \vec{c}$ (d) 0

Unit 1.4: Vectors (Gradient, Divergence, Curl)

1. The magnitude of the tangent vector at any point of the curve $x = 3\cos t, y = 3\sin t, z = 4t$ is
(a) 2 (b) 3
(c) 4 (d) 5

2. The unit tangent vector at point $t = 2$ on the curve $x = t^2 - 1, y = 4t - 3, z = 2t^2 - 6t$ is:-
(a) $\frac{1}{3}(2\hat{\imath} + 2\hat{\jmath} + \hat{k})$ (b) $\frac{1}{3}(2\hat{\imath} - 2\hat{\jmath} + \hat{k})$
(c) $\frac{1}{3}(2\hat{\imath} + 2\hat{\jmath} - \hat{k})$ (d) $\frac{1}{3}(2\hat{\imath} - 2\hat{\jmath} - \hat{k})$

3. If $\frac{d\vec{a}}{dt} = \vec{u} \times \vec{a}$ and $\frac{d\vec{b}}{dt} = \vec{u} \times \vec{b}$ then $\frac{d}{dt}[\vec{a} \times \vec{b}] = ?$
(a) $(\vec{u} \times \vec{a}) \times (\vec{u} \times \vec{b})$ (b) $\vec{u} \times \vec{a} \times \vec{b}$
(c) $\vec{u} \times (\vec{a} \times \vec{b})$ (d) $\vec{u} \cdot (\vec{a} \times \vec{b})$

4. The unit normal vector to the surface $x^2 + y^2 + z^2 = 5$ at point $(1, 2, 3)$ is:-
(a) $\frac{-\hat{\imath}-2\hat{\jmath}-3\hat{k}}{\sqrt{14}}$ (b) $\frac{\hat{\imath}-2\hat{\jmath}-3\hat{k}}{\sqrt{14}}$
(c) $\frac{\hat{\imath}-2\hat{\jmath}+3\hat{k}}{\sqrt{14}}$ (d) $\frac{\hat{\imath}+2\hat{\jmath}+3\hat{k}}{\sqrt{14}}$

5. The rate of change of $\phi = xyz$ in the direction normal to the surface $\psi = x^2 + y^2 + z^2 - 5$ at point $(1, 1, 1)$ is:-
(a) $\sqrt{1}$ (b) $\sqrt{2}$
(c) $\sqrt{3}$ (d) $\sqrt{4}$

6. The direction in which the directional derivation of $f(x, y) = \frac{x^2 - y^2}{xy}$ at $(1, 1)$ is zero is:-
(a) $\frac{\hat{\imath}+\hat{\jmath}}{\sqrt{2}}$ (b) $\frac{-\hat{\imath}+\hat{\jmath}}{\sqrt{2}}$
(c) $\frac{-\hat{\imath}-\hat{\imath}}{\sqrt{2}}$ (d) $\frac{\hat{\imath}-\hat{\jmath}}{\sqrt{2}}$

7. The greatest rate of increase of function $\phi = xyz$ at point $(1, 0, 3)$ is
(a) 1 (b) 2
(c) 3 (d) 4

8. If $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{z}$, then which of the following relation is valid?
(a) $grad\left(\frac{1}{x}\right) = \frac{-\vec{r}}{r^3}$ (b) $grad(r) = \frac{-\vec{r}}{r^2}$
(c) both (a) & (b) (d) none of the above

9. Consider a vector field $\vec{F}$. Which of the following conditions must be satisfied for $\vec{F}$ to be conservative?
(a) $\nabla^2\vec{F} = 0$ (b) $\vec{\nabla} \cdot \vec{F} = 0$
(c) $\vec{\nabla} \times \vec{F} = 0$ (d) $grad\ F = 0$

10. Which of the following statements is correct?
(a) Every irrotational field is conservative
(b) Every conservative field is irrotational
(c) Every solenoidal field is conservative
(d) None is correct

11. For a solenoidal field
(a) $\nabla^2 F = 0$ (b) $\vec{\nabla} \cdot \vec{\mathrm{F}} = 0$
(c) $\vec{\nabla} \times \vec{\mathrm{F}} = 0$ (d) $grad\ F = 0$

12. For the fluid motion $\vec{v} = (y + z)\hat{\imath} + (z + x)\hat{\jmath} + (x + y)\hat{k}$, the velocity potential is:-
(a) $x + y + z + c$ (b) $xyz + c$
(c) $xy + yz + zx + c$ (d) $x^2 + y^2 + z^2 + c$

