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FUNDAMENTAL CONCEPTS IN COMPLEX ANALYSIS

CHAPTER 1

1.1 INTRODUCTION TO COMPLEX NUMBER SYSTEM

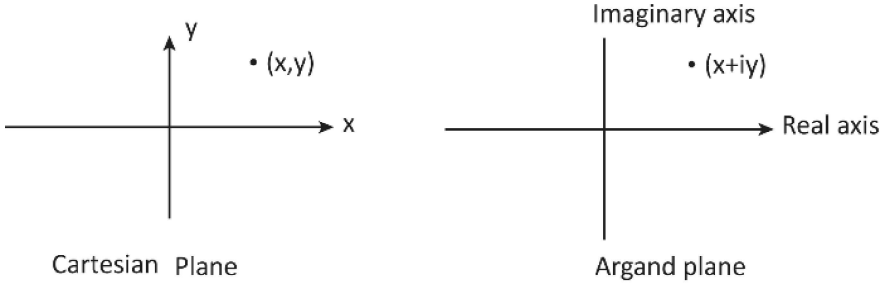
In 1777, Euler observed that equation $x^2 + 1 = 0$ has no solution in Real field .to allow solution of same equation real number system extended to the set of complex numbers.
Euler introduced symbol i with the property $i^2 = -1$ (Also called as imaginary unit).

1.1.1 Definition:

Set of all complex numbers of the form $x + iy; x, y$ real numbers are called as complex numbers.
$\mathbb{C} = \{z = x + iy | x, y \in \mathbb{R}\}$
$(\mathbb{C}, +, \cdot)$ is form's an unordered field (as complex number are not comparable)
Where '+' & $(\cdot)$ are defined as
$(a + ib) + (c + id) = (a + c) + i(b + d)$
$(a + ib) \cdot (c + id) = (ac - bd) + i(ad + bd)$

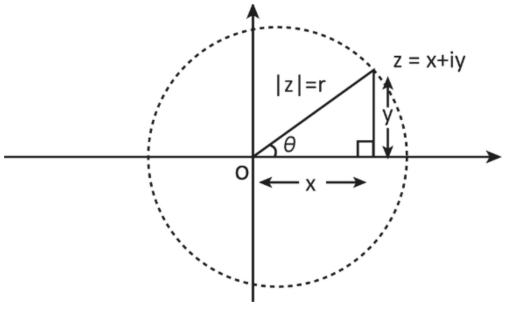
1.2 GEOMETRY OF COMPLEX NUMBERS:

1. Every element in $\mathbb{R}^2$ can be viewed as unique element of $\mathbb{C}$.



i.e.
$if\ D = \{z = x + iy | x + iy \in \mathbb{C}\} \subseteq \mathbb{C}\ then\ there\ corresponds\ a\ subset$
$$S = \{(x, y) | x + iy \in D\} \subseteq \mathbb{R}^2$$
$\therefore S$ is the region in$\mathbb{R}^2$ analogues to D in $\mathbb{C}$.

2. Let z=x+iy be located in argand plane as shown in diagram. then



$|z|$ = distance of complex number from origin = $\sqrt{x^2 + y^2}$=r(say)
Also by property of triangle we've $y = r \sin\theta$
$x = r \cos\theta$

$$\frac{\sin\theta = \frac{y}{r}}{\cos\theta = \frac{x}{r}} \Rightarrow \tan\theta = \frac{y}{x}$$

$\theta = \tan^{-1}\left(\frac{y}{z}\right)$ (called as argument)

3. $z = x + iy \quad ; \quad z = r\cos\theta + i\, r\sin\theta$
   $z = r[\cos\theta + i\sin\theta]$
   $z = r\, e^{i\theta}$

Remark:

(1) Polar representation of complex number is not unique .as a complex number can have different polar representation. $[Since\ \ (r,\theta)\ has\ infinite\ representation\ in\ the\ form\ (r,\theta + 2n\pi)]$

(2) '0' can't be represent in polar form (as $tan^{-1}\left(\frac{0}{0}\right)\ is\ not\ defined$)

1.3 MAGNITUDE & ARGUMENT OF COMPLEX NUMBERS

Every non-zero complex number $z = x + iy$ can be written in the form $z = r[\cos\theta + i\sin\theta]$

where r = $|z| = \sqrt{x^2 + y^2}$ and θ are called magnitude & argument respectively