13. For a vector field $\vec{A}$
(a) $\vec{\nabla} \cdot \vec{A} = 0$ (b) $\vec{\nabla} \times \vec{A} = 0$
(c) $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) = 0$ (d) none of the above

14. If $\vec{F}$ is a conservative force field, then the value of $curl\ \vec{F}$ is:-
(a) 1 (b) 0
(c) $\vec{\nabla}F$ (d) -1

15. If $\vec{R} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and $\vec{A}$ is a constant vector, then $curl(\vec{A} \times \vec{R})$ is:-
(a) $\vec{R}$ (b) $\vec{A}$
(c) $2\vec{R}$ (d) $2\vec{A}$

16. For what values of a, b, c the vector field $\vec{V} = (x + y + az)\hat{\imath} + (bx + 3y - z)\hat{\jmath} + (3x + cy + z)\hat{k}$ is irrotational?
(a) $a = 3, b = 1, c = -1$ (b) $a = 1, b = 3, c = -1$
(c) $a = 3, b = -1, c = 1$ (d) $a = 3, b = 1, c = 1$

17. For what value of k the field $\vec{A} = (y + z)\hat{\imath} + (kz + x)\hat{\jmath} + (x + y)\hat{k}$, is irrotational?
(a) 1 (b) 0
(c) 2 (d) -1

18. The value of $\nabla \cdot (\nabla\phi)$ is
(a) $\vec{\nabla} \times \vec{\phi}$ (b) $\nabla^2\phi$
(c) 0 (d) 1

19. The greatest rate of increase of $\phi = x^2 + y^2 = 2z^2$ at point $(5, 0, 4)$ is:-
(a) $\sqrt{256}$ (b) $\sqrt{356}$
(c) $\sqrt{456}$ (d) 0

20. If $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, then find $\vec{\nabla} \cdot \left(\frac{\vec{r}}{r^2}\right)$.
(a) $\frac{1}{r^2}$ (b) $\frac{1}{r}$
(c) 0 (d) r

21. The line integral $\phi\vec{A} \cdot d\vec{r}$ of $\vec{A} = (3x^2 + 6y)\hat{\imath} - 14yz\hat{\jmath} + 20xz\hat{k}$ along curve C from $(0, 0, 0)$ to $(1, 1, 1)$ is $[curve\ C \rightarrow x = t, y = t^2, z = t^3]$
(a) 2 (b) 3
(c) 4 (d) 5

22. The work done by a particle when force $\vec{F} = (x^2 - y^2 + z)\hat{\imath} - (2xy + y)\hat{\jmath}$ is applied to move it from origin to $(1, 1)$ along a parabola $y^2 = x$ is:-
(a) $\frac{1}{3}$ (b) $\frac{2}{3}$
(c) $\frac{1}{2}$ (d) 1

23. For a solenoidal vector point function $\vec{F}$
(a) $\iint(\vec{F} \cdot \hat{n})dS = 0$ (b) $\vec{F} = 0$
(c) $\vec{\nabla} \times \vec{F} = 0$ (d) none of the above

24. Evaluate $\oint(yzdx + zxdy + zydz)$ where C is the curve $x^2 + y^2 = 1, z = y^2$
(a) 0 (b) 1
(c) -1 (d) 2

25. The line integral $\oint \vec{F} \cdot d\vec{r}$ for $\vec{F} = (x^2 + y^2)\hat{\imath} - 2xy\hat{\jmath}$ bounded by a rectangle $x = \pm a$, $y = 0$ and $y = b$ is
(a) $4ab$ (b) $-4ab$
(c) $-4ab^2$ (d) $4ab^2$

Unit 1.5: Taylor Series

1. For a function $f(z)$ which is analytic at all points inside a circle C, with centre at point 'a' and radius R, then it's taylor's series expansion can be written as:-
(a) $f(z) = f(a) + f(a)(z - a) + \frac{f^4(a)}{2!}(z - a)^2 +$-------
(b) $f(z) = f(a) + f'(a)(z - a) + \frac{f''(a)}{2!}(z - a)^2 +$-----
(c) $f(z) = f'(a) + f'(a) + f'''(a) +$---------
(d) $f(z) = \frac{f(a)}{1!} + \frac{f'(a)(z-a)}{2!} + \frac{f''(a)(z-a)^2}{3!}$ ---------

2. The Taylor series expansion of e^z about $'a'$ is
(a) $e^0 + \frac{(z-a)}{1!}e^1 + \frac{(z-a)^2e^2}{2!} +$ ----------
(b) $1 + e^0 + \frac{(z-a)e^1}{1!} + \frac{(z-a)^2e^2}{2!} +$ -------
(c) $1 + e^a + \frac{(z-a)e^a}{1!} + \frac{(z-a)^2e^a}{2!} +$ -------
(d) $e^a + \frac{(z-a)e^a}{1!} + \frac{(z-a)^2e^a}{2!} +$ -------

3. The Taylor series expansion of $f(z) = \cos hz$ about πi is:-
(a) $\pi i + \frac{(\pi i)^2}{2!} + \frac{(\pi i)^3}{3!} +$ -------
(b) $\cosh \pi i + (2 - \pi i)\sinh \pi i + \frac{(z-\pi i)^2}{2!}\cosh(\pi i) +$ -------
(c) $\sinh \pi i + (2 - \pi i)\sinh \pi i + \frac{(z-\pi i)^2}{2!}\sinh(\pi i) +$ -------
(d) $\cosh \pi i + (z - \pi i)\cosh \pi i + \frac{(z-\pi i)^2}{2!}\cosh(\pi i) +$ -------

4. The radius of convergence R for a power series $\sum a_n z^n$ is given by:-
(a) $\frac{1}{R} = \lim_{n\to\infty}\left|\frac{a_n}{a_{(n+1)}}\right|$ (b) $\frac{1}{R} = \lim_{n\to\infty}\left|\frac{a_{(n+1)}}{a_n}\right|$
(c) $\frac{1}{R} = \lim_{n\to\infty}\left|\frac{a_n}{a_{(n+1)}}\right|$ (d) $\frac{1}{R} = \lim_{n\to\infty}|a_n + a_{(n+1)}|$

5. The Taylor series expansion of $\cos z$ about $z = \frac{\pi}{4}$ is
(a) $\frac{1}{\sqrt{2}}\left[1 - \left(z - \frac{\pi}{4}\right) - \frac{1}{2!}\left(z - \frac{\pi}{4}\right)^2 + \frac{1}{3!}\left(z - \frac{\pi}{4}\right)^3 + \ldots\right]$
(b) $\frac{1}{\sqrt{2}}\left[\left(z - \frac{\pi}{4}\right) + \frac{1}{2!}\left(z - \frac{\pi}{4}\right)^2 + \frac{1}{3!}\left(z - \frac{\pi}{4}\right)^3 + \ldots\right]$
(c) $\frac{1}{\sqrt{2}}\left[1 + \left(z - \frac{\pi}{4}\right) + \frac{1}{2!}\left(z - \frac{\pi}{4}\right)^2 + \frac{1}{3!}\left(z - \frac{\pi}{4}\right)^3 + \ldots\right]$
(d) $\frac{1}{\sqrt{2}}\left[\left(z - \frac{\pi}{4}\right) - \frac{1}{2!}\left(z - \frac{\pi}{4}\right)^2 + \frac{1}{\sqrt{3}!}\left(z - \frac{\pi}{4}\right)^3 + \ldots\right]$

Unit 1.6: "Integrals, stokes Theorem, green's Theorem"

1. Numerical type question – IIT JAM
   Consider a vector field $\vec{A} = (2y+3)\hat{\imath} + xz\,\hat{\jmath} + (yz - x)\hat{k}$. Caculate the line integral $\int \vec{A}.d\vec{r}$ along the path $c, x = 2t, y = t, z = t^3$ from $t - 0$ to $t = 1$.

2. The line integral for a field $\vec{A} = (\sin y)\hat{\imath} + x(1+\cos y)\hat{\jmath}$ over a circular path $x^2 + y^2 = a^2, z = 0$ is
   (a) π (b) $\frac{\pi a^2}{2}$
   (c) $\frac{\pi q^2}{4}$ (d) πa^2

3. For a line integral to be independent of the path
   (a) The curl of given field must be zero
   (b) The divergence of given field must be zero
   (c) Both (a) and (b)
   (c) None of the above

4. The physical significance of the term $\iint(\hat{f}.\hat{n})\ ds$ is :-
   (a) It represents the work done by particle in moving the given field.
   (b) It represents the potential in the given field.
   (c) It represents the velocity of liquid
   (d) None of the above