1.3.1 Properties on Magnitude:

Let z, z_1 & z_2 be complex numbers.$\overline{z}\ is\ conjugate\ of\ z$ Then we've

i. $|z| \geq 0$ (& it is real valued) & |z|=0 iff $z = 0$

ii. $-|z| \leq Re(z), Im(z) \leq |z|$

iii. $|z| = |\overline{z}|$

iv. $z.\overline{z} = |z|^2$

v. $|z_1 \cdot z_2| = |z_1| \cdot |z_2|$ & $|z^n| = |z|^n$

vi. $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|} \qquad z_2 \neq 0$

vii. $|z_1 + z_2|^2 = |z_1|^2 + 2Re(z_1 \cdot \overline{z_2}) + |z_2|^2$..... (a)

viii. $|z_1 - z_2|^2 = |z_1|^2 - 2Re(z_1 \cdot \overline{z_2}) + |z_2|^2$..... (b)

ix. $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2|z_1|^2 + 2|z_2|^2$ (parallelogram law)

x. $||z_1| - |z_2|| \leq |z_1 + z_2| \leq |z_1| + |z_2|$

1.3.2 Principal Argument of Complex Number

Let $z = x + iy \in \mathbb{C} - \{0\}$

Let $\propto = \tan^{-1}\left|\frac{y}{x}\right|$

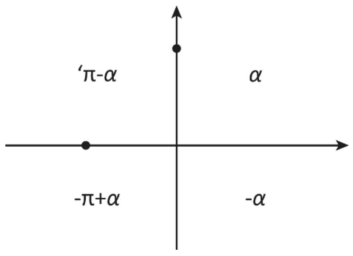
$0 \leq \propto < \pi/2$

Then θ is called principal argument of z and written as

So we can write

$$\text{Arg(z)} = \begin{cases} 0 & x > 0 \quad y = 0 \\ \propto & x > 0 \quad y > 0 \\ \pi/2 & x = 0 \quad y > 0 \\ \pi - \propto & x < 0 \quad y > 0 \\ \pi & x < 0 \quad y = 0 \\ -\pi + \propto & x < 0 \quad y < 0 \\ -\pi/2 & x = 0 \quad y < 0 \\ -\propto & x > 0 \quad y < 0 \end{cases}$$

Blueprint ✓



1.3.3 General Argument:

Let $z = x + iy \in \mathbb{C} - \{0\}$ then General argument of z is defined as $arg\ z = \{Arg\ z + 2n\pi | n \in \mathbb{Z}\}$

Remark:

(1) $-\pi < Arg\ z \leq \pi$

(2) Some author prefer interval $[0, 2\pi)$ for range of Arg (z)

(3) Principal argument is single valued function

(4) General argument is infinite valued

(5) $\arg(z_1\ z_2) = \arg(z) + \arg(z_2)$

(6) $\arg(\overline{Z}) = -\arg(Z)$

(7) $\text{Arg}(z_1\ z_2) \neq \text{Arg}(z_1) + \text{Arg}(z_2)$

$counter\ example$:

Take $z_1 = -1, z_2 = 5i$

$\text{Arg}(z_1) = \pi,\ \text{Arg}(z_2) = \frac{\pi}{2},$

$\text{Arg}(z_1\ z_2) = \text{Arg}(-5i) = -\frac{\pi}{2}$

$\text{Arg}(z_1) + \text{Arg}(z_2) = \frac{3\pi}{2}$

$\Rightarrow \text{Arg}(z_1) + \text{Arg}(z_2) \neq \text{Arg}(z_1.z_2)$

Ex 01: Find principal argument of $z = 1 + i$

Solution:

$z = 1 + i$ (lies in first quadrant)

here $x = 1 \qquad y = 1$

$\propto = \tan^{-1}\left|\frac{y}{x}\right| = \tan^{-1}\left|\frac{1}{1}\right| = \tan^{-1}(1) = \frac{\pi}{4}$

$\propto = \frac{\pi}{4}$

$\therefore$ Arg $(1 + i) = \propto = \frac{\pi^c}{4}$

Ex 02 : Find principal argument & General argument of

$z = -1 + 4i$ (given Arctan (4) =76^0)

Solution:

$z = -1 + 4i$ (lies in second quadrant)

$\propto = \tan^{-1}\left|\frac{4}{-1}\right|$

$\propto = \tan^{-1}(4)$

$\propto = 76°$

$\therefore$ Arg $z = \pi - \propto = 180 - 76° = 104°$

Arg $z = 104 \times \frac{\pi}{180}$ (in radian)

Arg z = $\frac{26\pi^c}{45}$

Ex 03 : If $|z_1| = |z_2|$ & $\arg(z_1) \sim \arg(z_2) = 0$ then, which of the following correct ?

(a) $z_1 = \frac{1}{z_2}$

(b) $z_1 = z_2$

(c) $z_1 z_2 = -1$

(d) $z_1 + z_2 = 0$

Solution:

Suppose, $z_1 = r_1 e^{i\theta_1}$; $|z_1| = r_1$ & $\arg z_1 = \theta_1$

$z_2 = r_2 e^{i\theta_2}$; $|z_2| = r_2$ & $\arg z_2 = \theta_2$
Given, $|z_1| = |z_2|$
suppose $|z_1| = |z_2| = r$
& $\arg z_1 \sim \arg z_2 = 0$
$\theta_1 \sim \theta_2 = 0$
$\Rightarrow \theta_1 = \theta_2 = \theta$ (say)
$\therefore z_1 = r_1 e^{i\theta_1} = re^{i\theta}$
$z_2 = r_2 e^{i\theta_2} = re^{i\theta}$
$\therefore z_1 = z_2$
Correct option (b)

Ex 04: If $\arg\left(\frac{z+1}{z-i}\right) = \pi/4$ then z lies on
(a) circle
(b) straight line
(c) ellipse
(d) parabola
Solution:
$\arg\left(\frac{z+1}{z-i}\right) = \pi/4$
$\Rightarrow \arg\left(\frac{(x+1)+iy}{x+i(y-1)}\right) = \pi/4$
$\Rightarrow \arg\left[\frac{((x+1)+iy)\,(x-i(y-1))}{x^2+(y-1)^2}\right] = \pi/4$
$\Rightarrow \arg\left[\frac{(x+1)x+y(y-1)+i(xy-(x+1)(y-1))}{x^2+(y-1)2}\right] = \pi/4$
$\Rightarrow \tan^{-1}\left(\frac{\frac{xy-(x+1)\,(y-1)}{x^2+(y-1)^2}}{\frac{(x+1)x+y(y-1)}{x^2+(y-1)^2}}\right) = \pi/4$
$\Rightarrow \frac{xy-xy+x-y+1}{x^2+x+y^2-y} = 1$
$\Rightarrow x^2 + x + y^2 - y = x - y + 1$
$\Rightarrow x^2 + y^2 = 1$

1.3.4 Principal Value of Logarithm

Let $z = x + iy \in \mathbb{C} - \{0\}$ then Principal value of Logarithm is defined as
Log $z = ln|z| + i\,Arg(z)$
For example
$Log(1+i) = In(1+i) + iArg(1+i)$
$Log\,(1+i) = In\sqrt{2} + \frac{\pi}{4}i$
And General logarithmic value $\log z = In|z| + i(\arg(z))$

1.3.5 Exponent

If $z_1, z_2 \in \mathbb{C}$ $z_1 \neq 0$
then $z_1^{z_2}$ may have infinite values.
$\therefore$ The principal value: $z_1^{z_2} = e^{z_2 Log z_1}$
The general value: $z_1^{z_2} = e^{z_2 Log z_1}$

Ex 01 : Find real part of principal value of $(1+i)^{(1-i)}$
Solution:
We know that $z_1^{z_2} = e^{z_2 Log z_1}$
$z_1 = 1 + i$, $z_2 = 1 - i$

$(1+i)^{1-i} = e^{(1-i)Log(1+i)}$

We know $Log\,(1+i) = In\sqrt{2} + \frac{\pi}{4}i$

$(1+i)^{1-i} = e^{(1-i)(In\sqrt{2}+\frac{\pi}{4}i)}$

$(1+i)^{1-i} = e^{In\sqrt{2}+i\frac{\pi}{4}-i\,In\sqrt{2}+\pi/4}$

$= e^{\left(In\sqrt{2}+\frac{\pi}{4}\right)+i\left(\frac{\pi}{4}-In\sqrt{2}\right)}$

$= e^{(In\sqrt{2}+\pi/4)}\ e^{i\left(\frac{\pi}{4}-In\sqrt{2}\right)}$

$= e^{In\sqrt{2}+\pi/4}\left[cos\left[\frac{\pi}{4}-In\sqrt{2}\right]+i\,sin[\frac{\pi}{4}-In\sqrt{2}]\right]$