5. The value of $\iint(yz\hat{\imath} + zx\,\hat{\jmath} + xy\hat{k}).d\vec{s}$
   Where s is surface of sphere $x^2 + y^2 + z^2 = a^2$ in first octant is :-
   (a) $\frac{3a^4}{8}$ (b) $\frac{3a^4}{4}$
   (c) $\frac{3a^2}{8}$ (d) $\frac{3a^2}{4}$

6. The line integral of a vector field around a closed path can always be written as :-
   (a) $\oint \vec{A}.d\vec{e} = \iint(\vec{\nabla}\vec{A})d\vec{s}$
   (b) $\oint \vec{A}.d\vec{l} = \iint(\vec{\nabla}\vec{A})d\vec{s}$
   (c) $\oint \vec{A}.d\vec{r} = \iint(\vec{\nabla}\times\vec{A})d\vec{s}$
   (d) $\oint \vec{A}.d\vec{r} = \iint(\vec{\nabla}\times\vec{A})d\vec{s}$

7. Which of the following represents an operator that can be operated on both scalar(ϕ) and vector $(\vec{A})$ fields ?
   (a) $\vec{\nabla}\vec{A}$ (b) $\vec{A}.\vec{\nabla}$
   (c) Δ (d) $A^2\nabla$

8. For two differential scalar fields U & ϕ, which of the following holds true ?
   (a) $\vec{\nabla}.(U\phi) = 0$ (b) $\vec{\nabla}(U\phi) = 0$
   (c) $\vec{\nabla}(\vec{\nabla}\,U\times\vec{\nabla}\phi) = 0$ (d) $\vec{\nabla}(\vec{\nabla}U.\vec{\nabla}\phi) = 0$

9. The gradient of a scalar field function represents :-
   (a) Solenoidal field (b) Gaussian field
   (c) Lamellar field (d) Rolational field

10. The divergence of curl of a vector is
    (a) 1 (b) 0
    (c) $\frac{1}{2}$ (d) $\frac{\pi}{2}$

11. The value of area (calculated by green's Theorem) in first quadrant bounded by $y = x, y = \frac{1}{x}$ and $y = \frac{x}{4}$ is
    (a) $\log 3$ (b) $\log 2$
    (c) $\log 4$ (d) 1

12. Evaluate the surface integral $\iint \vec{F}\,\hat{n}\,ds$ for $\vec{F} = 4xz\hat{\imath} - y^2\hat{\jmath} + yz\hat{k}$ and S is the surface of cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1\ is$
    (a) 0.5 (b) 1
    (c) 1.5 (d) 2

13. The value of $\iint(yz\,dy\,dz + zx\,dz\,dx + xy\,dy\,dx)$
    Where S is $x^2 + y^2 + z^2 = 4$, is :-
    (a) 3π (b) $\frac{32\pi}{3}$
    (c) 32π (d) 0

14. The value of $\oint \vec{F}.d\vec{r}$ for $\vec{F} = (x^2 + y^2)\hat{\imath} - 2xy\,\hat{\jmath}$ and c is boundary of the rectangle $x = \pm 1, y = 0$ and $y = 1$ is :-
    (a) $+4$ (b) -4
    (c) 0 (d) -1

15. The line integral $\int(\sin z\,dx - \cos x dy + \sin y dz)$ where C is bounded by $0 \le x \le \pi, 0 \le y \le 1, z = 3$ is.
    (a) 0 (b) 1
    (c) 2 (d) 3

Unit 1.7: Matrices

1. Let A be a nilpotent matrix whose index is 3, then which among the following is true ?
   (a) $A^3 = A$ (b) $A^2 = A$
   (c) $A^3 = 0$ (d) $A^2 = 0$

2. Any matrix 'A' will be idempotent if
   (a) $A^2 = I$ (b) $A^2 = A$
   (c) $A^2 = 0$ (d) $A = I$

3. The condition for a matrix A to be involuntary is:-
   (a) $A^2 = I$ (b) $A^2 = A$
   (c) $A^2 = 0$ (d) $A = R$

4. Choose the correct statement
   (a) In equal matrix, it is not necessary that the elements in two equal matrices are equal
   (b) Unit matrix is involuntary
   (c) For involuntary matrix, $A^2 = A$
   (d) For idempotent matrix, $A^2 = I$

5. Any square matrix can be represented as
   (a) Sum of symmetric and anti symmetric matrix
   (b) Difference of symmetric and anti symmetric matrix
   (c) Product of symmetric and anti symmetric matrix
   (d) Cannot be represented in the form of symmetric and anti symmetric matrix

6. Choose the wrong statement
   (a) All elements of unitary matrix are one
   (b) The determinant of singular matrix is zero
   (c) The diagonal elements of skew symmetric matrix are zero
   (d) All are correct

7. Let A be skew symmetric matrix, then
   (a) Determinant of A is zero if matrix is of even order
   (b) Determinant of A is zero or matrix is of add order
   (c) Determinant of A is always zero
   (d) None of the above

8. Consider the following statements:-
   (a) Area of triangle can be represented in the determinant form of matrix
   (b) For collinear points, the determinant is always zero.
   Which of the following is correct?
   (a) only A (b) only B
   (c) both A & B (d) neither A nor B

9. Which of the following statements is correct regarding orthogonal matrix "A"?
   (a) $AA' = I, |A| = I$ (b) $AA' = I, |A| = 0$
   (c) $AA' = I, |A| \neq 0$ (d) $AA' \neq I\ |A| = I$

10. Which of the following statements is wrong regarding orthogonal matrix A?
    (a) A^{-1} is also orthogonal
    (b) A^T is also orthogonal
    (c) Product of two orthogonal matrix is also orthogonal
    (d) All are correct statements

11. Consider the following statements.
    (I) The matrix A and its transpose A^T
    (II) The sum of the eigenvalues of matrix A is equal to the determinant of A
    Which one of the following is true?
    (a) Only I (b) Only II
    (c) Both I & II (c) Neither I nor II

12. Which of the following is wrong?
    (a) If λ is eigenvalue of orthogonal matrix, then $\frac{1}{\lambda}$ is also eigenvalue.
    (b) The eigenvalue of singular matrix is zero
    (c) Matrix and its transpose will have same eigenvalues
    (d) Matrix and its inverse will have same eigenvalues.

13. Two eigen vector A and B are called orthogonal vector if:-
    (a) $AB = 0$ (b) $A^T B = 0$
    (c) $\vec{A}B = 0$ (d) $A\vec{B} = 0$

14. If A and B be two matrices and $\vec{A}$ and $\vec{B}$ are their conjugate matrices, then which of the following is incorrect?
    (a) $\bar{\bar{A}} = A$ (b) $\overline{A+B} = \bar{A} + \bar{B}$
    (c) $k\,\bar{A} = \bar{k}\bar{A}$ (d) $\overline{AB} = \bar{B}\bar{A}$

15. A 3×3 matrix has elements such that its trace is 12 and its determinant is 42. The eigenvalues of this matrix are positive integers. The largest eigenvalue of the matrix is
    (a) 6 (b) 7
    (c) 9 (d) 11

16. Which of the following statements is incorrect regarding similar matrices A and B?
    (a) They have same determinant.
    (b) They have same value of trace.
    (c) They have same eigenvalues.
    (d) They have same eigenvectors.

17. The eigenvalues of the matrix $A = \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix}$ are
    (a) Real and distinct
    (b) Real and same
    (c) Complex but not conjugate of each other
    (d) Complex and conjugate of each other

18. Which of the following is incorrect regarding unitary matrix?
    (a) Unitary matrices are complex analog of real orthogonal matrix.
    (b) $AA^{\theta} = I$
    (c) $A^{-1} = A^{\theta}$
    (d) All statements are correct.

19. If a matrix A will have all imaginary eigenvalues. Then the matrix will necessarily be:-
(a) Unitary (b) Orthogonal
(c) Hermitian (d) Anti-Hermitian

20. If a matrix A has $0, 1+i$, and $1-i$ as its eigenvalues, then
(a) A is Hermitian (b) A is unitary
(c) A is anti-Hermitian (d) $|A| = 0$

21. The eigenvalues of the matrix $A = \begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$ are
(a) equal and real (b) real and distinct
(c) imaginary (d) zero

22. A 3×3 singular matrix has eigenvalues ± 1 The other eigenvalue will be:-
(a) 0 (b) +1
(c) – 1 (d) i

23. The eigenvalues of matrix $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ are
(a) 6, 1 (b) 2, 5
(c) 6, 1 (d) 2, 5