$= e^{In\sqrt{2}+\frac{\pi}{4}}\ cos\left(\frac{\pi}{4}-In\sqrt{2}\right)+i\,e^{In\sqrt{2}+\frac{\pi}{4}}sin\left(\frac{\pi}{4}-In\sqrt{2}\right)$

$\therefore Re\left((1+i)^{(1-i)}\right)$

$=e^{In\sqrt{2}+\frac{\pi}{4}}cos\left(\frac{\pi}{4}-In\sqrt{2}\right)$

$=\sqrt{2}e^{\frac{\pi}{4}}cos\left(\frac{\pi}{4}-In\sqrt{2}\right)$

Ex 02: Find principal value of i^i

Solution:

METHOD -I

$i^i = e^{i\,\mathrm{Log}\,i}$

$= e^{i(In|i|+iArgi)}$

$=e^{i(0+i\frac{\pi}{2})}$ (∵ln 1=0)

$=e^{-\pi/2}$

Solution:

METHOD -II (shortcut)

Note that $e^{i\,\pi/2}$ =i

$i^i = (e^{i\,\pi/2})^i=(e^{\,\pi/2})^{i^2}=e^{-\pi/2}$

Multiple Choice Questions (MCQ)

1. If z_1 and z_2 are distinct complex numbers such that $|z_1| = |z_2| = 1$ and $z_1 + z_2 = 1$, then the triangle in the complex plane with z_1, z_2 and -1 as vertices

(a) Must be equilateral

(b) Must be right-angled

(c) Must be isosceles, but not necessarily equilateral

(d) Must be obtuse angled

Explanation: (a)

Since $z_1 + z_2 = 1$,

$implies\ Re(z_1) + Re(z_2) = 1$

$and\ Im(z_1) + Im(z_2) = 0, i.e., Im(z_1) = -Im(z_2)$

Let $z_1 = x_1 + iy$ and $z_2 = x_2 - iy$

Since $|z_1| = |z_2|$ implies $\sqrt{x_1^2 + y^2} = \sqrt{x_2^2 + y^2}$

Which gives $|x_1| = |x_2|$ and $x_1 + x_2 = 1$ implies $x_1 = x_2 = \frac{1}{2}$ or $x_1 = x_2 = -\frac{1}{2}$

If $x_1 = \frac{1}{2}$ then $y = \frac{\sqrt{3}}{2}$ as $|z_1| = 1$

Take $z_1 = \frac{1+\sqrt{3}i}{2}$ implies $z_2 = \frac{1-\sqrt{3}i}{2}$

$|z_1| = \sqrt{\frac{1}{4}+\frac{3}{4}} = 1$ and $|z_2| = \sqrt{\frac{1}{4}+\frac{3}{4}} = 1$

$\therefore |z_1| = |z_2| = 1$

Also $z_1 + z_2 = 1$

Now,

If we consider a triangle ABC with vertices z_1, z_2 and -1 then $AB = AC = BC = \sqrt{3}$

$\Rightarrow$ triangle ABC is an equilateral triangle.

Multiple Select Questions (MSQ)

1. Fix positive real number c, consider the locus of all points $z \in \mathbb{C}$ such that $\left|\frac{z-i}{z+i}\right| = c$, Which of the following are true?

(a) $if\ \ c > 1$, the locus is a circle centered on imaginary axis

(b) $if\ \ c < 1$, the locus is a circle cantered on the real axis

(c) $if\ \ c = 1$, the locus is a straight line parallel to the imaginary axis

(d) $if\ c = 1$, the locus is a straight line not passing through the origin

Explanation: (a)

If $c = 1$; $\left|\frac{z-i}{z+i}\right| = 1$

$\Rightarrow |z-i|^2 = |z+i|^2$

$\Rightarrow (z-i)(\bar{z}+i) = (z+i)(\bar{z}-i)$

$\Rightarrow\ z\bar{z} + iz - i\bar{z} + 1 = z\bar{z} - iz + i\bar{z} + 1$

$\Rightarrow\quad i(z-\bar{z}) = -i(z-\bar{z})$

$\Rightarrow\ \ 2i(z-\bar{z}) = 0$

$\Rightarrow\ \ (z-\bar{z})$=0

$z = \bar{z} \Rightarrow\quad y = 0\ i, e, x - axis$

If $c = 1$, the locus is straight line passing through origin and is not parallel to imaginary axis