24. The matrix $A = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$, what will be the eigenvalues of A^{-1}?
(a) 1, 6 (b) $1, \frac{1}{6}$
(c) $-1, -6$ (d) $-1, \frac{-1}{6}$

25. Find the eigenvectors of $A = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$
(a) $\binom{1}{2}, \binom{2}{1}$ (b) $\binom{1}{2}, \binom{2}{-1}$
(c) $\binom{1}{-2}, \binom{2}{1}$ (d) $\binom{1}{-2}, \binom{-2}{1}$

26. The sum of eigenvalue of the matrix $\begin{bmatrix} 3 & -3 & 3 \\ 2 & 1 & 1 \\ 1 & 5 & 6 \end{bmatrix}$ is
(a) 18 (b) 0
(c) 10 (d) 12

27. The eigenvalues of matrix representing the following pair of linear equations
$$x - iy = 0$$
$$-ix + y = 0$$
(a) $1+i, 1-i$ (b) $-1+i, -1-i$
(c) $1, i$ (d) $0, 1$

28. Find the determinant of $A = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{vmatrix}$
(a) 0 (b) 1
(c) 18 (d) 36

29. The area of triangle having vertices $(1,2)$, $(3,6)$ and $(-1,-8)$ is (sq units)
(a) 12 (b) 6
(c) 18 (d) 24

30. If area of a triangle is 15 sq units with vertices $(-2,-3), (3,k)$ and $(-1,-8)$, then what will be the value of k?
(a) 1 (b) 2
(c) 3 (d) 4

31. Which of the following forms a set of linearly independent vectors?
(a) $(1,-1,1)$, $(2,1,1)$, $(3,0,2)$
(b) $(1,2,-2)$, $(-1,3,0)$, $(0,-2,1)$
(c) $(3,1,-4)$, $(2,2,-3)$, $(0,-4,1)$
(d) $(1,-2)$, $(2,1)$, $(3,2)$

32. The vectors $A_1 = (a_1, b_1)$, $A_2 = (a_2, b_2)$ are linearly dependent if:-
(a) $a_1a_2 - b_1b_2 = 0$ (b) $a_1a_2 - b_1b_2 \neq 0$
(c) $a_1b_2 - a_2b_1 = 0$ (d) $a_1b_1 - a_2b_2 = 0$

33. The value of 'k' if $(1,-1,1), (2,1,1)$ and $(3,0,k)$ forms a set of linearly dependent vectors is:-
(a) 1 (b) 2
(c) 0 (d) 3

34. If eigenvalues of matrix A are 1, 2, 3 then one of the eigenvalues
(a) 1 (b) 3
(c) 2 (d) 0

35. Let A and B be two matrices with ranks r_1 and r_2 respectively, then
(a) Rank $(AB) = r_1.r_2$ (b) Rank $(AB) \leq r_1$
(c) Rank $(AB) > r_1$ (d) Rank $(AB) = r_1 + r_2$

36. The points $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) are collinear, then,
(a) rank of matrix they form is less than 2
(b) rank of matrix they form is greater then 3
(c) rank of matrix they form is greater than 3
(d) rank cannot be relatable with collinearity

37. The rank of matrix is said to be 'r' if f
(I) it has at least one non-zero minor of order 'r'
(II) every minor of A of order higher than 'r' is zero.

(a) only I (b) only II
(c) both I & II (d) none of these

38. For which value of 'b' the rank of the matrix $A = \begin{vmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ b & 13 & 10 \end{vmatrix}$ is 2
(a) 1 (b) 2
(c) 3 (d) 4

39. The rank of matrix $A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$ is
(a) 0 (b) 1
(c) 2 (d) 3

40. Let A be a matrix and C be its augmented matrix, then the system of equations A forms will have consistent solution if
(a) Rank A = Rank C (b) Rank A ≠ Rank C
(c) Rank A = 1 (d) Rank C = 1

41. If A and B are two matrices such that A, B = B, A then
(a) $(A\,B)^n = B^n A^n$ (b) $(AB)^n = A^n B^n$
(c) $(AB)^n = n\,(A\,B)$ (d) None of the above

42. If A and B are two matrices such that $A.B = 0$, then
(I) either $A = 0$ or $B = 0$
(II) it is not necessary that $A = 0$ or $B = 0$
Which of the following is true?
(a) I is true
(b) II is true
(c) I is true only when II is true
(d) none of true

43. If A, B and C are matrices such that $AB = AC$, then
(a) $B = C$ (b) $B \neq C$
(c) $B = I$ (d) $A = I$

44. Let A and B are two matrices then which of the following is not true?
(a) $AI = IA = A$ (b) $A(B + C) = AB + AC$
(c) $AB = BA$ (d) $(AB)^1 = B^1 A^1$

Unit 1.8: Algebra of Complex Number

1. The phase of complex number $-1 + i$ in the polar representation is
(a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$
(c) π (d) $\frac{\pi}{2}$

2. The value of i^{10^2} *is*
(a) 1 (b) -1
(c) i (d) $-i$

3. If z_1 and z_2 are two complex numbers then $\frac{z_1}{z_2}$ can be simplified as-
(a) $\frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) - i\sin(\theta_1 - \theta_2]$
(b) $r_4(\cos\theta_1 + i\sin\theta_1 - r_2(\cos\theta_2 + i\sin\theta_2)$
(c) $r_1(\cos\theta_1 + i\sin\theta_1 + r_2(\cos\theta_2 + i\sin\theta_2)$
(d) $\frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2]$

4. $(3 + 4i)(2 + i)$ can be simplified as-
(a) $\frac{13-qi}{2}$ (b) $\frac{13+qi}{2}$
(c) $\frac{q+13i}{2}$ (d) $\frac{q-13i}{2}$

5. The modulus of $(4 + 2i)\left(-3 - \sqrt{2i}\right)$ is
(a) $2\sqrt{55}$ (b) $\sqrt{55}$
(c) $10\sqrt{11}$ (d) $\sqrt{11}$

6. The principal argument of $\frac{(1+i)^2}{1-i}$ is
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$
(c) $\frac{3\pi}{4}$ (d) π

7. If z, and z_2 are two complex number then $|Z_1 + |Z_2 + |Z_1 - |Z_2$ is
(a) $2[|Z_1|^2 - |Z_2|^2]$ (b) $2[|Z_1|^2 + |Z_2|^2]$
(c) $[|Z_1|^2 - |Z_2|^2]$ (d) $|Z_1|^2 + |Z_2|^2$

8. Which of the following is not true?
(a) $|Z_1 + |Z_2| \leq |Z_1| + |Z_2|$
(b) $|Z_1 - |Z_2| \leq |Z_1| - |Z_2|$
(c) $\left|\frac{Z_1}{Z_2}\right| = \frac{|Z_1|}{|Z_2|}$
(d) $|Z_1.|Z_2| =) |Z_1.|Z_2|$

9. Polynomial of degree 'n' has a hole of order 'n' at (for $n > 0$)
(a) zero (b) anywhere
(c) curer C:$|Z| = 1$ (d) ∞

10. Let 'a' be an isolated singularity of $f(Z)$and if $if(Z)$ is bounded on some neighbourhood of 'a' then 'a' is
(a) Removable singularity
(b) Essential singularity
(c) isolated singularity
(d) non isolated singularity

11. The function $f(Z) = |Z|^2$ is _____ (Z: complex number)
    (a) Continuous and differentiable at all points
    (b) Continuous at all points but not differentiable
    (c) Continuous at all point but not differentiable except at organ.
    (d) Discontinuous

12. Choose the wrong statement
    (a) A differentiable function is always continuous.
    (b) A continuous function is always differentiable.
    (c) Analytic function is always differentiable.
    (d) Analytic function is always continuous.

13. Choose the wrong statement for a function $f(z) = u + iv$
    (a) Cauchy- Riemann condition are necessary but not sufficient for analytic function
    (b) Cauchy- Riemann condition are sufficient if the partial derivatives are discontinuous
    (c) if a function is analytic in a domain then u, v satisfy C-R on at all points in
    (d) all are correct

14. The polar form of C-R equations
    (a) $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$, $\frac{\partial u}{\partial \theta} = r\frac{\partial v}{\partial r}$
    (b) $\frac{\partial u}{\partial r} = \frac{-1}{r}\frac{\partial v}{\partial \theta}$, $\frac{\partial u}{\partial \theta} = r\frac{\partial v}{\partial r}$
    (c) $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$, $\frac{\partial u}{\partial \theta} = -r\frac{\partial v}{\partial r}$
    (d) $\frac{\partial u}{\partial \theta} = \frac{1}{r}\frac{\partial v}{\partial \theta}$, $\frac{\partial u}{\partial r} = -r\frac{\partial v}{\partial r}$