If $c\ > 1$, $\left|\frac{z-i}{z+i}\right| =\ c > 1$

$|z-i| = c|z+i|$

$(x^2 + (y-i)^2) = c(x^2 + (y+i)^2)$

$(c-1)x^2 + (c-1)y^2 + 2i(c+1)y + (c-1) = 0$

$x^2 + y^2 + \frac{2i(c+1)}{c-1}y + 1 = 0$

Centre is $\left(0, \frac{2i(c+1)}{c-1}\right)$

$\Rightarrow$ Centre is always on imaginary axis when $c \neq 1$

2. Consider the set: $S \coloneqq \{\exp(2\pi i\theta) : \theta$ is a rational number$\}$

For each $z \in S$ the set $\{z^{n!} : n$ is a positive integer$\}$ is

(a) Countable

(b) Countable infinite

(c) Uncountable

(d) Finite

Explanation: (a), (d)

$S = \left\{\exp(2\pi i\theta) : \theta = \frac{\mathrm{p}}{\mathrm{q}}\right\}$

Let $z_1 = e^{2\pi i\frac{p}{q}}$

$z_1^{n!} = e^{2\pi i\frac{p}{q}n!}$

$= \cos 2\pi\frac{p}{q}n! + i\sin 2\pi\frac{p}{q}n!$

When $n = q$

$\cos 2\pi\frac{p}{q}.q! + i\sin 2\pi\frac{p}{q}.q!\ \frac{p}{q}.q!$ is integer,

$z_1^{n!} = e^{2\pi i}\frac{p}{q}n! = 1$

Also, $\forall n \geq q\ \ z_1^{n!} = 1$

So, there are only distinct value of $z_1^{n!}\ \forall n < q$

$\Rightarrow$There are finitely many elements in the set.

3. Let $\mathrm{Re}(z), \mathrm{Im}(z)$ denote the real and imaginary parts of $z \in \mathbb{C}$, respectively. Consider the domain $\Omega = \{z \in \mathbb{C}: \mathrm{Re}(z) > |\mathrm{Im}(z)|\}$ and let $f_n(z) = \log z^n$, Where $n \in \{1,2,3,4\}$ and where $\log: \mathbb{C}\backslash(-\infty, 0] \to \mathbb{C}$.
Defines the principal branch of logarithm. Then which of the following are true?
(a) $f_1(\Omega) = \{z \in \mathbb{C}: 0 \leq |\mathrm{Im}(z)| < \pi/4\}$
(b) $f_2(\Omega) = \{z \in \mathbb{C}: 0 \leq |\mathrm{Im}(z)| < \pi/2\}$
(c) $f_3(\Omega) = \{z \in \mathbb{C}: 0 \leq |\mathrm{Im}(z)| < 3\pi/4\}$
(d) $f_4(\Omega) = \{z \in \mathbb{C}: 0 \leq |\mathrm{Im}(z)| < \pi\}$
Explanation: (a), (b), (c), (d)
Given domain $\Omega = \{z \in \mathbb{C}: \mathrm{Re}(z) > |\mathrm{Im}(z)|\}$
Since, $Re(z) > |Im(z)| \geq 0$
$\Rightarrow Re(z) > 0 \Rightarrow Re(z) = |Re(z)|$
And $\Omega = \{z \in \mathbb{C}: |\mathrm{Re}(z)| > |\mathrm{Im}(z)|\}$
Where $z = x + iy, so, Re(z) = x, Im(z) = y$
$\Rightarrow \Omega = \left\{z \in \mathbb{C}: 1 > \frac{|y|}{|x|} > 0\right\}$
$\Rightarrow \Omega = \left\{z \in \mathbb{C}: \tan^{-1} 1 > \tan^{-1}\frac{|y|}{|x|} > 0\right\}$
$\Rightarrow \Omega = \left\{z \in \mathbb{C}: 0 < Arg(z) < \frac{\pi}{4}\right\}$.....(1)
Now, we know that, $logz = log|z| + i\arg(z)$
So, $f_n(z) = \log z^n = \log|z^n| + i\arg(z^n) = nlog|z| + in\arg z$
From (1),
$f_n(\Omega) = \left\{z \in \mathbb{C}: 0 < nArg(\boldsymbol{z}) < \boldsymbol{\frac{n\pi}{4}}\right\}$
$\Rightarrow \boldsymbol{f_n(\Omega) = \left\{z \in \mathbb{C}: 0 < |Im(z)| < \frac{n\pi}{4}\right\}}$
So,
$\boldsymbol{f_1(\Omega) = \left\{z \in \mathbb{C}: 0 < |Im(z)| < \frac{\pi}{4}\right\}}$
$\boldsymbol{f_2(\Omega) = \left\{z \in \mathbb{C}: 0 < |Im(z)| < \frac{\pi}{2}\right\}}$
$\boldsymbol{f_3(\Omega) = \left\{z \in \mathbb{C}: 0 < |Im(z)| < \frac{3\pi}{4}\right\}}$
$\boldsymbol{f_4(\Omega) = \{z \in \mathbb{C}: 0 < |Im(z)| < \pi\}}$
So, option (a), (b), (c), (d) are correct.

ASSIGNMENT:

1. Principle value of $\log\ (1+i)^2$ is _____
(1) ln 3 + i $\frac{\pi}{2}$ (2) ln 2 + i $\frac{\pi}{2}$
(3) ln 2 + i $\frac{\pi}{4}$ (4) Θ=0

2. If 1, ω, ω^2, ... ω^{n-1} are n^{th} roots of unity, then (11- ω)(11- ω^2).....(11- ω^{n-1})=
(1) ω (2) ω -1
(3) 0 (4) (11^n-1)/10

3. Find the imaginary part of square roots of $\sqrt{3} + 3i$.
(1) $\left(\frac{\sqrt{3}}{2}\right)^{1/2}$ (2) $\left(\frac{\sqrt{3}}{3}\right)^{1/2}$
(3) -1 (4) 0

4. If A, B are roots of the equations $x^2 - 2x + 4 = 0$,
$A^n + B^n$ is equal to
(1) $2^{n+1}\cos\left(\frac{n\pi}{3}\right)$ (2) $2^n\cos\left(\frac{n\pi}{3}\right)$
(3) $2^{n+1}\sin\left(\frac{n\pi}{3}\right)$ (4) $2^n\sin\left(\frac{n\pi}{3}\right)$

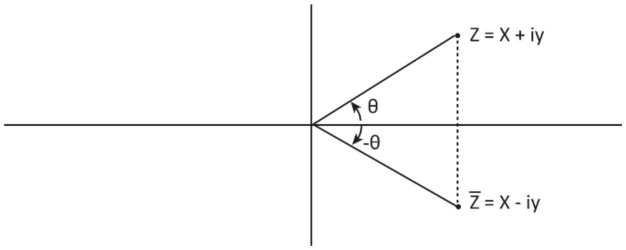
5. If m & M are minimum and maximum value of $|z_1 - Z_2|$ where $z, w \in \mathbb{C}$ such that $|z_1| = 11$ and $|Z_2 + 4 + 3i| = 5$ is respectively then m + M are
(1) 22 (2) 24
(3) 20 (4) 14

Answer key

1	2	3	4	5
2	4	1	1	1

1.4 CONJUGATE OF A COMPLEX NUMBER

1.4.1 Definition: - For a complex number
$z = x + iy, x, y \in \mathbb{R}$. Then its conjugate is defined as $\overline{z} = x - iy$



$z = x + iy$ ……(1)
$\overline{z} = x - iy$ ……(2) after addition
$z + \overline{z} = 2x$
$\Rightarrow x = Re(z) = \frac{z+\overline{z}}{2}$ (called, real part of z)
$similarly\ , y = Im(z) = \frac{z-\overline{z}}{2i}$ (called, imaginary part of z)
geometrically conjugate $\overline{z} = x - iy\ is\ the\ reflection\ of\ \ z = x + iy\ when\ real\ axis\ is\ considered\ as\ mirror$.

1.4.2 Properties of Conjugate of Complex Number
If z, z_1 & z_2 are complex number's then.
We've following result.
i) $\overline{(\overline{z})} = z$
ii) $\overline{z_1 \pm z_2} = \overline{z_1} \pm \overline{z_2}$
iii) $\overline{z_1 \cdot z_2} = \overline{z_1} \cdot \overline{z_2}$
iv) $\left(\frac{z_1}{z_2}\right) = \frac{\overline{z_1}}{\overline{z_2}} \qquad (z_2 \neq 0)$
v) $(\overline{z})^n = \overline{z^n}$
vi) $z_1\overline{z_2} + \overline{z_1} \cdot z_2 = 2Re(z_1\overline{z_2}) = 2Re(\overline{z_1}z_2)$

Problem : If Which of the following are correct ?
(a) $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$
(b) $\overline{(\overline{z_1})} = z_1$
(c) $\overline{z_1}\ \overline{z_2} = \overline{z_1 z_2}$
(d) $\overline{z_1}/\ \overline{z_2} = \overline{(z_1/z_2)}$