15. Which of the following is analytic function
    (a) $(Z+2)^2$
    (b) $Z\bar{Z}$
    (c) $Z\bar{Z} + \bar{z}^2$
    (d) $Z\bar{Z} - \bar{Z}^2$

16. The Cauchy Riemann equation for $f(z) = u(x_1 y) + iv(x_1 y)$ to be analytic are,
    (a) $\frac{\partial u}{\partial x} = \frac{-\partial v}{\partial y}$, $\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$
    (b) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$, $\frac{\partial u}{\partial y} = \frac{-\partial v}{\partial x}$
    (c) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$, $\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$
    (d) $\frac{\partial u}{\partial x} = \frac{-\partial v}{\partial y}$, $\frac{\partial u}{\partial y} = \frac{-\partial v}{\partial x}$

17. Which of the following is analytic function?
    (a) $(Z+2)^2$
    (b) $Z\bar{Z}$
    (c) $Z\bar{Z} + \bar{Z}^2$
    (d) $Z\bar{Z} - \bar{Z}^2$

18. The Cauchy Riemann equation for $f(z) = u(x_1 y) + iv(x_1 y)$ to be analytic are:-
    (a) $\frac{\partial u}{\partial x} = -\frac{\partial u}{\partial y}$, $\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$
    (b) $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y}$, $\frac{\partial u}{\partial y} = \frac{-\partial v}{\partial x}$
    (c) $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y}$, $\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$
    (d) $\frac{\partial u}{\partial x} = \frac{-\partial u}{\partial y}$, $\frac{\partial u}{\partial y} = \frac{-\partial v}{\partial x}$

19. The carvers C_1 and C_2 will be orthogonal if
    (a) The product of their slopes is one
    (b) The product of their slopes is -1
    (c) The product of their slopes is 0
    (d) None of the above

20. If u and v are harmonic function, then
    (a) $f(z) = u = iv$ need not to be analytic
    (b) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x^2} = 0$ & $\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial y^2} = 0$
    (c) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ & $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}$
    (d) u and v satisfies , $\frac{\partial^2 u}{\partial x^2} = \frac{-\partial^2 v}{\partial x \partial y}$

21. Which of the following are not harmonic function
    (a) $\frac{1}{2}\log(x^2 + y^2)$
    (b) $x^2 + y^2$
    (c) $x^2 + y^2 + 2y$
    (d) $2x^2 y^2$

22. For what values of 'k' the function $f(x, y) = x^2 + xy + y^2$ is harmonic?
    (a) 1
    (b) 0
    (c) -1
    (d) -2

23. The harmonic conjugate of $u = 3x - 2xy$ is
    (a) $x^2 + 3y + y^2$
    (b) $x^2 - 3y + y^2$
    (c) $x^2 - 3y - y^2$
    (d) $x^2 + 3y - y^2$

24. If $u = x^3 - 3xy^2 + 3x^2 - 3y^2$ is real part of an analytic funcition then its imaginary part is-
    (a) $3x^2 y + 6xy - y^3$
    (b) $3x^2 y - 6xy - y^3$
    (c) $3x^2 y - 6xy + y^3$
    (d) $3x^2 y + 6xy + y^3$

25. If $u = x^2 - y^2$, then corresponding analytic function is
    (a) Z
    (b) Z^2
    (c) Z^3
    (d) $Z + 2Z$

26. If imaginary part of an analytic function $f(z) = u + uv$ is $v = l^x(x \sin y + y \cos y)$, then function $f(z)$ is
    (a) Z^2
    (b) l^Z
    (c) Zl^Z
    (d) $Z^2 l^Z$

27. If $|f(z)|$ is constant $|f(z) = u + iv|$, then $f(z)$ is
    (a) variable
    (b) constant
    (c) partially variable
    (d) none of these

28. If $f(z)$is analytic function within and on a closed curve C and if 'a' is any point within C then
    (a) $f(a) = \left(\frac{f|z|}{z-a}\right)dz$
    (b) $f(a) = \left(\frac{f|z|}{z+a}\right)dz$
    (c) $f(a) = \frac{1}{2\pi i}\left(\frac{f|z|}{z-a}\right)dz$
    (d) $f(a) = \frac{-1}{2\pi i}\left(\frac{f|z|}{z-a}\right)dz$